THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 12 (Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P264-265: 2, 4, 9;

2 Use residues to derive the integration formulas

$$\int_0^\infty \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$$

.

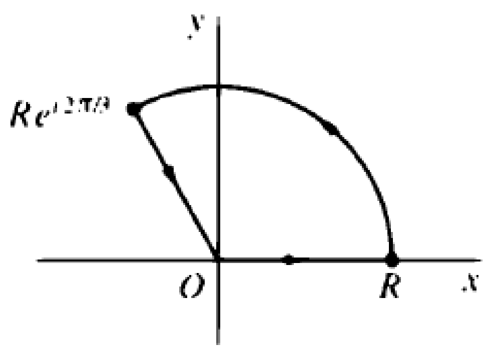
4 Use residues to derive the integration formulas

$$\int_0^\infty \frac{x^2 dx}{x^6+1} = \frac{\pi}{6}$$

.

9 Use a residue and the contour shown in the follwing, where $R > 1$, to establish the integration formula

$$\int_0^\infty \frac{dx}{x^3+1} = \frac{2\pi}{3\sqrt{3}}.$$



P273: 3, 5, 8, 12;

3 Use residues to derive the integration formulas

$$\int_0^\infty \frac{\cos ax dx}{(x^2+b^2)^2} = \frac{\pi}{4b^3}(1+ab)\,e^{-ab} \quad (a > b > 0).$$

5 Use residues to derive the integration formulas

$$\int_{-\infty}^\infty \frac{x^3 \sin ax}{x^4+4} dx = \pi e^{-a}\cos a \quad (a > 0).$$

8 Use residues to find the Cauchy principal values of the improper integrals

$$\int_{-\infty}^{\infty} \frac{\sin x dx}{x^2 + 4x + 5}.$$

12 Follow the steps below to evaluate the Fresnel integrals, which are important in diffraction theory:

$$\int_0^{\infty} \cos x^2 dx = \int_0^{\infty} \sin x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

.

P282-283: 1-4;

1 Use the function $f(z) = (e^{iaz} - e^{ibz})/z^2$ and the indented contour in Fig. 108 (Sec. 89) to derive the integration formula

$$\int_0^{\infty} \frac{\cos ax - \cos bx}{x^2} dx = \frac{\pi}{2}(b - a) \quad (a, b \geq 0).$$

Then, with the aid of the trigonometric identity $1 - \cos(2x) = 2\sin^2 x$, point out how it follows that

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \pi/2.$$

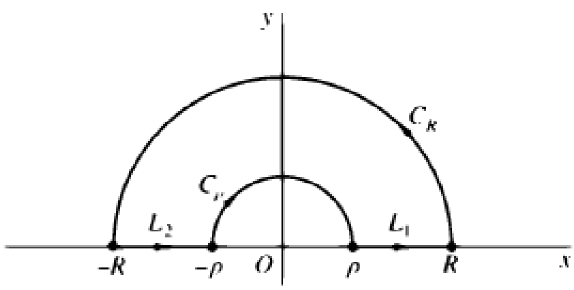


FIGURE 108

2 Derive the integration formula

$$\int_0^{\infty} \frac{dx}{\sqrt{x}(x^2 + 1)} = \frac{\pi}{\sqrt{2}}$$

by integrating the function

$$f(z) = \frac{z^{-1/2}}{z^2 + 1} = \frac{e^{(-1/2)\log z}}{z^2 + 1} \quad (|z| > 0, \pi/2 < \arg z < 3\pi/2)$$

over the indented contour appearing in Fig. 109.

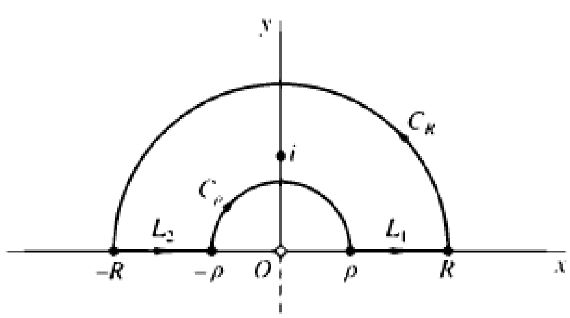


FIGURE 109

3 Derive the integration formula obtained in Exercise 2 by integrating the branch

$$f(z) = \frac{z^{-1/2}}{z^2+1} = \frac{e^{(-1/2)\log z}}{z^2+1} \quad (|z| > 0, 0 < \arg z < 2\pi)$$

of the multiple-valued function $\dfrac{z^{-1/2}}{z^2+1}$ over the closed contour in Fig. 110

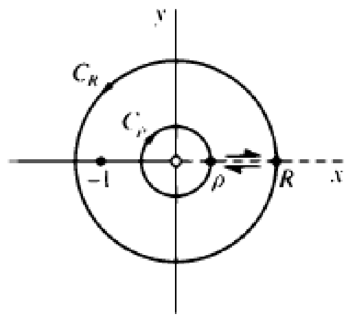


FIGURE 110

4 Derive the integration formula

$$\int_0^\infty \frac{x^{1/3}}{(x+a)(x+b)}dx = \frac{2\pi}{\sqrt{3}}\frac{a^{1/3}-b^{1/3}}{a-b} \quad (a > b > 0)$$

using the function

$$f(z) = \frac{z^{1/3}}{(z+a)(z+b)} = \frac{e^{(1/3)\log z}}{(z+a)(z+b)} \quad (|z| > 0, 0 < \arg z < 2\pi)$$

and a closed contour similar to the one in Fig. 110 (Sec. 91). but where $\rho < b < a < R$.

P287: 1-4;

Use residues to establish the following integration formulas:

1 $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = \frac{2\pi}{3}.$

2 $\displaystyle\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta} = \sqrt{2}\pi.$

3 $\displaystyle\int_0^{2\pi} \frac{\cos^2 3\theta d\theta}{5-4\cos 2\theta} = \frac{3\pi}{8}.$

4 $\displaystyle\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}} \quad (-1 < a < 1).$

P293-294: 6-8.

6 Determine the number of zeros, counting multiplicities, of the polynomial
(a) $z^6 - 5z^4 + z^3 - 2z$; (b)$2z^4 - 2z^3 + 2z^2 - 2z + 9$; (c) $z^7 - 4z^3 + z - 1$, inside the circle $|z| = 1$.

7 Determine the number of zeros, counting multiplicities, of the polynomial (a) $z^4 - 2z^3 + 9z^2 + z - 1$; (b) $z^5 + 3z^3 + z^2 + 1$, inside the circle $|z| = 2$.

8 Determine the number of zeros, counting multiplicities, of the equation

$$2z^5 - 6z^2 + z + 1 = 0$$

in the annulus $1 \leq |z| < 2$.