UGEB2530 Game and strategic thinking
Quiz 1

Name:____________________________________ID:____________________Marks:__________/40

Time allowed: 60 mins Answer all questions.

1. (4 marks) Circle all pure Nash equilibria of the games.

   (a) $\begin{pmatrix} (3,-1) & (1,0) \\ (5,2) & (0,-1) \end{pmatrix}$

   (b) $\begin{pmatrix} (1,0) & (6,4) & (3,5) \\ (4,2) & (-1,0) & (2,-1) \end{pmatrix}$

2. (4 marks) In a game there is a bag which contains 3 red balls and 2 blue balls. Two balls are chosen randomly from the bag without replacement. 3 marks will be given for each red ball and 1 mark will be given for each blue ball.

   (a) Find the probability that the total marks is 4.

   (b) Find the expected total marks of the game.

3. (4 marks) Circle all saddle points of the following game matrices.

   (a) $\begin{pmatrix} 3 & 0 & -2 & 2 \\ 4 & 2 & 1 & 3 \\ 1 & -1 & 0 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} -4 & 0 & -3 & -3 \\ 0 & -1 & -2 & 1 \\ 3 & -2 & -3 & -5 \\ 2 & 1 & -4 & 2 \end{pmatrix}$

4. (8 marks) John calls out a number '1' or '2' and Peter calls out a number '3' or '4' simultaneously. If the sum of the two numbers is even, Peter pays John the sum. If the sum of the two numbers is odd, John pays Peter the sum.

   (a) Write down the payoff matrix for John.

   (b) Suppose John chooses '1' with a probability of 0.4 and Peter chooses '3' with a probability of 0.2. Find the expected payoff of John.

   (c) What is the best strategy of Peter if John chooses '1' with a probability of 0.4?

   (d) Find the strategy of John such that his payoff is fixed no matter how Peter plays.

   (e) Find the value of the game.

5. (4 marks) Solve the 2×2 zero sum game with game matrix

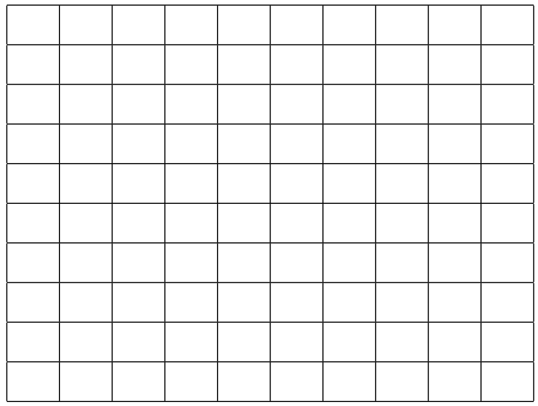
$$\begin{pmatrix} -3 & 4 \\ 3 & 0 \end{pmatrix}$$

that is, find a maximin strategy, a minimax strategies, and the value of the game.

6. (6 marks) Solve the zero sum game with the game matrix

$$\begin{pmatrix} 1 & -1 & 3 & -3 \\ -2 & 3 & -3 & 4 \end{pmatrix}$$

Solution:



7. (10 marks) Consider the 3×3 zero sum game with game matrix

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

where a is a real number.

(a) Find the value of the game matrix A if $a \leq 0$.

(b) Suppose $a = 1$.

(i) What is the best strategy of the column player if the row player uses mixed strategy $(0.2, 0.3, 0.5)$?

(ii) Find the maximin strategy for the row player.

(iii) Find the value of the game matrix A.

Solution.

-End of Paper-