THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5000 Analysis I (Fall 2015)
Quiz 1

Time allowed: 90 minutes
Total points: 25 points

1. Prove or disprove the following statements.

   (a) If A and B are nonempty subsets of $\mathbb{R}$ such that $a < b$ for all $a \in A$ and $b \in B$, then $\sup A < \inf B$.

   (b) Suppose that A_n, $n \in \mathbb{N}$ is a sequence of nonempty subsets of $\mathbb{R}$.
   If A_n is bounded below for each $n \in \mathbb{N}$, then $\displaystyle\bigcup_{n=1}^{\infty} A_n$ is bounded below.

   (c) If A is a nonempty subset of $\mathbb{R}$ such that A is bounded, then $\inf A \leq \sup A$.

(9 Points)

2. Suppose that $\{x_n\}$ is a sequence of real numbers such that $0 < x_n < 1$ for all $n \in \mathbb{N}$.
   Define $A = \displaystyle\bigcup_{n=1}^{\infty} (0, x_n)$.

   (a) Show that A is bounded.

   (b) Show that $\inf A = 0$ and $\sup A = \sup\{x_n \,:\, n \in \mathbb{N}\}$.

(4 Points)

3. Let $S = \{\dfrac{m\sqrt{3}}{n} \,:\, m,\, n \in \mathbb{Z}\}$.
   Show that S is a dense subset of $\mathbb{R}$, i.e. for all real numbers $x,\, y \in \mathbb{R}$ with $x < y$, there exists $s \in S$ such that $x < s < y$.

(6 Points)

4. (a) State without proof of the **Nested Interval Property**.

   (b) Suppose I_n, $n \in \mathbb{N}$ is a nested sequence of closed bounded intervals and I_{n_r} is a subsequence of I_n.

      (i) Prove that I_{n_r} is a nested sequence of closed bounded intervals.

      (ii) Suppose that $\xi \in \mathbb{R}$ and $\xi \in I_{n_r}$ for all $r \in \mathbb{N}$. Show that $\xi \in I_n$ for all $n \in \mathbb{N}$.

(6 Points)