

## 2050 Optional Test (F1-2 on 2 Dec 2016)

1. Show by  $\epsilon$ - $\delta$  terminology that  $\lim_{x \rightarrow -2} \frac{x-3}{x+1} = 5$ .
2. Let  $f: (a, b) \rightarrow \mathbb{R}$  be uniformly continuous. Show that one can assign a value of  $f$  at  $a$  such that  $f: [a, b) \rightarrow \mathbb{R}$  is then continuous at  $a$ . Is  $f$  uniformly continuous on  $[a, b)$ ? Give your reasoning.
3. Give a 'self-contained' proof for the following result (that is, use only definitions: if you need results about intervals or sequences you should provide their proofs or via other methods directly from definitions):  
if  $f: [a, b] \rightarrow \mathbb{R}$  is continuous and  $k \in \mathbb{R}$  are such that  
$$f(a) < k < f(b)$$
then there exists  $c$  with  $a < c < b$  such that  
$$f(c) = k.$$