THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5220 Complex Analysis and its Applications 2014-2015
Test 2, 18 Mar, 2015

- Time allowed: 45 minutes
- Answer all questions.
- Show your work clearly and concisely in your answer book.
- Write down your name and student ID number on the front page of your answer book.
- You are allowed to use a calculator in this test.

1. Find

   (a) $\displaystyle\int_C e^z \sin z \cos 2z \, dz$

   (b) $\displaystyle\int_C \frac{\sin z}{z} \, dz$

   (c) $\displaystyle\int_C \frac{1}{z^2(z^2-4)e^z} \, dz$

   where C denotes the unit circle oriented positively.

(30 points)

2. With the aid of series, prove that the function f defined by

$$f(z) = \begin{cases} \dfrac{e^z - 1}{z} & \text{if } z \neq 0 \\ 1 & \text{if } z = 0 \end{cases}$$

   is entire.

(20 points)

3. (a) Find the Taylor series for the function $\dfrac{e^z}{1-z}$ about the point $z = 0$ and write down the disk of convergence of the Taylor series.

   (b) Hence, find the Taylor series for the function $\dfrac{(2-z)e^z}{(1-z)^2}$ about the point $z = 0$.

(20 points)

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4. (a) Suppose $f(z)$ is analytic in the disk $\{|z-z_0| \leq R\}$ and $|f(z)| \leq M$ for $|z-z_0| = R$. Prove that

$$\left|f^{(n)}(z_0)\right| \leq \frac{n!}{R^n} M$$

for any natural number n.

(b) Suppose $f(z)$ is an entire function which is bounded. Prove that $f(z)$ is a constant function.

(c) A function $f(z)$ is a doubly periodic function if there exist ω_0 and ω_1 that are linearly independent so that $f(z+\omega_0) = f(z+\omega_1) = f(z)$ for all $z \in \mathbb{C}$. Prove that the only entire functions that are doubly periodic are constant functions.

(30 points)