"HOMEWORK"

The following facts will be needed for the exam.

(1) Let U and V be open subsets of $\mathbb{R}^m$ and $\mathbb{R}^k$, respectively. Let $\varphi : U \to V$ be a smooth map. Let α and β be two differential forms defined on V. Show that

$$\varphi^*(\alpha \wedge \beta) = (\varphi^*\alpha) \wedge (\varphi^*\beta).$$

(2) Let M be a manifold in U. If α_1, α_2, β_1, and β_2 are differential forms such that

$$\alpha_1\Big|_{(T_xM)^k} = \alpha_2\Big|_{(T_xM)^k} \text{ and } \beta_1\Big|_{(T_xM)^m} = \beta_2\Big|_{(T_xM)^m}$$

for all x in M. Show that $\alpha_1 \wedge \beta_1\Big|_{(T_xM)^{k+m}} = \alpha_2 \wedge \beta_2\Big|_{(T_xM)^{k+m}}$ for all x in M.

(3) Let α and β be a k-form and m-form, respectively, on a manifold M. Show that $\alpha \wedge \beta = (-1)^{km}\beta \wedge \alpha$ and $d(\alpha \wedge \beta) = (d\alpha) \wedge \beta + (-1)^k \alpha \wedge d\beta$.

(4) Let α, β, and γ be differential forms on M. Show that $(\alpha \wedge \beta) \wedge \gamma = \alpha \wedge (\beta \wedge \gamma)$.

(5) Let $f : M \to N$ be a smooth map. Show that $d(f^*\alpha) = f^*d\alpha$ and $f^*(\alpha \wedge \beta) = f^*\alpha \wedge f^*\beta$.

(6) Definition of integral on manifolds and Stokes' theorem.