THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5000 Analysis I (Fall 2014)
Homework 2
Due Date: 13, 2014

Name: _____ Student No.: _____

Class: _____ Final Result: _____

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/ I also understand that I may be asked to explain my solutions by the instructors. _____ _____ Signature Date

1. Find the following limits, and establish your results according to the $\epsilon-\delta$ definition.

   (a) $\lim\limits_{(x,y)\to(1,-4)} x^2y$.

   (b) $\lim\limits_{(x,y)\to(-2,3)} \frac{\sin(9x+2y^2)}{9x+2y^2}$.

2. For each of the following functions, determine whether it tends to a real number or to $\pm\infty$ as (x,y) tends to $(0,0)$, by using the $\epsilon-\delta$ definition.

   (a) $g(x,y) \stackrel{def}{=} \frac{x^3+y^2}{2x^4+3y^2}$.

   (b) $h(x,y) \stackrel{def}{=} \frac{x^2y}{x+y^2}$.

3. For each $(x,y) \in U \stackrel{def}{=} \{(s,t) \in \mathbb{R}^2 : 3s^2+5t^2 > 7\}$, find an $r > 0$ (r depends on (x,y)) such that the ball $B_r((x,y)) \subset U$ (according to the usual metric on $\mathbb{R}^2$).

4. Find necessary and sufficient conditions (and establish your result) for a metric ρ on a real linear space X to be induced by a norm on X.

5. Is there a complete metric space in which every convergent sequence $(a_n)_{n\in\mathbb{N}}$ is eventually constant (i.e. there exists $N \in \mathbb{N}$ such that $a_n = a_N$ for all $n > N$)? Substantiate your answer.