THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5540 Advanced Geometry 2016-2017
Quiz 1
Date: 2 Mar, 2017

- Time allowed: 60 minutes
- Total points: 20 points

Recall the axioms of incidence and axioms of betweenness:

**I1**. For any distinct points $A$, $B$, there exists a unique line $l_{AB}$ containing $A$, $B$.

**I2**. Every line contains at least two points.

**I3**. There exist three noncollinear points.

**B1**. If a point $B$ is between two points $A$ and $C$ (written as $A * B * C$), then $A$, $B$ and $C$ are distinct points on a line, and also $C * B * A$.

**B2**. For any two distinct points $A$ and $B$, there exists a point $C$ such that $A * B * C$.

**B3**. Given three distinct points on a line, one and only one of them is between the other two.

**B4**. Let $A$, $B$ and $C$ be three noncollinear points, and let $l$ be a line not containing any of $A$, $B$ and $C$. If $l$ contains a point $D$ lying between $A$ and $B$, then it must also contain either a point lying between $A$ and $C$ or a point lying between $B$ and $C$, but not both.

A geometry $(\mathcal{S}, \mathcal{L})$, where $\mathcal{S}$ is the set of points and $\mathcal{L}$ is a collection of subsets of $\mathcal{S}$ which are called lines, is said to be an incidence geometry if it satisfies **I1**, **I2** and **I3**.

1. For an incidence geometry, determine whether each of the following statements is true or not. Justify your answers with brief explanations.

   (a) It cannot have a line containing all the points.

   (b) It contains at least 3 lines.

   (c) Every pair of lines must have at least one intersection point.

(6 points)

2. (a) State the definition of parallelism of two lines.

   (b) If $(\mathcal{S}, \mathcal{L})$ is an incidence geometry, does parallelism give an equivalence relation on $\mathcal{L}$? You can explain your answer by giving a proof or a counterexample.

(4 points)

3. Let $\sim$ be a relation defined on $\mathbb{R}$ which is given by $a \sim b$ if $b - a$ is an integer.

   (a) Show that $\sim$ is an equivalence relation.

   (b) Write down the elements of $\mathbb{R}/\sim$.

(4 points)

4. Using the axioms of incidence and betweenness, prove that

   (a) for any line $l$, there exists at least one point $P$ that does not lie on $l$;

   (b) every line has infinitely many distinct points;

   (c) for any point $A$, there are infinitely many lines passing through the point $A$.

(6 points)