UGEB2530 Game and strategic thinking
Solution to Assignment 4

1.) Explain whether the following bimatrix games can be transformed to a zero sum game.

   Solution:

   (a) If it can be transformed to a 0 sum game, then there are α and β such that: αA+βI=-B.
   where: A=$\begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}$
   B=$\begin{pmatrix} -8 & -2 \\ 7 & 1 \end{pmatrix}$
   I=$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
   This equation has solution that: α=3, β=-1. So this game can be transformed to a zero sum game.

   (b) If it can be transformed to a 0 sum game, then there are α and β such that: αA+βI=-B.
   where: A=$\begin{pmatrix} 2 & -2 \\ -4 & 3 \end{pmatrix}$
   B=$\begin{pmatrix} 2 & 4 \\ 5 & 1 \end{pmatrix}$
   I=$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
   This equation has no solution, So this game cannot be transformed to a zero sum game.

2. Find all pure Nash equilibrium of the games with the following game bimatrices and state whether they are Pareto optimal.

   Solution:

   (a) The Nash equilibrium are $(4,6)$ and $(2,4)$, with $(4,6)$ is a Pareto optimal and $(2,4)$ is not a Pareto optimal.

   (b) The Nash equilibrium are $(3,3)$ and $(4,2)$, with both $(3,3)$ and $(4,2)$ are not Pareto optimal.

3. **Solution:**

   (a) The prudential strategy for player I is $(\frac{1}{5}, \frac{4}{5})$ and the prudential strategy for player I is $(\frac{1}{2}, \frac{1}{2})$. So the payoff of each player using the strategy are:

$$v_I = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = 3.4.$$

$$v_{II} = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = 2.5.$$

(b) The Nash equilibrium for player I is $(\frac{1}{4}, \frac{3}{4})$ and the prudential strategy for player I is $(\frac{2}{5}, \frac{3}{5})$. So the payoff of each player using the strategy are:

$$v_I = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = 3.4.$$

$$v_{II} = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = 2.5.$$

4. **Solution:**

(a) The prudential strategy for player I is $(1, 0)$ and the prudential strategy for player I is $(\frac{2}{5}, \frac{3}{5})$. So the payoff of each player using the strategy are:

$$v_I = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = 3.2.$$

$$v_I I = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = 1.2.$$

(b) The Nash equilibrium for player I is $(1, 0)$ and the prudential strategy for player I is $(0, 1)$. So the payoff of each player using the strategy are:

$$v_I = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2.$$

$$v_I I = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 4.$$