

# Exercise on Integration

## 1.1 Substitution

Use a suitable substitution to evaluate the following integral.

1.  $\int \frac{dx}{\sqrt{2-5x}}$
2.  $\int \frac{e^{3x} + 1}{e^x + 1} dx$
3.  $\int \frac{x}{\sqrt{1-x^2}} dx$
4.  $\int x^2 \sqrt[3]{1+x^3} dx$
5.  $\int \frac{x dx}{(1+x^2)^2}$
6.  $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$
7.  $\int \frac{1}{x^2} \sin \frac{1}{x} dx$
8.  $\int x e^{-x^2} dx$
9.  $\int \frac{(\ln x)^2}{x} dx$
10.  $\int \frac{e^x dx}{2+e^x}$
11.  $\int \frac{\cos x dx}{\sqrt{1-\sin x}}$
12.  $\int \frac{\sec^2 x}{\sqrt{1+\tan x}} dx$
13.  $\int \frac{dx}{\sqrt{x-1}}$
14.  $\int \frac{dx}{e^x + e^{-x}}$
15.  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$
16.  $\int \sin x \sec^3 x dx$
17.  $\int \tan x dx$
18.  $\int \sec^2 x \cot x dx$
19.  $\int \frac{dx}{1+e^x}$
20.  $\int x(x^2+2)^8 dx$
21.  $\int \frac{x}{\sqrt{25-x^2}} dx$
22.  $\int \frac{x}{\sqrt{3x^2+1}} dx$
23.  $\int \frac{x^2}{\sqrt{9-x^3}} dx$
24.  $\int x(x+2)^7 dx$
25.  $\int \frac{x dx}{\sqrt{4x+5}}$
26.  $\int x \sqrt{x-1} dx$
27.  $\int \frac{(1-\sin x) \ln(1+\sin x)}{\cos x} dx$
28.  $\int (x+2) \sqrt{x-1} dx$
29.  $\int \frac{x dx}{\sqrt{x+9}}$
30.  $\int x^3(1+3x^2)^{\frac{1}{2}} dx$

## 1.2 Trigonometric Integrals

Evaluate

$$1. \int \cos 6x \sin 4x dx$$

$$2. \int \frac{dx}{1 - \cos x}$$

$$3. \int \sin^5 x \cos x dx$$

$$4. \int \sin 3x \sin 5x dx$$

$$5. \int \cos \frac{x}{2} \cos \frac{x}{3} dx$$

$$6. \int \cos^3 x dx$$

$$7. \int \sin^4 x dx$$

$$8. \int (\sin x + \sec x)^2 dx$$

$$9. \int \sec^2 x \tan^2 x dx$$

$$10. \int \sec x \tan^3 x dx$$

$$11. \int \frac{\tan x}{1 + \sec x} dx$$

$$12. \int \cot^2 x dx$$

$$13. \int \frac{dx}{\cos x \sin^2 x}$$

$$14. \int \frac{\sin x \cos^3 x}{1 + \cos^2 x} dx$$

$$15. \int \tan^5 x dx$$

$$16. \int \frac{1 - \tan x}{1 + \tan x} dx$$

$$17. \int \frac{dx}{\sin^4 x \cos^4 x} dx$$

$$18. \int \sin 5x \cos x dx$$

$$19. \int \cos x \cos 2x \cos 3x dx$$

$$20. \int \cos^5 x \sin^3 x dx$$

$$21. \int \cos^5 x \sin^4 x dx$$

$$22. \int \sin^2 x \cos^4 x dx$$

## 1.3 Integration By Parts

$$1. \int \ln x dx$$

$$2. \int x^2 \ln x dx$$

$$3. \int \frac{(\ln x)^2}{x^2} dx$$

$$4. \int x e^{-x} dx$$

$$5. \int x^2 e^{-2x} dx$$

$$6. \int x \cos x dx$$

$$7. \int x^2 \sin 2x dx$$

$$8. \int x^5 e^{x^3} dx$$

$$9. \int x \sin x \cos x dx$$

$$10. \int (\ln x)^2 dx$$

- |                                       |                                      |
|---------------------------------------|--------------------------------------|
| 11. $\int \sin^{-1} x dx$             | 18. $\int x \cos^{-1} x dx$          |
| 12. $\int x \tan^{-1} x dx$           | 19. $\int \tan^{-1} x dx$            |
| 13. $\int \sin \sqrt{x} dx$           | 20. $\int x^4 \ln x dx$              |
| 14. $\int \ln(x + \sqrt{1 + x^2}) dx$ | 21. $\int \frac{\ln x}{x^4} dx$      |
| 15. $\int x \sin^2 x dx$              | 22. $\int x \sec^2 x dx$             |
| 16. $\int \sin(\ln x) dx$             | 23. $\int e^{2x} \cos 3x dx$         |
| 17. $\int x \sin 4x dx$               | 24. $\int \sqrt{x} \cos \sqrt{x} dx$ |

## 1.4 Reduction Formula

Prove the following reduction formulas.

- $I_n = \int x^n e^{ax} dx; I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}, n \geq 1$
- $I_n = \int \cos^n x dx; I_n = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, n \geq 2$
- $I_n = \int \frac{1}{\sin^n x} dx; I_n = -\frac{\cos x}{(n-1) \sin^{n-1} x} + \frac{n-2}{n-1} I_{n-2}, n \geq 2$
- $I_n = \int x^n \cos x dx; I_n = x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}, n \geq 2$
- $I_n = \int \tan^n x dx; I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$
- $I_n = \int \frac{dx}{(x^2 - a^2)^n}; I_n = -\frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)} I_{n-1}, n \geq 1$
- $I_n = \int \frac{x^n dx}{\sqrt{x+a}}; I_n = \frac{2x^n \sqrt{x+a}}{2n+1} - \frac{2an}{2n+1} I_{n-1}, n \geq 1$
- $I_n = \int (\ln x)^n dx; I_n = x(\ln x)^n - n I_{n-1}, n \geq 1$
- $I_n = \int_0^1 x^n \sqrt{1-x} dx; I_n = \frac{2n}{2n-3} I_{n-1}, n \geq 2$

## 1.5 Trigonometric Substitution

Evaluate the following integrals by trigonometric substitution.

$$1. \int \frac{x^2}{1+x^2} dx$$

$$7. \int \frac{dx}{\sqrt{4+x^2}}$$

$$2. \int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$$

$$8. \int x^2 \sqrt{16-x^2} dx$$

$$3. \int \sqrt{4-9x^2} dx$$

$$9. \int \frac{dx}{x^2 \sqrt{x^2+4}}$$

$$4. \int \sqrt{\frac{1+x}{1-x}} dx$$

$$10. \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$5. \int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$$

$$11. \int (1-x^2)^{\frac{3}{2}} dx$$

$$6. \int \frac{x^2 dx}{\sqrt{9-x^2}}$$

$$12. \int \frac{dx}{(2x-x^2)^{\frac{3}{2}}}$$

## 1.6 Rational Functions

Evaluate the following integrals of rational functions.

$$1. \int \frac{x^2 dx}{1-x^2}$$

$$10. \int \frac{x^2+1}{(x+1)^2(x-1)} dx$$

$$2. \int \frac{x^2+2x-1}{x-1} dx$$

$$11. \int \frac{x^2}{(x^2-3x+2)^2} dx$$

$$3. \int \frac{x^3}{x+3} dx$$

$$12. \int \frac{x^2+5x+4}{x^4+5x^2+4} dx$$

$$4. \int \frac{(1+x)^2}{1+x^2} dx$$

$$13. \int \frac{dx}{(x+1)(x^2+1)}$$

$$5. \int \frac{x(x-6)}{(x-3)^2} dx$$

$$14. \int \frac{2x^3-4x^2-x-3}{x^2-2x-3} dx$$

$$6. \int \frac{dx}{x^2+2x-3}$$

$$15. \int \frac{4-2x}{(x^2+1)(x-1)^2} dx$$

$$7. \int \frac{x^2+1}{x^4-2x^2+1} dx$$

$$16. \int \frac{dx}{x(x^2+1)^2}$$

$$8. \int \frac{dx}{(x^2-2)(x^2+3)}$$

$$17. \int \frac{x^2 dx}{(x-1)(x-2)(x-3)}$$

$$9. \int \frac{x+1}{x^2+4x+8} dx$$

$$18. \int \frac{xdx}{x^2(x^2-2x+2)}$$

## 1.7 $t$ -method

Use  $t$ -substitution to evaluate the following integrals.

1.  $\int \frac{dx}{\sin^3 x}$

5.  $\int \frac{\cos x}{1 + \cos x} dx$

2.  $\int \frac{dx}{1 + \sin x}$

6.  $\int \frac{1 - \cos x}{3 + \cos x} dx$

3.  $\int \frac{dx}{\sin x \cos^4 x}$

7.  $\int \frac{dx}{4 \sin x + 3 \cos x}$

4.  $\int \frac{dx}{2 + \sin x}$

8.  $\int \frac{1 + \cos x}{\sin x + \cos x} dx$

## 1.8 Miscellaneous

Evaluate the following integrals.

1.  $\int \frac{x^3}{\sqrt{1-x^2}} dx$

12.  $\int \frac{\sin 2x}{1 + \cos^2 x} dx$

2.  $\int x(\ln x)^2 dx$

13.  $\int \frac{e^{\frac{1}{x}}}{x^2} dx$

3.  $\int \frac{x+4}{(x+1)^2} dx$

14.  $\int \frac{\sin x}{\cos^2 x} dx$

4.  $\int \frac{\cos^3 x}{\sin^2 x} dx$

15.  $\int x \sec x \tan x dx$

5.  $\int \frac{xdx}{(1+x^2)^2}$

16.  $\int x \tan^2 x dx$

6.  $\int \frac{x^4}{4-x^2} dx$

17.  $\int \frac{\cot x}{1 + \sin x} dx$

7.  $\int \frac{x^3}{4+x^2} dx$

18.  $\int \frac{\tan^3 x}{1 + \sec x} dx$

8.  $\int \frac{e^{2x} dx}{1 + e^x}$

19.  $\int \frac{x^3 dx}{x^2 - 1}$

9.  $\int \frac{dx}{x(1 + 2 \ln x)}$

20.  $\int \frac{dx}{e^{2x} + e^x - 2}$

10.  $\int \frac{dx}{2^x}$

21.  $\int \frac{\ln x}{x\sqrt{1 + \ln x}} dx$

11.  $\int \cos^2 x \sin^3 x dx$

22.  $\int \sqrt{x^2 + 1} dx$

23.  $\int \frac{\sqrt{9-x^2}}{x^2} dx$

24.  $\int \frac{x^2}{x^2+1} dx$

25.  $\int \frac{x^2 dx}{\sqrt{x^2+9}}$

26.  $\int (4+x^2)^{\frac{3}{2}} dx$

27.  $\int \frac{x}{1+\sqrt{1-x^2}} dx$

28.  $\int \frac{dx}{1+\sqrt{1-x^2}}$

29.  $\int \frac{\cos^3 x}{\sin x} dx$

30.  $\int \frac{x^2+8}{x^2-5x+6} dx$

31.  $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx$

32.  $\int \frac{x dx}{\sqrt{x-2}}$

33.  $\int \frac{dx}{\sqrt{1+e^x}}$

34.  $\int \cos(\ln x) dx$

35.  $\int x^2 \sqrt{1-x^2} dx$

36.  $\int \frac{x^2+2}{x^4+4} dx$

37.  $\int x \cos^2 3x dx$

38.  $\int \frac{dx}{1+\tan x}$

39.  $\int \frac{dx}{\sqrt{e^x-1}}$

40.  $\int \frac{4 dx}{x^2 \sqrt{4-x^2}}$

41.  $\int \frac{dx}{x \sqrt{x^2-1}}$

42.  $\int \frac{x+1}{x^2(x-1)} dx$

43.  $\int \sec^3 x \tan x dx$

44.  $\int \tan^4 x dx$

45.  $\int \sec x \tan^2 x dx$

46.  $\int x^3 \sqrt{x^2+1} dx$

47.  $\int \cos 2x \sin 3x dx$

48.  $\int \frac{x^4+x^2-1}{x^3+x} dx$

49.  $\int \frac{x^3 dx}{\sqrt{x^2+4}}$

50.  $\int \frac{dx}{(x^2-1)^2}$

51.  $\int \frac{dx}{1+\sqrt{x}}$

52.  $\int \cos \sqrt{x} dx$

53.  $\int \ln(1+x^2) dx$

54.  $\int \frac{dx}{\sqrt{x}(x-1)}$

55.  $\int x^2 \tan^{-1} x dx$

56.  $\int \tan^{-1} \sqrt{x} dx$

57.  $\int \frac{\sqrt{x} dx}{\sqrt{1-x}}$

58.  $\int \frac{\sqrt{x+1}}{x} dx$

59.  $\int \frac{1+\sqrt{x}}{1+x} dx$

60.  $\int \sqrt{x} \sqrt{1-x} dx$

### Section 1.1: Substitution

1.  $-\frac{2}{5}\sqrt{2-5x} + C$
2.  $\frac{1}{2}e^{2x} - e^x + x + C$
3.  $-\sqrt{1-x^2} + C$
4.  $\frac{1}{4}(1+x^3)^{\frac{4}{3}} + C$
5.  $-\frac{1}{2(1+x^2)} + C$
6.  $2\ln(1+\sqrt{x}) + C$
7.  $\cos \frac{1}{x} + C$
8.  $-\frac{1}{2}e^{-x^2} + C$
9.  $\frac{1}{3}(\ln x)^3 + C$
10.  $\ln(2+e^x) + C$
11.  $-2\sqrt{1-\sin x} + C$
12.  $2\sqrt{1+\tan x} + C$
13.  $2\sqrt{x} + 2\ln|\sqrt{x}-1| + C$
14.  $\tan^{-1} e^x + C$
15.  $2\sin \sqrt{x} + C$
16.  $\frac{1}{2}\sec^2 x + C$
17.  $-\ln|\cos x| + C$
18.  $\ln|\tan x| + C$
19.  $x - \ln(1+e^x) + C$
20.  $\frac{(x^2+2)^9}{18} + C$
21.  $-\sqrt{25-x^2} + C$
22.  $\frac{1}{3}\sqrt{3x^2+1} + C$
23.  $-\frac{2}{3}\sqrt{9-x^3} + C$
24.  $\frac{(x+2)^9}{9} - \frac{(x+2)^8}{4} + C$
25.  $\frac{1}{12}(2x-5)\sqrt{4x+5} + C$
26.  $\frac{2}{15}(x-1)^{3/2}(3x+2) + C$
27.  $\frac{1}{2}(\ln(1+\sin x))^2 + C$
28.  $\frac{2}{5}(x-1)^{3/2}(x+4) + C$
29.  $\frac{2}{3}(x-18)\sqrt{x+9} + C$
30.  $\frac{1}{135}(3x^2+1)^{3/2}(9x^2-2) + C$

### Section 1.2: Trigonometric Integrals

1.  $\frac{1}{4}\cos 2x - \frac{1}{20}\cos 10x + C$
2.  $-\cot \frac{x}{2} + C$
3.  $\frac{1}{6}\sin^6 x + C$
4.  $\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$
5.  $3\sin \frac{x}{6} + \frac{3}{5}\sin \frac{5x}{6} + C$
6.  $\sin x - \frac{1}{3}\sin^3 x + C$
7.  $\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
8.  $\frac{x}{2} - \frac{\sin 2x}{4} - 2\ln|\cos x| + \tan x + C$
9.  $\frac{1}{3}\tan^3 x + C$
10.  $\frac{1}{3}\sec^3 x - \sec x + C$
11.  $-\ln(1+\cos x) + C$
12.  $-x - \cot x + C$
13.  $-\frac{1}{\sin x} + \frac{1}{2}\ln \frac{1+\sin x}{1-\sin x} + C$
14.  $-\frac{1}{2}\cos^2 x + \frac{1}{2}\ln(1+\cos^2 x) + C$
15.  $\frac{\tan^4}{4} - \frac{\tan^2 x}{2} - \ln|\cos x| + C$
16.  $\ln|\sin x + \cos x| + C$
17.  $-8\cot 2x - \frac{8}{3}\cot^3 2x + C$
18.  $-\frac{1}{8}\cos 4x - \frac{1}{12}\cos 6x + C$
19.  $\frac{x}{4} + \frac{\sin 2x}{8} + \frac{\sin 4x}{16} + \frac{\sin 6x}{24} + C$
20.  $\frac{\cos^8(x)}{8} - \frac{\cos^6(x)}{6} + C$

$$21. \frac{\sin^9(x)}{9} - \frac{2\sin^7(x)}{7} + \frac{\sin^5(x)}{5} + C$$

$$22. -\frac{1}{6} \cos^5 x \sin x + \frac{1}{24} \cos^3 x \sin x + \frac{1}{16} \cos x \sin x + \frac{1}{16} x + C.$$

### Section 1.3: Integration By Parts

1.  $x \ln x - x + C$
2.  $\frac{x^3}{3}(\ln x - \frac{1}{3}) + C$
3.  $-\frac{1}{x}((\ln x)^2 + 2 \ln x + 2) + C$
4.  $-(x+1)e^{-x} + C$
5.  $-\frac{e^{-2x}}{4}(2x^2 + 2x + 1) + C$
6.  $x \sin x + \cos x + C$
7.  $-\frac{2x^2-1}{4} \cos 2x + \frac{x}{2} \sin 2x + C$
8.  $\frac{1}{2}x^3 e^{x^3} - \frac{1}{3}e^{x^3} + C$
9.  $\frac{1}{8} \sin 2x - \frac{1}{4}x \cos 2x + C$
10.  $x(\ln x)^2 - 2x \ln x + 2x + C$
11.  $x \sin^{-1} x + \sqrt{1-x^2} + C$
12.  $-\frac{x}{2} + \frac{1+x^2}{2} \tan^{-1} x + C$
13.  $2 \sin \sqrt{x} - 2\sqrt{x} \cos \sqrt{x} + C$
14.  $x \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C$
15.  $\frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + C$
16.  $\frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C$
17.  $\frac{1}{16} \sin 4x - \frac{1}{4}x \cos 4x + C$
18.  $\frac{x^2 \cos^{-1} x}{2} + \frac{\sin^{-1} x}{4} - \frac{x\sqrt{1-x^2}}{4} + C$
19.  $x \tan^{-1} x - \frac{1}{2} \log(x^2 + 1) + C$
20.  $\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$
21.  $-\frac{\ln x}{3x^3} - \frac{1}{9x^3} + C$
22.  $x \tan x + \ln(\cos x) + C$
23.  $\frac{1}{13}e^{2x}(3 \sin 3x + 2 \cos 3x) + C.$
24.  $2(x-2) \sin \sqrt{x} + 4\sqrt{x} \cos \sqrt{x} + C$

### Section 1.5: Trigonometric Substitution

1.  $x - \tan^{-1} x + C$
2.  $\frac{x}{\sqrt{1-x^2}} + C$
3.  $\frac{x\sqrt{4-9x^2}}{2} + \frac{2}{3} \sin^{-1} \frac{3x}{2} + C$
4.  $-\sqrt{1-x^2} + \sin^{-1} x + C$
5.  $\frac{x}{\sqrt{1+x^2}} + C$
6.  $\frac{9}{2} \sin^{-1} \frac{x}{3} - \frac{x}{2} \sqrt{9-x^2} + C$
7.  $\ln|x + \sqrt{4+x^2}| + C$
8.  $\sqrt{16-x^2} \left(\frac{x^3}{4} - 2x\right) + 32 \sin^{-1} \left(\frac{x}{4}\right) + C$
9.  $-\frac{\sqrt{x^2+4}}{4x} + C$
10.  $-\sqrt{x}\sqrt{1-x} - \tan^{-1} \frac{\sqrt{1-x}}{\sqrt{x}} + C$
11.  $\frac{x(1-x^2)^{\frac{3}{2}}}{4} + \frac{3x(1-x^2)^{\frac{1}{2}}}{8} + \frac{3}{8} \sin^{-1} x + C$
12.  $\frac{x-1}{\sqrt{2x-x^2}}$

### Section 1.6: Rational Functions

1.  $-x + \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$
2.  $\frac{x^2}{2} + 3x + 2 \ln|x-1| + C$
3.  $9x - \frac{3}{2}x^2 + \frac{1}{3}x^3 - 27 \ln|x+3| + C$
4.  $x + \ln(1+x^2) + C$
5.  $x + \frac{9}{x-3} + C$
6.  $\frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + C$



7.  $\frac{x}{1-x^2} + C$
8.  $\frac{1}{10\sqrt{2}} \ln \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{1}{5\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C$
9.  $\frac{1}{2} \ln(x^2 + 4x + 8) - \frac{1}{2} \tan^{-1} \frac{x+2}{2} + C$
10.  $\frac{1}{x+1} + \frac{1}{2} \ln |x^2 - 1| + C$
11.  $-\frac{5x-6}{x^2-3x+2} + 4 \ln \left| \frac{x-1}{x-2} \right| + C$
12.  $\tan^{-1} x + \frac{5}{6} \ln \frac{x^2+1}{x^2+4} + C$
13.  $\frac{1}{2} \tan^{-1} x + \frac{1}{4} \ln \frac{(x+1)^2}{x^2+1} + C$
14.  $x^2 + 2 \ln |x + 1| + 3 \ln |x - 3| + C$
15.  $\tan^{-1} x - \frac{1}{x-1} + \ln \frac{x^2+1}{(x-1)^2} + C$
16.  $\frac{1}{2(x^2+1)} + \ln |x| - \frac{1}{2} \ln(x^2 + 1) + C$
17.  $\frac{9}{2} \ln(x-3) - 4 \ln(x-2) + \frac{1}{2} \ln(x-1) + C$
18.  $\frac{1}{4} \ln \left( \frac{x^2}{x^2-2x+2} \right) - \frac{1}{2} \tan^{-1}(1-x) + C$

### Section 1.7: *t*-method

1.  $-\frac{\cos x}{2\sin^2 x} + \frac{1}{2} \ln \left| \tan \frac{x}{2} \right| + C$
2.  $\tan x - \sec x + C$
3.  $\frac{1}{\cos x} + \frac{1}{3\cos^3 x} + \ln \left| \tan \frac{x}{2} \right| + C$
4.  $\frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2 \tan(\frac{x}{2}) + 1}{\sqrt{3}} \right) + C$
5.  $x - \tan \frac{x}{2} + C$
6.  $2\sqrt{2} \tan^{-1} \left( \frac{\tan(\frac{x}{2})}{\sqrt{2}} \right) - x + C$
7.  $\frac{1}{5} \ln |3 \tan \frac{x}{2} + 1| - \frac{1}{5} \ln | \tan \frac{x}{2} - 3| + C$
8.  $\frac{1}{2}(x + \ln(\sin x + \cos x + 3)) - \frac{1}{\sqrt{7}} \tan^{-1} \left( \frac{2 \tan(\frac{x}{2}) + 1}{\sqrt{7}} \right) + C$

### Section 1.8: Miscellaneous

1.  $-\frac{(x^2+2)\sqrt{1-x^2}}{3} + C$
2.  $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$
3.  $\ln |x + 1| - \frac{3}{x+1} + C$
4.  $-\frac{1}{\sin x} - \sin x + C$
5.  $-\frac{1}{2(1+x^2)} + C$
6.  $4 \ln |x + 2| - 4 \ln |x - 2| - \frac{x^3}{3} - 4x + C$
7.  $\frac{x^2}{2} - 2 \ln(4 + x^2) + C$
8.  $e^x - \ln(1 + e^x) + C$
9.  $\frac{1}{2} \ln |1 + 2 \ln x| + C$
10.  $-\frac{1}{2^x \ln 2} + C$
11.  $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$
12.  $-\ln(1 + \cos^2 x) + C$
13.  $-e^{\frac{1}{x}} + C$
14.  $\sec x + C$
15.  $x \sec x - \ln |\sec x + \tan x| + C$
16.  $-\frac{x^2}{2} + x \tan x + \ln \cos x + C$
17.  $-\ln |1 + \csc x| + C$
18.  $\sec x + \ln |\cos x| + C$
19.  $\frac{1}{2}x^2 + \frac{1}{2} \ln |x^2 - 1| + C$
20.  $-\frac{x}{2} + \frac{1}{3} \ln |e^x - 1| + C$
21.  $-\frac{4}{3}\sqrt{1 + \ln x} + \frac{2}{3}(\ln x)\sqrt{1 + \ln x} + C$
22.  $\frac{1}{2}(x\sqrt{x^2 + 1} + \ln |x + \sqrt{x^2 + 1}|) + C$
23.  $-\frac{\sqrt{9-x^2}}{x} - \sin^{-1} \frac{x}{3} + C$
24.  $x - \tan^{-1} x + C$

25.  $\frac{x\sqrt{x^2+9}}{2} - \frac{9}{2} \ln|x + \sqrt{x^2+9}| + C$
26.  $\frac{x(x^2+10)\sqrt{x^2+4}}{4} + 6 \ln(x + \sqrt{x^2+4}) + C$
27.  $\ln(1 + \sqrt{1-x^2}) - \sqrt{1-x^2} + C$
28.  $\sin^{-1} x - \frac{1-\sqrt{1-x^2}}{x} + C$
29.  $\ln|\sin x| - \frac{1}{2} \sin^2 x + C$
30.  $x + 17 \ln|x-3| - 12 \ln|x-2| + C$
31.  $x - 2\sqrt{x} + 2 \ln(1 + \sqrt{x}) + C$
32.  $\frac{2}{3}(x-2)^{\frac{3}{2}} + 4(x-2)^{\frac{1}{2}} + C$
33.  $x - 2 \ln(1 + \sqrt{1+e^x}) + C$
34.  $\frac{x}{2}(\cos(\ln x) + \sin(\ln x)) + C$
35.  $\frac{1}{4}x(1-x^2)^{\frac{3}{2}} + \frac{3}{8}x\sqrt{1-x^2} + \frac{3}{8} \sin^{-1} x + C$
36.  $\frac{1}{2} \tan^{-1}(x+1) + \frac{1}{2} \tan^{-1}(x-1) + C$
37.  $\frac{x^2}{9} + \frac{x \sin 6x}{12} + \frac{\cos 6x}{72} + C$
38.  $\frac{x}{2} + \frac{1}{2} \ln|\sin x + \cos x| + C$
39.  $-2 \sin^{-1} e^{-\frac{x}{2}} + C$
40.  $-\frac{\sqrt{4-x^2}}{x} + C$
41.  $\sin^{-1} \frac{1}{x} + C$
42.  $\frac{1}{x} - 2 \ln|x| + 2 \ln|x-1| + C$
43.  $\frac{1}{3} \sec^3 x + C$
44.  $\frac{1}{3} \tan^3 x - \tan x + x + C$
45.  $\frac{1}{2} \sec x \tan x - \frac{1}{2} \ln|\sec x + \tan x| + C$
46.  $\frac{1}{3}x^2(x^2+1)^{\frac{3}{2}} - \frac{2}{15}(x^2+1)^{\frac{5}{2}} + C$
47.  $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$
48.  $\frac{1}{2}x^2 - \ln|x| + \frac{1}{2} \ln(x^2+1) + C$
49.  $\frac{1}{3}(x^2+4)^{\frac{3}{2}} - 4\sqrt{x^2} + C$
50.  $\frac{1}{4} \ln|x+1| - \frac{1}{4} \ln|x-1| - \frac{x}{2(x^2-1)} + C$
51.  $2\sqrt{x} - 2 \ln(1 + \sqrt{x}) + C$
52.  $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$
53.  $-2x + x \ln(1+x^2) + 2 \tan^{-1} x + C$
54.  $\ln|\sqrt{x}-1| - \ln|\sqrt{x}+1| + C$
55.  $\frac{1}{3}x^3 \tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6} \ln(x^2+1) + C$
56.  $(x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + C$
57.  $\sin^{-1} \sqrt{x} - \sqrt{x}\sqrt{1-x} + C$
58.  $2\sqrt{x+1} + \ln|\sqrt{x}-1| - \ln|\sqrt{x}+1| + C$
59.  $2\sqrt{x} + \ln|1+x| - 2 \tan^{-1} \sqrt{x} + C$
60.  $\frac{1}{4} \sin^{-1} \sqrt{x} - \frac{1}{4} \sqrt{x}\sqrt{1-x}(1-2x) + C$