Suggested solutions of HW3

Ex. 16.1

20. (a) Now $r(t) = t\,i + 4t\,j$, $0 \le t \le 1$ and $\frac{dr}{dt} = i + 4j$ so $\left|\frac{dr}{dt}\right| = \sqrt{17}$

Then $\int_C \sqrt{x+2y}\, ds = \int_0^1 \sqrt{t+8t}\,\sqrt{17}\, dt = 3\sqrt{17}\int_0^1 \sqrt{t}\, dt = 2\sqrt{17}$.

(b) $C_1: r(t) = t\,i$ and $\left|\frac{dr}{dt}\right| = 1$; $C_2: r(t) = i + t\,j$ and $\left|\frac{dr}{dt}\right| = 1$.

Then $\int_C \sqrt{x+2y}\, ds = \int_{C_1} \sqrt{x+2y}\, ds + \int_{C_2} \sqrt{x+2y}\, ds = \int_0^1 \sqrt{t+0}\, dt + \int_0^2 \sqrt{1+2t}\, dt$

$= \frac{2}{3} + \left(\frac{5\sqrt{5}}{3} - \frac{1}{3}\right) = \frac{5\sqrt{5}+1}{3}$

26. $C_1: r(t) = t\,i$ and $\left|\frac{dr}{dt}\right| = 1$; $C_2: r(t) = i + t\,j$ and $\left|\frac{dr}{dt}\right| = 1$.

$C_3: r(t) = (1-t)i + j$ and $\left|\frac{dr}{dt}\right| = 1$; $C_4: r(t) = (1-t)j$ and $\left|\frac{dr}{dt}\right| = 1$.

So $\int_C \frac{1}{x^2+y^2+1}\, ds = \int_0^1 \frac{dt}{t^2+1} + \int_0^1 \frac{dt}{t^2+2} + \int_0^1 \frac{dt}{(1-t)^2+2} + \int_0^1 \frac{dt}{(1-t)^2+1}$

$= [\tan^{-1} t]_0^1 + \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^1 + \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\frac{t-1}{\sqrt{2}}\right)\right]_0^1 + [-\tan^{-1}(1-t)]_0^1$

$= \frac{\pi}{2} + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$

32. Since the curve is $2x+3y=6$, then $r(t) = t\,i + \left(2 - \frac{2}{3}t\right)j$ and $\left|\frac{dr}{dt}\right| = \frac{\sqrt{13}}{3}$, $0 \le t \le 6$. Then $\int_C f(x,y)\, ds = \int_C 4+3x+2y\, ds = \int_0^6 \left(4+3t+2\left(2-\frac{2}{3}t\right)\right)\frac{\sqrt{13}}{3}\, dt$

$= \frac{\sqrt{13}}{3}\int_0^6 \left(8 + \frac{5}{3}t\right) dt = 26\sqrt{13}$.

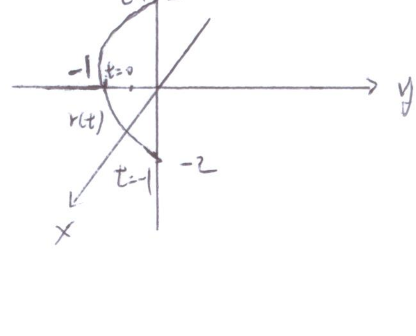
34. $r(t) = (t^2-1)j + 2t\,k$, $-1 \le t \le 1$ so $\left|\frac{dr}{dt}\right| = 2\sqrt{t^2+1}$

Then $M = \int_C \delta(x,y,z)\, ds = \int_{-1}^1 \left(15\sqrt{(t^2-1)+2}\right)\left(2\sqrt{t^2+1}\right) dt = \int_{-1}^1 30(t^2+1)\, dt = 80$.

Thus $M_{xz} = \int_C y\,\delta(x,y,z)\, ds = \int_{-1}^1 (t^2-1)[30(t^2+1)]\, dt = \int_{-1}^1 30(t^4-1)\, dt$

$= -48$, so $\bar{y} = \frac{M_{xz}}{M} = -\frac{3}{5}$. And $M_{yz} = \int_C x\,\delta\, ds = 0 = M_{xy}$

So $(\bar{x}, \bar{y}, \bar{z}) = \left(0, -\frac{3}{5}, 0\right)$



Ex 16.2

16. C_1: $x=t,\ y=3t,\ 0\le t\le 1 \Rightarrow dx=dt$; C_2: $x=1-t,\ y=3$, so $dx=-dt$

C_3: $x=0,\ y=3-t,\ 0\le t\le 3$; so $dx=0$

Then $\int_C \sqrt{x+y}\,dx = \int_0^1 \sqrt{t+3t}\,dt + \int_0^1 \sqrt{(1-t)+3}\,(-dt) + \int_0^3 \sqrt{0+(3-t)}\cdot 0$

$= \frac{4}{3} + (2\sqrt{3} - \frac{16}{3}) = 2\sqrt{3} - 4$.

26. $r(t) = \cos t\, i + \sin t\, j,\ 0\le t\le \frac{\pi}{2}$, and $F = y i - x j$. So $F = \sin t\, i - \cos t\, j$

So $F\cdot \frac{dr}{dt} = -\sin^2 t - \cos^2 t = -1$. Then $\int_C F\cdot dr = \int_0^{\frac{\pi}{2}} (-1)\,dt = -\frac{\pi}{2}$

38. (a) C_1: $r(t) = (1-t)i + j,\ 0\le t\le 2$, so $\frac{dr}{dt} = -i$ and $F\cdot\frac{dr}{dt} = -1$.

C_2: $r(t) = -i + (1-t)j,\ 0\le t\le 2$, so $\frac{dr}{dt} = -j$ and $F\cdot\frac{dr}{dt} = 2t-1$.

C_3: $r(t) = (t-1)i - j,\ 0\le t\le 2$, so $\frac{dr}{dt} = i$ and $F\cdot\frac{dr}{dt} = -1$.

C_4: $r(t) = i + (t-1)j,\ 0\le t\le 2$, so $\frac{dr}{dt} = j$ and $F\cdot\frac{dr}{dt} = 2t-1$.

Then $\int_C F\cdot\frac{dr}{dt}\,dt = \int_0^2 (-1)\,dt + \int_0^2 (2t-1)\,dt + \int_0^2 (-1)\,dt + \int_0^2 (2t-1)\,dt$

$= -2+2-2+2 = 0$.

(b) For $x^2+y^2=4$, so $r(t) = 2\cos t\, i + 2\sin t\, j,\ 0\le t\le 2\pi$

then $F\cdot\frac{dr}{dt} = (2(\sin t)i + (2\cos t + 2(2\sin t))j)\cdot((-2\sin t)i + (2\cos t)j)$

$= -4\sin^2 t + 4\cos^2 t + 8\sin t\cos t = 4\cos 2t + 4\sin 2t$.

Finally $\int_C F\cdot\frac{dr}{dt}\,dt = \int_0^{2\pi} (4\cos 2t + 4\sin 2t)\,dt = 0$.

(c) One possible path:

C_1: $r(t) = t i,\ 0\le t\le 1$ and $F\cdot\frac{dr}{dt} = 0$;

C_2: $r(t) = (1-t)i + tj,\ 0\le t\le 1$ and $F\cdot\frac{dr}{dt} = 1$;

C_3: $r(t) = (1-t)j,\ 0\le t\le 1$ and $F\cdot\frac{dr}{dt} = 2t-1$;

Then $\int_C F\frac{dr}{dt}\,dt = \int_0^1 0\,dt + \int_0^1 1\,dt + \int_0^1 (2t-1)\,dt$

$= 0$.

44. (a) $-\dfrac{xi+yj}{\sqrt{x^2+y^2}}$ is a unit vector through (x,y) pointing toward the origin and we want $|F|$ to have magnitude $\sqrt{x^2+y^2}$.

Then $F = \sqrt{x^2+y^2}\left(-\dfrac{xi+yj}{\sqrt{x^2+y^2}}\right) = -xi-yj$

(b) We want $|F| = \dfrac{C}{\sqrt{x^2+y^2}}$, where $C\neq 0$ is a constant

So $F = \dfrac{C}{\sqrt{x^2+y^2}}\cdot\left(-\dfrac{xi+yj}{\sqrt{x^2+y^2}}\right) = -C\left(\dfrac{xj+yj}{x^2+y^2}\right)$

45. The answer is yes. $\int_C F\cdot dr = \int_C y\,i\,dr = \int_b^a [f(t)i]\cdot[i+\frac{df}{dt}j]dt$

$= \int_a^b f(t)dt =$ Area under the curve.

46. Now $r = xi+yj = xi+f(x)j$ and $\frac{dr}{dx} = i+f'(x)j$

Because $F = \dfrac{k}{\sqrt{x^2+y^2}}(xi+yj)$ has magnitude k,

$F\cdot\frac{dr}{dx} = k\frac{d}{dx}\sqrt{x^2+[f(x)]^2}$. So $\int_C F\cdot T ds = \int_C F\cdot\frac{dr}{dx}dx = \int_a^b k\frac{d}{dx}\sqrt{x^2+[f]^2}\,dx$

$= k\left[\sqrt{x^2+f^2}\right]_a^b$

$= k\left(\sqrt{b^2+[f(b)]^2} - \sqrt{a^2+[f(a)]^2}\right).$