1, i, position vector $p = (x, y, x^2+y^2)$

$$P_x = (1, 0, 2x)$$

$$P_y = (0, 1, 2y)$$

We can take $N = P_x \times P_y$

$$= (-2x, -2y, 1)$$

---

or using polar coordinate.

$$p = (r\cos\theta, r\sin\theta, r^2)$$

$$P_r = (\cos\theta, \sin\theta, 2r)$$

$$P_\theta = (-r\sin\theta, r\cos\theta, 0)$$

take $N = P_r \times P_\theta$

$$= (-2r^2\cos\theta, -2r^2\sin\theta, r)$$

---

1 ii, for x-y coor.

$$|N| = \sqrt{(-2x)^2 + (-2y)^2 + 1^2} = \sqrt{4x^2+4y^2+1}$$

$$\hat{N} = \left(\frac{-2x}{\sqrt{4x^2+4y^2+1}}, \frac{-2y}{\sqrt{4x^2+4y^2+1}}, \frac{1}{\sqrt{4x^2+4y^2+1}} \right)$$

---

for polar coor.

$$|N| = \sqrt{(-2r^2\cos\theta)^2 + (-2r^2\sin\theta)^2 + r^2}$$

$$= \sqrt{4r^4+r^2} = r\sqrt{4r^2+1}$$

$$\hat{N} = \left(\frac{-2r\cos\theta}{\sqrt{4r^2+1}}, \frac{-2r\sin\theta}{\sqrt{4r^2+1}}, \frac{1}{\sqrt{4r^2+1}} \right)$$

1.iii. for x-y coor.

$$dS = |N| \, dx\, dy$$

$$= \sqrt{4x^2+4y^2+1} \; dx\, dy.$$

---

for polar coor.

$$dS = r\sqrt{4r^2+1} \; dr\, d\theta.$$

2. No, it is a cone

(0,0) is the corner where is not smooth.

4.

$$\int_{-1}^{1}\int_{-1}^{1} -2x-2xy+y \, dx\,dy$$

$$= \int_{-1}^{1} -x^2-yx^2+yx \Big|_{-1}^{1} \, dy$$

$$= \int_{-1}^{1} 2y \, dy$$

$$= y^2 \Big|_{-1}^{1}$$

$$= 0.$$