HW 8 Due Apr 13, 2017

1. Determine the number of zeros, counting multiplicities, of

   (a) $z^6 - 6z^4 + 2z^2 - z$ inside $|z| = 1$.

   (b) $z^5 - 3z^3 - z + 1$ inside $|z| = 2$.

2. Prove that $z = 1 - e^{-z}$ has exactly one root in the right half-plane.

3. Show that the composition of two linear fractional transformations is a linear fractional transformation

4. Show that the complex inversion $w = \frac{1}{z}$ maps

   (a) a line to a line or a circle;

   (b) a circle to a line or a circle.

5. Show that for any linear fractional transformation f,

   $(z, z_1, z_2, z_3) = (f(z), f(z_1), f(z_2), f(z_3))$.