THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 4

1. Let $\{(X_n, \mathfrak{T}_n)\}_{n\in\mathbb{N}}$ be a countable collection of topological spaces.

   Consider the product space $\prod_{n\in\mathbb{N}} X_n$ with the product topology $\mathfrak{T}_{\text{prod}}$.

   Show that if each $(X_n, \mathfrak{T}_n)$ is Hausdorff, then $(\prod_{n\in\mathbb{N}} X_n, \mathfrak{T}_{\text{prod}})$ is also Hausdorff.

2. Let $I = [0,1]$ and $Y_x = \mathbb{R}$ for all $x \in I$. Consider the infinite product space

$$F = \prod_{x\in[0,1]} Y_x = \prod_{x\in[0,1]} \mathbb{R} = \{f : I \to \mathbb{R}\}$$

   Define a sequence of elements $\{f_n\}_{n\in\mathbb{N}} \subset F$ by $f_n(x) = \dfrac{1}{n}$ and an element $f \in F$ by $f(x) = 0$.

   (a) Show that f_n converges to f if F is equipped with the product topology.

   (b) * Show that f_n does not converges to f if F is equipped with the box topology.