# Assignment 1

Hand in no 3 and 5 by Jan 24.

1. Prove the formula

$$\cos\theta + \cos 2\theta + \cdots + \cos N\theta = \frac{\sin\left(N+\frac{1}{2}\right)\theta - \sin\frac{1}{2}\theta}{2\sin\frac{\theta}{2}}, \quad \theta \neq 0.$$

   Hint: Multiply both sides of the formula by $\sin(\theta/2)$ and then use the compound angle formula.

2. Verify that the Fourier series of every even function is a cosine series and the Fourier series of every odd function is a sine series. A $2\pi$-periodic function is even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$ for $x \in [-\pi, \pi]$.

3. Show that the Fourier series of $f_1(x) = x^2, x \in [-\pi, \pi]$, is given by

$$f_1(x) \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx.$$

4. Show that the Fourier series of $f_2(x) = -1, x \in [-\pi, 0)$ and $= 1, x \in [0, \pi]$, is given by

$$f_2(x) \sim \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} .$$

5. Show that the Fourier series of $f_3(x) = e^{bx}, x \in [-\pi, \pi], b \in \mathbb{R}$, is given by

$$f_3(x) \sim \frac{\sinh b\pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{b - in} e^{inx} .$$

   Here the hyperbolic sine function is given by $\sinh\theta = (e^{\theta} - e^{-\theta})/2$.