Tutorial 7

Sect. 7.5 $\boxed{12}$. $x' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} x$ $\qquad (*)$

find the general sol.

Sol: $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$, then $|A - \lambda I| = \begin{vmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{vmatrix}$

$$= -(\lambda+1)^2(\lambda-8).$$

Thus the eigenvalues are $r_1 = -1$, $r_2 = -1$, $r_3 = 8$.

The eigenvecter ξ ~~for~~ corresponding to $r = -1$ satisfies.

$$A\xi = -\xi,$$

that is,
$$\begin{cases} 3\xi_1 + 2\xi_2 + 4\xi_3 = -\xi_1 \\ 2\xi_1 + 2\xi_3 = -\xi_2 \\ 4\xi_1 + 2\xi_2 + 3\xi_3 = -\xi_3 \end{cases}$$

$$\Rightarrow 2\xi_1 + \xi_2 + 2\xi_3 = 0. \qquad (**)$$

Hence two linealy independent vectors satisfying $(**)$

are $\left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)^T$, $\left(0, -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right)^T$.

A similiar calculation gives that the eigenvector corresponding to

eigenvalue $r = 8$. is $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)^T$

So a fundamental set of solutions of (✱) is

$$x^{(1)}(t)=\begin{pmatrix}-\frac{\sqrt{2}}{2}\\ 0\\ \frac{\sqrt{2}}{2}\end{pmatrix}e^{-t},\quad x^{(2)}(t)=\begin{pmatrix}0\\ -\frac{2\sqrt{5}}{5}\\ \frac{\sqrt{5}}{5}\end{pmatrix}e^{-t},\quad x^{(3)}(t)=\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ \frac{2}{3}\end{pmatrix}e^{8t}$$

and the general solution is

$$x=c_1\begin{pmatrix}-\frac{\sqrt{2}}{2}\\ 0\\ \frac{\sqrt{2}}{2}\end{pmatrix}e^{-t}+c_2\begin{pmatrix}0\\ -\frac{2\sqrt{5}}{5}\\ \frac{\sqrt{5}}{5}\end{pmatrix}e^{-t}+c_3\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\\ \frac{2}{3}\end{pmatrix}e^{8t}$$ □

Sect. 7.6 $\boxed{12}$ Express the general solution. Choose an initial point and draw the corresponding trajectory in the x_1x_2-plane.

$$x'=\begin{pmatrix}-\frac{4}{5} & -1\\ 2 & \frac{6}{5}\end{pmatrix}x$$

Sol. ——. So the eigenvalues are $r=\frac{1}{5}\pm i$

and the eigenvector of $r=\frac{1}{5}+i$ is

$$\left(-\frac{1-i}{2},1\right)^T$$

the eigenvector of $r=\frac{1}{5}-i$ is

$$\left(-\frac{1+i}{2},1\right)^T$$

So the general solution is $x(t)=c_1\begin{pmatrix}-\frac{1-i}{2}\\ 1\end{pmatrix}e^{(\frac{1}{5}+i)t}+c_2\begin{pmatrix}-\frac{1+i}{2}\\ 1\end{pmatrix}e^{(\frac{1}{5}-i)t}$

If the initial data is $x(0) = (a, b)^T$, $a, b \in \mathbb{R}$.

then $$c_1 \begin{pmatrix} -\frac{1-i}{2} \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -\frac{1+i}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \begin{cases} c_1 = \frac{b}{2} - (a + \frac{b}{2})\, i \\ c_2 = \frac{b}{2} + (a + \frac{b}{2})\, i \end{cases}$$

$$x(t) = \begin{pmatrix} a \\ b \end{pmatrix} e^{\frac{1}{5}t} \cos t + \begin{pmatrix} (a+b)i \\ -(2a+b)i \end{pmatrix} e^{\frac{1}{5}t} \cdot i \sin t$$

$$= e^{\frac{1}{5}t} \begin{pmatrix} a \cos t - (a+b) \sin t \\ b \cos t + (2a+b) \sin t \end{pmatrix}$$