Tutorial 5.

☆ Lecture Notes 5. Page 25.

The particular sol. of $ay'' + by' + cy = g(t) = g_1(t) + \cdots + g_n(t)$

$g_i(t)$	$Y_i(t)$
$P_n(t) = a_0 t^n + a_1 t^{n-1} + \cdots + a_0$	$t^s(A_0 t^n + A_1 t^{n-1} + \cdots + A_n)$
$P_n(t) e^{\alpha t}$	$t^s(A_0 t^n + A_1 t^{n-1} + \cdots + A_n) e^{\alpha t}$
$P_n(t) e^{\alpha t} \begin{cases} \sin \beta t \\ \cos \beta t \end{cases}$	$t^s [(A_0 t^n + \cdots + A_n) e^{\alpha t} \cos \beta t + (B_0 t^n + \cdots + B_n) e^{\alpha t} \sin \beta t]$

$S = 0, 1, 2$ is to make sure no term in $Y_i(t)$ is a sol. of the corresponding homogeneous eqn.

---

to higher order ODE. $S = 0, 1, 2, \cdots, \underline{\underline{m}}$ (the order of ODE)

☆ Section. 4.4.

$$Y(t) = \sum_{m=1}^{n} y_m(t) \int_{t_0}^{t} \frac{g(s) W_m(s)}{W(s)} ds$$

$$W(t) = c \exp\left[- \int p_1(t)\, dt \right]$$

W_m is the determinant obtained from W by replacing the ~~mth~~ m-th column by the column $(0, 0, \cdots, 0, 1)$

Problems

Sect. 4.2. $\boxed{29}$ find the solution of the given initial value prob.

$$y''' + y' = 0 ; \; y(0) = 1 \; . \; y'(0) = 1 \; . \; y''(0) = 2 .$$

Sol. The characteristic eqn for $y''' + y' = 0$ is

$$r^3 + r = 0 .$$

$$\Rightarrow r_1 = 0 \; . \; r_2 = i \; . \; r_3 = -i$$

So the fundamental set of sol. of eqn is

$$y_1(t) = 1 \; , \; y_2(t) = \sin t \; . \; y_3(t) = \cos t .$$

Suppose the sol. is $Y(t) = k_1 y_1(t) + k_2 y_2(t) + k_3 y_3(t)$.

Then the initial values give

$$\begin{cases} k_1 + k_3 = 1 \\ k_2 = 1 \\ -k_3 = 2 \end{cases} \Rightarrow (k_1, k_2, k_3) = (3, 1, -2)$$

□

Sect. 4.3 $\boxed{13}$ Determine a suitable form for $Y(t)$

if the method of undetermined coefficients

is to be used.

$$y''' - 2y'' + y' = 3t^3 + 2e^t$$

Sol: Consider the original eqn as the sum of particular solutions of

$$y''' - 2y'' + y' = 3t^3, \quad y''' - 2y'' + y' = 2e^t$$

Suppose the particular solution $Y_1(t)$, $Y_2(t)$ are the following seperately:

$$Y_1(t) = t(A_0 t^3 + A_1 t^2 + A_2 t + A_3)$$

$$Y_2(t) = B t^2 e^t$$

here note that a constant is a solution of the homogeneous eqn. e^t, te^t are also ——

The constants are determined by substituting into the individual differential eqns.

$$A_0 = \frac{3}{4}, \quad A_1 = 6, \quad A_2 = 27, \quad A_3 = \cancel{36}\ 72, \quad B = 1.$$

So the sol is

$$Y(t) = \frac{3}{4}t^4 + 6t^3 + 27t^2 + 72t + t^2 e^t$$

□