Proof of Product Rule (with each $\lim_{x \to x_0} f_i(x) = l_i$ & $x_0 \in D^c \cdots$).

Assuming the local-boundedness result can be used, take $M > 0$ and $\delta_0 > 0$ s.t. for each i $|l_i| \leq M$ and

$$|f_i(x)| \leq M \quad \forall x \in V_{\delta_0}(x_0) \cap (D \setminus \{x_0\})$$

Let $\varepsilon > 0$. Define $\varepsilon_i := \dfrac{\varepsilon}{2M}$ $(i = 1, 2)$. By assumption, $\exists\, \delta_i > 0$ such that

$$|f_i(x) - l_i| < \varepsilon_i \quad \forall x \in V_{\delta_i}(x_0) \cap (D \setminus \{x_0\})$$

Let $\delta = \min\{\delta_0, \delta_1, \delta_2\}$, and let $x \in V_\delta(x) \cap (D \setminus \{x_0\})$.

Then $|f_i(x)| \leq M$ and $|f_i(x) - l_i| < \varepsilon_i$ $\forall i = 1, 2$. Hence

$$|f_1(x) f_2(x) - l_1 l_2| = |f_1(x) f_2(x) - l_1 f_2(x) + l_1 f_2(x) - l_1 l_2|$$

$$\leq |f_1(x) - l_1| \cdot |f_2(x)| + |l_1|\,|f_2(x) - l_2|$$

$$< M\varepsilon_1 + M\varepsilon_2 = \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Therefore $\lim_{x \to x_0} (f_1(x) f_2(x)) = l_1 l_2$.

Note. If not allow to apply the local-boundedness result then you need either provide the proof of the result or modify the above proof as follows.

Let $M := \max\{|l_1|, |l_2|\} + 1$. Let $\varepsilon > 0$. Let

$$\varepsilon_i := \min\left\{\frac{\varepsilon}{2M}, 1\right\} \quad (>0)$$

Take $\delta_i > 0$ such that

$$|f_i(x) - l_i| < \varepsilon_i \quad \forall x \in V_{\delta_i}(x_0) \cap (D \setminus \{x_0\}).$$

Let $\delta := \min\{\delta_1, \delta_2\}$ (>0). Let $x \in V_\delta(x_0) \cap (D \setminus \{x_0\})$.

Then each i satisfies

$$|f_i(x)| = |f_i(x) - l_i| + |l_i| < \varepsilon_i + |l_i| \leq 1 + |l_i| \leq M$$

and

$$|f_1(x) f_2(x) - l_1 l_2| \leq |f_1(x) - l_1| \cdot M + M\,|f_2(x) - l_2| \quad \text{(as before)}$$

Proof of Quotient Rule (with $\lim_{x\to x_0} f_i(x) = l_i$, $i = 1, 2$ & $l_2 \neq 0$, $f_2(x) \neq 0 \ \forall x$)

Let $\varepsilon > 0$. With positive $m := \frac{|l_2|^2}{2}$ and $M := \max\{|l_1|+1, |l_2|+1\}$, let

$$\varepsilon_1 := \min\left\{\frac{m}{M}(\varepsilon/2), \frac{|l_2|}{2}, 1\right\}$$

$$\varepsilon_2 := \min\left\{\frac{m}{M}\left(\frac{\varepsilon}{2}\right), \frac{|l_2|}{2}, 1\right\}$$

Then, by assumptions, $\exists\ \delta_1, \delta_2 > 0$ such that

$$|f_i(x) - l_i| < \varepsilon_i \quad \forall\ x \in V_{\delta_i}(x_0) \cap (D \setminus \{x_0\}).$$

Take $\delta := \min\{\delta_1, \delta_2\}$ (>0). Then, $\forall x \in V_\delta(x_0) \cap (D \setminus \{x_0\})$, we have

$$|f_i(x) - l_i| < \varepsilon_i \quad \left(\leq 1, \frac{|l_2|}{2}, \frac{\varepsilon}{2}\frac{m}{M}\right)$$

Since $\pm(|f_i(x)| - |l_i|) \leq |f_i(x) - l_i|$ it follows that

$$|f_i(x)| \leq |l_i| + 1 \leq M \qquad (i = 1, 2)$$

and

$$|l_2| - |f_2(x)| < \frac{|l_2|}{2} \quad \left(\text{so } \frac{|l_2|}{2} < |f_2(x)|\right).$$

as well as

$$\left|\frac{f_1(x)}{f_2(x)} - \frac{l_1}{l_2}\right| = \frac{|f_1(x) l_2 - l_1 l_2 + l_1 l_2 - l_1 \cdot f_2(x)|}{|f_2(x) \cdot l_2|}$$

$$\leq \frac{|f_1(x) - l_1| \cdot |l_2| + |f_2(x) - l_2| \cdot |l_1|}{|f_2(x)| \cdot |l_2|}$$

$$< \frac{M\varepsilon_1 + M\varepsilon_2}{\frac{|l_2|}{2} \cdot |l_2|} = \frac{M}{m}\varepsilon_1 + \frac{M}{m}\varepsilon_2 \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon,$$

valid $\forall\ x \in V_\delta(x_0) \cap (D \setminus \{x_0\})$. $\therefore \lim_{x\to x_0} \frac{f_1(x)}{f_2(x)} = \frac{l_1}{l_2}$.