

UGEB2530 Games and Strategic Thinking

1. Alan, Betty and Carl each has to buy a book on Game Theory. The list price is \$130. There is a discount if you buy in batch: one book for \$130, two books for \$220 and three for \$300. Alan has a discount card that allows him to save extra \$10 for each book he buys in batch

Number of books	1	2	3
Without discount card	\$130	\$220	\$300
With discount card	\$120	\$200	\$270

- a) Complete the following table

Coalition	Cost	Save
{A}	\$120	\$0
{B}	\$130	\$0
{C}	\$130	\$0
{A,B}	\$200	\$50
{A,C}	\$200	\$50
{B,C}	\$220	\$40
{A,B,C}	\$270	\$110

- b) Find the Shapley's value of the players.

Shapley's value of Alan is

$$\begin{aligned} \phi_A &= \frac{v(\{A, B\}) + v(\{A, C\}) - 2v(\{B, C\}) + 2v(\{A, B, C\})}{6} \\ &= \frac{50 + 50 - 2 \times 40 + 2 \times 110}{6} \\ &= 40 \end{aligned}$$

Shapley's value of Betty and Carl:

$$\begin{aligned} \phi_B &= \frac{v(\{A, B\}) + v(\{B, C\}) - 2v(\{A, C\}) + 2v(\{A, B, C\})}{6} \\ &= \frac{50 + 40 - 2 \times 50 + 2 \times 110}{6} \\ &= 35 \end{aligned}$$

$$\phi_C = \phi_B = 35$$

- c) How should they divide the cost?

Alan pays: \$80; Betty pays: \$95 ; Carl pays: \$95