Review for Final Examination

I. Differentiation Theory (Chapter 6 of Text)

- Definition of the derivative of a function. Examples and basic properties. In particular, should know how to determine differentiability for functions like $|f(x)|$.
- Mean-Value Theorem, its consequences and Taylor's Expansion Theorem. Should know the precise statements well (but not the proofs).
- L'Hospital Rules and Convexity can be skipped.

II. Integration Theory (Notes 2)

- Should be familiar with the definitions of Riemann integrals, Riemann sums, Darboux upper and lower sums, and Riemann upper and lower integrals.
- The two integrability criteria.
- First and Second Fundamental Theorems of Calculus. Statements and proofs.
- Integration by parts and change of variables. Statements and proofs.
- Improper integrals.

III. Uniform Convergence (Notes 3) This is the most important chapter of the course.

- Several methods to determine uniform convergence of sequences of functions. study again those examples in Notes and exercises.
- Theorems 3.5, 3.6, 3.8 and the remark following Theorem 3.8 for sequences of functions. Important.
- The corresponding Theorems 3.6' and 3.8' for series of functions. Important.
- Weierstrass M-Test. This is the common tool in proving uniform convergence for series of functions. Important.
- Elementary functions (3.4). It suffices to understand how they are defined. Proofs not so relevant.

IV. Infinite Series of Numbers and functions (Chapter 9 of Text)

- All convergence tests for absolute convergence: Ratio, Root, Raabe's, and Integral Tests.
- Dirichlet's and Abel's Tests for conditional convergence. No proof.
- Power series. Radius of convergence. The statement of Cauchy-Hadamard Theorem.

Some Basic Limits.

I.

$$\lim_{n\to\infty} n^{1/n} = 1 \ ,$$

$$\lim_{n\to\infty} a^{1/n} = 1, \quad a > 0 \ .$$

II.

$$\lim_{x\to\infty} \frac{x^a}{e^x} = 0 \ , \quad a > 0 \ .$$

$$\lim_{x\to\infty} \frac{\log x}{x^a} = 0 \ , \quad a > 0 \ .$$

III.

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ .$$

We also have

$$\frac{\sin x}{x} < 1, \quad x \neq 0 \ .$$

IV.

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \ .$$

In fact, the sequence $\{(1 + 1/n)^n\}$ increases to e.