Solution to Assignment 3

1. Let X be the random variable of the number of defective items.

$$P[X] = 1 \cdot \frac{C_1^4 C_2^{16}}{C_3^{20}} + 2 \cdot \frac{C_2^4 C_1^{16}}{C_3^{20}} + 3 \cdot \frac{C_3^4}{C_3^{20}} = \frac{3}{5}$$

If you want to use the fact that it is a Bernoulli trial, you need to show that it is indeed one since it is not entirely apparent.

2. Let X be the random variable of the amount of winning.

(a)

$$P[X] = 1.1 \cdot \frac{C_2^5 + C_2^5}{C_2^{10}} - 1 \cdot \frac{5 \cdot 5}{C_2^{10}} = -\frac{1}{15}$$

(b)

$$\mathrm{Var}(X) = E[(X-\mu)^2] = \left(1.1 - \frac{1}{15}\right)^2 \cdot \frac{C_2^5 + C_2^5}{C_2^{10}} - \left(1 - \frac{1}{15}\right)^2 \cdot \frac{5 \cdot 5}{C_2^{10}} = \frac{49}{45}$$

3. (a)

$$E[(2+X)^2] = E[X^2] + 4E[X] + 4 = \mathrm{Var}(X) + 5E[X] + 4 = 14$$

(b)

$$\mathrm{Var}(4+3X) = \mathrm{Var}(3X) = 3^2\mathrm{Var}(X) = 45$$

4.

$$\sum_{i=1}^{\infty} P\{N \geq i\} = \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} P\{N = k\} = \sum_{k=1}^{\infty}\sum_{i=1}^{k} P\{N = k\} = \sum_{k=1}^{\infty} kP\{N = k\} = E[N]$$

5. (a)

$$E\left[\frac{X-\mu}{\sigma}\right] = \frac{1}{\sigma}(E[X] - \mu) = 0$$

(b)

$$\mathrm{Var}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}\mathrm{Var}(X) = 1$$

6.

$$P(X = n+k | X > n) = \frac{P(X = n+k)}{P(X > n)} = \frac{(1-p)^{n+k-1}p}{\sum_{i=n+1}^{\infty}(1-p)^{i-1}p} = (1-p)^{k-1}p = P(X = k)$$

Since all trials are independent, if we are given that the first n trials are failed, to get a success in the $(n+k)$-th trial is same as getting a success in the upcoming k-th trial.