MATH1010 University Mathematics 2014-2015
Assignment 4
Due: 14 Nov 2014 (Friday)

Answer all questions.

1. Find the Taylor polynomial of order 4 (up to the term in x^4) for the following function at the specific point c:

   (a) $f(x) = \sqrt{x+1}$ at $c = 0$

   (b) $f(x) = \frac{1+x+x^2}{1-x+x^2}$ at $c = 0$

   (c) $f(x) = \sin(\sin x)$ at $c = 0$

   (d) $f(x) = x^4 + x^2 + 1$ at $c = -2$

2. Evaluate the following limit using Taylor's Theorem:

   (a) $\displaystyle\lim_{x\to 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4}$

   (b) $\displaystyle\lim_{x\to 0} \frac{e^x \sin x - x(1+x)}{x^3}$

   (c) $\displaystyle\lim_{x\to +\infty} x^{\frac{3}{2}}(\sqrt{x+1} + \sqrt{x-1} - 2\sqrt{x})$

   (d) $\displaystyle\lim_{x\to \infty} \left[x - x^2 \ln\left(1 + \frac{1}{x}\right)\right]$

3. Let $f : (a, b) \to \mathbb{R}$ be a differentiable function and $x_0 \in (a, b)$. For any $h > 0$ small, there exists $\theta \in (0, 1)$, depending on h, such that

   $$f(x_0 + h) = f(x_0) + hf'(x_0 + \theta h).$$

   If f is twice differentiable at x_0 with $f''(x_0) \neq 0$, prove that

   (a) $\displaystyle\lim_{h\to 0} \frac{f(x_0 + h) - f(x_0) - f'(x_0)h}{h^2} = \frac{f''(x_0)}{2}$.

   (b) $\displaystyle\lim_{h\to 0} \theta = \frac{1}{2}$.

   (Note that we do not assume that f is twice differentiable in (a, b).)

End