Sample Solutions of Assignment 3 for MAT3270B:4.1-4.3

Note: Any problems to the sample solutions, email Ms.Rong Zhang (rzhang@math.cuhk.edu.hk) directly.

Section 4.1

In each problems 11 throuth 16,verify that the given functions are solutions of the differential equation,and determine their Wronskian.

13.$y''' + 2y'' - y' - 2y = 0$;$e^t$,$e^{-t}$,$e^{-2t}$

Answer:

$y = e^t, e^t + 2e^t - e^t - 2e^t = 0$

$y = e^{-t}, -e^{-t} + 2e^{-t} + e^{-t} - 2e^{-t} = 0$

$y = e^{-2t}, -8e^{-2t} + 8e^{-2t} + 2e^{-2t} - 2e^{-2t} = 0$

$$W(e^t, e^{-t}, e^{-2t}) = \begin{vmatrix} e^t & e^{-t} & e^{-2t} \\ e^t & -e^{-t} & -2e^{-2t} \\ e^t & e^{-t} & 4e^{-2t} \end{vmatrix} = -6e^{-2t}$$

15.$xy''' - y'' = 0$;1,x,x^3

Answer:

$y = 1, 0 - 0 = 0$

$y = x, 0 - 0 = 0$

$y = x^3, 6x - 6x = 0$

$$W(1, x, x^3) = \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & 3x^2 \\ 0 & 0 & 6x \end{vmatrix} = 6x$$

Section 4.2

In each of the problem find the general solution of the given differential equation.

11. $y''' - y'' - y' + y = 0$

15. $y^{(6)} + y = 0$

21. $y^{(8)} + 8y^{(4)} + 16y = 0$

Answer:

11. The characteristic equation is

$$r^3 - r^2 - r + 1 = (r-1)^2(r+1) = 0$$

The roots are $r_1 = r_2 = 1$ and $r_3 = -1$. Thus the general solution is

$$y(t) = (c_1 + c_2 t)e^t + c_3 e^{-t}$$

15. The characteristic equation is

$$r^6 + 1 = 0$$

The roots are $r_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, $r_2 = \frac{\sqrt{3}}{2} - \frac{1}{2}i$, $r_3 = i$, $r_4 = -i$, $r_5 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$, and $r_6 = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$. Thus the general solution is

$$y(t) = e^{\frac{\sqrt{3}t}{2}}(c_1 \cos\frac{t}{2} + c_2 \sin\frac{t}{2}) + c_3 \cos t + c_4 \sin t + e^{-\frac{\sqrt{3}t}{2}}(c_5 \cos\frac{t}{2} + c_6 \sin\frac{t}{2})$$

21. The characteristic equation is

$$r^8 + 8r^4 + 16 = (r^4 + 4)^2 = 0$$

The roots are $r_1 = r_2 = 1 + i$, $r_3 = r_4 = -1 + i$,$r_5 = r_6 = -1 - i$, and $r_7 = r_8 = 1 - i$. Thus the general solution is

$$y(t) = e^t(c_1 \cos t + c_2 t \cos t + c_3 \sin t + c_4 t \sin t) + e^{-t}(c_5 \cos t + c_6 t \cos t + c_7 \sin t + c_8 t \sin t)$$

Section 4.3

Determine the general solution of the given differential equation.

5.$y^{(4)} - 4y'' = t^2 + e^t$

6.$y^{(4)} + 2y'' + y = 3 + \cos 2t$

Answer: 5.The characteristic equation of homogeneous equation is

$$r^4 - 4r^2 = 0,$$

the roots are $r_1 = r_2 = 0, r_3 = 2, r_4 = -2$. So the general solution of homogeneous equation is

$$y(t) = c_1 + c_2 t + c_3 e^{2t} + c_4 e^{-2t}.$$

Let the particular solution is $Y(t) = t^2(A + Bt + Ct^2) + De^t$, by computation, we have $A = -\frac{1}{16}, B = 0, C = -\frac{1}{48}, D = -\frac{1}{3}$. The general solution is

$$y = c_1 + c_2 t + c_3 e^{2t} + c_4 e^{-2t} + t^2(-\frac{1}{16} - \frac{1}{48}t^2) - \frac{1}{3}e^t.$$

Answer: 6.The characteristic equation of homogeneous equation is

$$r^4 + 2r^2 + 1 = 0,$$

the roots are $r_1 = i, r_2 = -i, r_3 = i, r_4 = -i$. So the general solution of homogeneous equation is

$$y(t) = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t.$$

Let the particular solution is $Y(t) = A \cos 2t + B \sin 2t + C$, by computation, we have $A = \frac{1}{9}, B = 0, C = 3$. The general solution is

$$y = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t + \frac{1}{9} \cos 2t + 3.$$

Find the solution of the given initial value problem.Then plot a graph of the solution.

11. $y''' - 3y'' + 2y' = t + e^t;\ y(0) = 1, y'(0) = -\frac{1}{4}, y''(0) = \frac{3}{2}$

Answer: The characteristic equation of homogeneous equation is

$$r^3 - 3r^2 + 2r = 0$$

the roots are $r_1 = 0, r_2 = 1, r_3 = 2$. So the general solution of homogeneous equation is

$$Y(t) = c_1 + c_2 e^t + c_3 e^{2t}.$$

Since $g = t + e^t$, we can assume a particular solution is

$$Y = t(A + Bt) + Cte^t$$

by comptutation,we have $A = \frac{3}{4}, B = \frac{1}{4}, C = -1$.

The general solution is

$$y = c_1 + c_2 e^t + c_3 e^{2t} + \frac{3}{4}t + \frac{1}{4}t^2 - te^t$$

By the initial condition,we have $c_1 = 1, c_2 = 0, c_3 = 0$,hence the solution is

$$y = 1 + \frac{3}{4}t + \frac{1}{4}t^2 - te^t$$

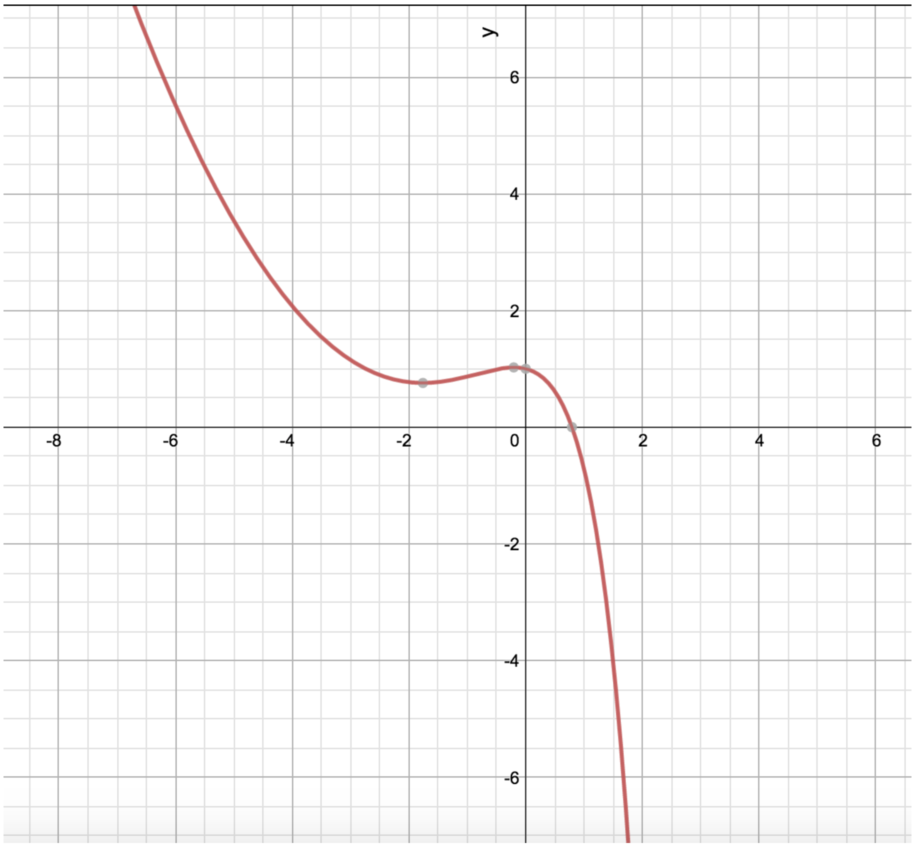

FIGURE 1. Question 11:graph of solution