MATH 2010E ADVANCED CALCULUS I HOMEWORK 3

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Due at 5:00pm on Friday, 22th May, 2015

When you work on homework problems, please do not simply write down the numerical answers, but also the **step-by-step deduction** for each problem. In your homework, mid-term and final exam, your step-by-step deductions will take up an important proportion of your final scores.

Problems that need to be submitted 12.6 1–12. 13.1 2, 4, 7, 12, 18, 20, 23e, 25.

Problem 1. Let

 $25x^2 + 60xy + 27y^2 + 20x + 30y = 18$

be the equation of a conic section. Let M be the symmetric matrix representing the quadratic portion of the equation. Find all real numbers λ such that $\det(M - \lambda I) = 0$, and determine whether the conic section is an ellipse, a hyperbola, or a parabola.

Problem 2. Let

 $5x^{2} + 3y^{2} + 9z^{2} + 4xy - 2xz + 8yz - 6x + 7y - 6z = 100$

be the equation of a quadric surface. Let M be the symmetric matrix representing the quadratic portion of the equation.

(a) Compute the leading principal minors D_1 , D_2 , and D_3 , where D_i is the determinant of submatrix formed by the first *i* rows and *i* columns of *M*.

(b) By noticing that the numbers of positive, negative and zero entries in $\{D_1, \frac{D_2}{D_1}, \frac{D_3}{D_2}\}$ is the same as those in the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$ of M, determine the type of quadric surface of the above equation (You may want to refer to Theorem 5 in Lecture Note 3).

Other problems

- 12.6 19, 23, 25, 27, 29, 31 (You are encouraged to use MATLAB or Mathematica to check your answers.)
- $13.1 \quad 32.$

Date: Tuesday, 19^{th} May, 2015.