A mapping $f : A \to B$ is said to be ***bijective***, if it satisfies the following two conditions:

(1) $f(x) \neq f(y)$ whenever $x \neq y$ in A,

(2) for each $b \in B$, there exists $a \in A$ such that $b = f(a)$.

Such a map f is called a ***bijection***.

Suppose a mapping $f : A \to B$ is bijective. Then for each $b \in B$, there exists a unique $a \in A$ such that $b = f(a)$. Thus, a mapping $\tilde{f}$ from B into A is defined by: $\tilde{f}(b) = a$.

We have:

$\tilde{f} \circ f(a) = a$ for every $a \in A$,

$f \circ \tilde{f}(b) = b$ for every $b \in B$.

This map $\tilde{f}$ is called the **inverse map** of f, and is usually denoted by f^{-1}.

Example 1.

(i) The mapping $g : [0, \pi] \to [0, 1]$ given by:

$g(x) = \sin x, \;\; x \in [0, \pi]$.

is not bijective, because it does not satisfy condition (1) (e.g $g(\pi/4) = g(3\pi/4)$, though it satisfies condition (2)).

(ii) The mapping $h : [-\pi/2, \pi/2] \to [-1, 2]$ given by:

$h(x) = \sin x, \;\; x \in [-\pi/2, \pi/2]$.

is not bijective, because it does not satisfy condition (2) ($2 \neq h(x)$ for all $x \in [-\pi/2, \pi/2]$, though it satisfies condition (1)).

(iii) The mapping $f : [-\pi/2, \pi/2] \to [-1, 1]$ given by:

$f(x) = \sin x, \;\; x \in [-\pi/2, \pi/2]$

is bijective.

The inverse $f^{-1} : [-1, 1] \to [-\pi/2, \pi/2]$ is usually denoted by arcsin, and for each $y \in [-1, 1]$, $\arcsin(y) = x$, where x is the unique number in $[-\pi/2, \pi/2]$ such that $y = \sin x$.

(iv) The mapping $g : [\pi/2, 3\pi/2] \to [-1, 1]$ given by:

$g(x) = \sin x, \;\; x \in [\pi/2, 3\pi/2]$

is bijective.

The inverse g^{-1} is a map from $[-1,1]$ into $[\pi/2, 3\pi/2]$, and for an $y \in [-1,1]$, $g^{-1}(y) = x$, where x is the unique number in $[\pi/2, 3\pi/2]$ such that $y = \sin x$.