THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520C University Mathematics for Applications
Suggested Solution to Assignment 3

Exercise 5.2 In 29 and 35, find the indicate integral and check your answer by differentiation.

Exercise 5.2: 29.

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$$

Solution: Letting $u = e^x - e^{-x}$, then $du = (e^x + e^{-x})\,dx$. Therefore

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|e^x - e^{-x}| + C$$

Check: $\dfrac{d}{dx}(\ln|e^x - e^{-x}| + C) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Exercise 5.2: 35.

$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$$

Solution: Letting $u = \sqrt{x} + 1$, $du = \dfrac{1}{2\sqrt{x}}\,dx$

$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx = \int \frac{2}{u}\,du = 2\ln u + C = 2\ln(\sqrt{x}+1) + C$$

Check: $\dfrac{d}{dx}(2\ln(\sqrt{x}+1) + C) = 2 \cdot \dfrac{1}{2}\dfrac{\frac{1}{\sqrt{x}}}{\sqrt{x}+1} = \dfrac{1}{\sqrt{x}(\sqrt{x}+1)}$

Exercise 6.1: 3.

$$\int (1-x)e^x\,dx$$

Solution: Applying integration by parts

$$\begin{aligned}\int (1-x)e^x\,dx &= (1-x)e^x + \int e^x\,dx \\ &= (1-x)e^x + e^x + C = (2-x)e^x + C\end{aligned}$$

Exercise 6.1: 5.

$$\int t\ln 2t\,dt$$

Solution:

$$\begin{aligned}\int t\ln 2t\,dt &= \frac{1}{2}t^2\ln 2t - \frac{1}{2}\int t^2\cdot\frac{1}{t}\,dt\\ &= \frac{1}{2}t^2\ln 2t - \frac{1}{4}t^2 + C\end{aligned}$$

Exercise 6.1: 13

$$\int \frac{x}{\sqrt{x+2}}\,dx$$

Solution:

$$\begin{aligned}\int \frac{x}{\sqrt{x+2}}\,dx &= 2x\sqrt{x+2} - 2\int \sqrt{x+2}\,dx\\ &= 2x\sqrt{x+2} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C\end{aligned}$$

1. Evaluate the following indefinite integrals.

   (a)

   $$\int \frac{x^2-5x+3}{x-3}\,dx$$

   (b)

   $$\int \frac{3x+2}{x^2-9}\,dx$$

Solution: (a) Since $x^2-5x+3 = x(x-3)-2(x-3)-3$

$$\begin{aligned}\int \frac{x^2-5x+3}{x-3}\,dx &= \int x-2-\frac{3}{x-3}\,dx\\ &= \frac{1}{2}x^2-2x-3\ln|x-3|+C\end{aligned}$$

(b) Since $x^2-9=(x+3)(x-3)$, we can suppose that $\dfrac{3x+2}{x^2-9} = \dfrac{A}{x+3}+\dfrac{B}{x-3}$, where A and B are constants.

$\Longrightarrow 3x+2 = A(x-3)+B(x+3)$

Letting $x=3$, $11=6B \Longrightarrow B=\dfrac{11}{6}$.

Letting $x=-3$, $-7=-6A \Longrightarrow A=\dfrac{7}{6}$.

Therefore

$$\begin{aligned}\int \frac{3x+2}{x^2-9}\,dx &= \frac{7}{6}\int \frac{1}{x+3}\,dx + \frac{11}{6}\int \frac{1}{x-3}\,dx\\ &= \frac{7}{6}\ln|x+3| + \frac{11}{6}\ln|x-3| + C\end{aligned}$$