MATH 2550
Course Work 1
(Suggested Solution)

Typo: This should be "0"!

1. Recall that the equation of a plane is given by $0 = Ax + By + Cz + D$. Find the constants A, B, C, D if the plane passes through the points with coordinates $(1,2,1), (0,1,1), (1,-2,3)$.

(Suggested solution)
Discussions of ideas: "since (in general) 3 points pass through a plane,
$\Longrightarrow$ (each) of the 3 points, $(1,2,1), (0,1,1), (1,-2,3)$ has to satisfy the equation $0 = Ax + By + Cz + D$.
$\Longrightarrow$ (Difficulty) We have 4 equations, 3 unknowns.
$\Longrightarrow$ (Way to overcome it) Divide the equation through by A (or B, C, D, if it is non-zero!). Then we get the "new" equation $0 = x + \left(\frac{B}{A}\right) y + \left(\frac{C}{A}\right) z + \left(\frac{D}{A}\right)$.

- Letting $b = \frac{B}{A}, c = \frac{C}{A}, d = \frac{D}{A}$, we obtain $0 = x + by + cz + d$.
- Now we get 3 equations in the 3 unknowns b, c, d:

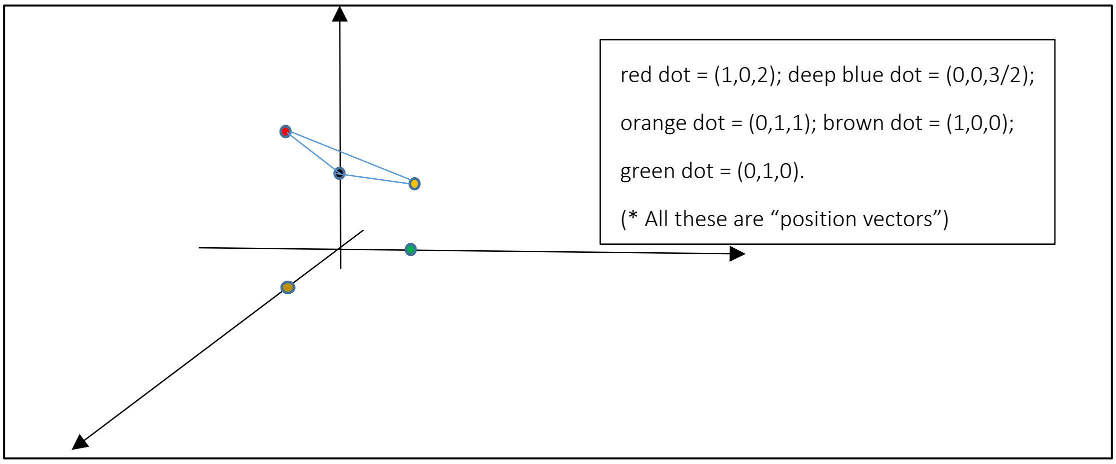
$$\begin{aligned} 0 &= 1 + 2b + c + d \\ 0 &= 0 + b + c + d \\ 0 &= 1 - 2b + 3c + d \end{aligned}$$

$$b = -1, c = -2, d = 3$$

Concl: $\frac{B}{A} = -1, \frac{C}{A} = -2, \frac{D}{A} = 3 \Longrightarrow B = -A, C = -2A, D = 3A$ $(A \neq 0)$ are the solutions. They all represent the same plane, given by $0 = x - y - 2z + 3$.
(I used Symbolab, an online solver, to work it out).

Comment: There are infinitely many solutions, all representing the same plane!

2. Sketch the plane in question 1. (Sol.) $z = \frac{x-y+3}{2} \Longrightarrow x = 1, y = 0, \text{then } z = 2;\ x = 0, y = 1 \Longrightarrow z = 1$. Finally, $x = 0, y = 0 \Longrightarrow z = \frac{3}{2}$. Now we can plot these 3 points on the diagram and get a triangle, which is part of the plane.



3. Compute the following partial derivatives:

(a) $\frac{\partial \sin(xy^2)}{\partial x}$, (b) $\frac{\partial \ln(x^2+y^2)}{\partial x}$, (c) $\frac{\partial\sqrt{x^2+y^2}}{\partial y}$

(Sol.) (a) $\cos(xy^2)y^2$, (b) $\frac{1}{x^2+y^2}2x$, (c) $y(x^2+y^2)^{-\frac{1}{2}}$

4. Recall that the <u>equation</u> of <u>tangent plane</u> to the <u>surface</u> $z = f(x,y)$ at a point (a,b) is given by the equation $z = f_x(a,b)\cdot(x-a) + f_y(a,b)\cdot(y-b) + f(a,b)$. Find equation to the tangent plane of the surface $z = x^2 - y^2$ at the point $(1,2)$

(Sol.) $z = f(1,2) + f_x(1,2)\cdot(x-1) + f_y(1,2)\cdot(y-2)$. Now, $f(1,2) = 1^2 - 2^2 = -3$. $f_x = 2x, \implies f_x(1,2) = 2\cdot 1 = 2; f_y = -2y \implies f_y(1,2) = -4$. Putting these into the formula for the tg. pl., we obtain $z = -3 + 2(x-1) - 4(y-2)$.

5. In question 4, sketch the tangent plane if $(a,b) = (0,0)$.

Sol: Use the idea for Q2!