MATH2550

Home Work 1

Due Date: 20 Feb. 2017

1. Let $\vec{p} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{q} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$ be two vectors in $\mathbb{R}^3$, find (if any) the unit vector with positive $y-$coordinate and which is orthogonal to both $\vec{p}$ and $\vec{q}$.
2. Compute the following vectors:
   i. $(\hat{p} \times \hat{q}) \cdot \hat{k} - (\hat{p} \times \hat{k}) \cdot \hat{q}$, where $\vec{p}$ and $\vec{q}$ are the two vectors given in question 1 above.
   ii. Show that for any two non-zero vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$, the vector $\vec{v} \cdot (\vec{v} \times \vec{w})$ is a zero vector.
3. Find the cylindrical coordinates of the point $\mathbb{R}^3$ whose rectangular coordinates are given by $x = -5,\ y = 1,\ z = 4$.
4. Find the rectangular coordinates of the point $\mathbb{R}^3$ whose cylindrical coordinates are given by $r = 1, \theta = \pi/3, h = -2$.
5. Suppose $\vec{r}(t) = \cos(2t)\ \hat{\imath} + \sin(2t) + (t^2 + 1)\hat{k}$ represents the position of a certain particle in $\mathbb{R}^3$ at time t (You may assume $t \geq 0$).
   i. What is the value of t when the height is 5?
   ii. What is the speed (i.e. the magnitude of the velocity vector) when the height is 5?
   iii. What are the x, y coordinates of the particle at height 5?
   iv. Suppose that at height 5, the particle leaves its trajectory and moves along a straight line to the origin. Find the length of this straight line.
6. Compute the following line integral $\int_C f ds$, where $f(x, y) = 2x + 3y + 4z$ and C is the curve given by the vector-valued function $\vec{r} : [0,2\pi] \to \mathbb{R}^3$, with $\vec{r}(t) = \cos(2t)\ \hat{\imath} + \sin(2t)\ \hat{\jmath} + t\hat{k}$.
7. Compute the following line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = 2x\vec{\imath} + 3y\vec{\jmath}$ and C is the rectangle joining the points $(0,0), (0,1), (2,1)$ and $(2,0)$. (in that order).