

# MATH 2221A Mathematics Laboratory II

## Lab Assignment 10

Name: \_\_\_\_\_

Student ID.: \_\_\_\_\_

In this assignment, you are asked to run MATLAB demos to see MATLAB at work. The color version of this assignment can be found in your own **H:**\ drive.

### Instructions

1. Start MATLAB, until you see a window with the MATLABprompt ">>". This window is called the **Command Window**.
2. After you started have MATLAB, you will automatically be in the directory **H:**\. Please enter "**diary on**" after the MATLABprompt >> only once to record all your work in **H:**\diary. No marks will be given if no diary is found.
3. You should write your results on the lab sheet provided, and save the figures in the **H:** drive, in your personal drive.
4. Please read and sign the following declaration before handing in your assignment. Otherwise, no marks will be given.

I declare that the assignment here submitted is original except for source material explicitly acknowledged. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the website <http://www.cuhk.edu.hk/policy/academichonesty/>

\_\_\_\_\_  
Signature

\_\_\_\_\_  
Date

1 (25 marks)		3 (25 marks)	
2 (25 marks)		4 (25 marks)	
		Total	/100 pts

Please read the following carefully:

**General Guidelines for Lab Assignment Submission.**

- Please sign and date the statement of Academic Honesty.
- Please go to the class and lab indicated by your registered course code via the CUSIS system. If you go to a different lab than the one you are registered for, you will not receive credit for the assignment even if you completed it.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your lab). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on a double-sided printout of this pdf file. Try to fit your answers inside the available space.
- The use of computers/cellular phones/graphing calculators/iPads will NOT be permitted during tests and lab assignments. Please do not use our lab computer to recharge your cellular phone battery.
- In order to make it fair for all students, during the labs and tests, if you touch/press any icons on your cellular phone, our TA will check your phone to determine whether or not you are exchanging messages with another student. If you are found cheating (in the tests or in the lab or on homework assignments), you will automatically get an F grade in this course and your act will be reported to the Department for necessary disciplinary actions.

## Exercises

**Note:** Remember to add ";" if you want to suppress the output. Marks will be deducted if there is any unnecessary output. Unless specified in the question, you should not print anything on the screen.

- (25 marks) In this question we will revisit the built-in functions taught in last lecture. Write a script named `q1.m` to finish the following tasks:
  - Define one symbolic variable  $x$ .
  - Expand  $p(x) = (x - 2)^2(x - 7)^3$  in ascending order.
  - Compute  $p(2)$ , and **show your result on the screen**.
  - Use a column vector `prime` to store all prime numbers that is greater than 1 and smaller than 500. **Hint:** A for-loop may help.
  - Given two rotational matrices

$$A = \begin{pmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{pmatrix} \quad B = \begin{pmatrix} \cos(b) & -\sin(b) \\ \sin(b) & \cos(b) \end{pmatrix}$$

Here  $a, b$  are two symbolic variables. Compute  $AB$  and **show your result on the screen**. Write down the answer in the space below. What is your conclusion?

**Note:**

- $\cos A \cos B - \sin A \sin B = \cos(A + B)$
- $\sin A \cos B + \cos A \sin B = \sin(A + B)$

```
[ cos(a)*cos(b) - cos(b)*sin(a), - sin(a)*sin(b) - cos(a)*sin(b)]
[ cos(a)*cos(b) + cos(b)*sin(a),  sin(a)*sin(b) - cos(a)*sin(b)]

% Rotation of A degree followed by a rotation of B degree
% equals to a rotations of (A+B) degree
```

## Solution

```
syms x

p = (x-2)^2*(x-7)^3;
expand(p);
subs(p,2)

prime = [];
for i = 2:500
    if isprime(i)
        prime = [prime;i];
    end
end

syms a b
A = [cos(a),-sin(a);cos(a),sin(a)];
B = [cos(b),-sin(b);cos(b),sin(b)];
A*B
```

2. (25 marks) In lecture, we compared the speed of computing a finite sum. In this question, you need to compare the speed of solving a system numerically and symbolically. Write a script named `q2.m` to finish the following tasks:
- Use the function `vander` to create an  $5 \times 5$  matrix  $A$ , where  $a_{ij} = (1 + 0.1(i - 1))^{5-j}$ .
  - Create  $b = (1, 1, 1, 1, 1)^T$ .
  - Use `tic,toc` to calculate the time used for solving the system  $Ax = b$  numerically.
  - Define 5 symbolic variables  $x,y,z,w,v$ , use `solve` to solve the system symbolically, use `tic,toc` to calculate the time used.

Which way is faster? Write down that ratio

$$\frac{\text{Time used for symbolic calculation}}{\text{Time used for numerical calculation}}$$

in the space below.

Around 2.4473e+03.

### Solution

```
clear all;

A = vander(1:.1:1.4);
b = ones(5,1);
tic
A\b;
toc

syms x y z w v

lhs = A*[x;y;z;w;v]
tic
solve(lhs==b);
toc
```

3. (25 marks) Create a MATLAB function `[l,nmin] = ComputeLimit(g,e)`. In this function, you need to first use a suitable function to check whether  $l$ , the limit of the sequence  $g$ , exists. Secondly,

- (a) If  $l$  is a finite number, compute  $nmin$ , minimum  $n$  such that  $|g(n) - l| < e$ , and display

The limit is  $l$ , and minimum  $n$  is  $nmin$

- (b) If  $l = \infty$  or  $l$  doesn't exist, display the following sentence:

Limit does not exist!

and set  $l = []$ ,  $nmin = \infty$ .

## Sample Output

```
>> [l,nmin] = ComputeLimit(1/n,0.1);  
The limit is 0, and minimum n is 11  
>> [l,nmin] = ComputeLimit(n,0.01);  
Limit does not exist!
```

Write down the output of the following:

```
>> [l,nmin] = ComputeLimit(1/n^2,1e-3);  
The limit is 0, and minimum n is 32  
  
>> [l,nmin] = ComputeLimit(n^3,1e-1);  
Limit does not exist!
```

## Solution

```
function [l,nmin] = ComputeLimit(g,e)  
    lim = limit(g,inf);  
    if isfinite(lim)  
        l = double(lim);  
        nmin = 1;  
        while abs(subs(g,nmin) - l) >= e  
            nmin = nmin+1;  
        end  
        display(['The limit is ' num2str(l) ', '  
            'and minimum n is ' num2str(nmin) ]) %They are on the same line  
    else  
        display('Limit does not exist!')  
        l = [];  
        nmin = inf;  
    end  
end
```

4. (25 marks) Create a MATLAB function `l = RatioTest(a)` to check whether an infinite series is convergent by using **Ratio Test**.

**Note:** Let  $s = \sum_{n=1}^{\infty} a_n$ , define

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

If

- (a)  $L < 1$ , the series is absolutely convergent.
- (b)  $L > 1$ , the series is divergent.
- (c)  $L = 1$ , this test has no conclusion.

The input variable `a` is a symbolic variable depending on the symbolic variable `n`. (For example, `a=1/n`). It represents  $a_n$  in the infinite series  $s$ . By using ratio test, check whether  $s = \sum_{n=1}^{\infty} a_n$  is convergent. If

- (a)  $L < 1$ , compute the limit and display "The limit is ?". and set the output to ?. Here ? is the limit of the infinite series.
- (b)  $L > 1$ , display "The series is divergent.", and set the output to [].
- (c)  $L = 1$ , display "No conclusion.", and set the output to [].

**Hint:** Given a symbolic variable, you can substitute another symbolic variable into it by using the function `subs`. For example, let `k` be a symbolic variable, `a = n`, `subs(a,1/k)` gives  $1/k$ .

### Sample Output

```
>> l = RatioTest(1/factorial(n));  
The limit is 1.7183  
>> l = RatioTest(factorial(n));  
The series is divergent.  
>> l = RatioTest(n);  
No conclusion.
```

Write down the output of the following:

```
>> l = RatioTest(n^2);  
No conclusion  
  
>> l = RatioTest(factorial(n)/(5^n));  
The series is divergent.
```

### Solution

```
function l = RatioTest(a)  
    syms k  
  
    ratio = abs(subs(a,k+1)/subs(a,k));  
    L = limit(ratio,inf);  
    if L<1  
        l = double(symsum(a,1,inf));  
        display(['The limit is ' num2str(l)])  
    elseif L>1  
        display('The series is divergent.')        l = [];  
    else  
        display('No conclusion')        l = [];  
    end  
end  
end
```