

MATH1010 University Mathematics 2014-2015
Assignment 2
Due: 3 Oct 2013 (Friday)

Answer all questions.

1. Evaluate the following limits.

(a) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - 2x - 8}$

(d) $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{1-x}} - \frac{1}{\sqrt{1+x}} \right)$

(b) $\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x^3 - 27}$

(e) $\lim_{x \rightarrow 0} \frac{\tan^2 x}{\sin(x^2)}$

(c) $\lim_{x \rightarrow 4} \frac{8 - x^{\frac{3}{2}}}{16 - x^2}$

(f) $\lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \sqrt{\cos x}}$

2. Let $f(x)$ be a function. Prove that if $\lim_{x \rightarrow a} |f(x)| = 0$, then $\lim_{x \rightarrow a} f(x) = 0$.

3. Use definition to evaluate the derivatives of the following functions.

(a) $y = \frac{3}{x^2}$

(b) $y = 2\sqrt{x} - 1$

4. Find $\frac{dy}{dx}$ if

(a) $y = x^4 \cos 5x$

(d) $y = \frac{x}{\sqrt{x^2 + 1}}$

(g) $y = \cos \left(\frac{1}{\cosh x} \right)$

(b) $y = \frac{e^{-x}}{\sqrt{x}}$

(e) $y = \sec^2 x$

(h) $y = \sqrt{\frac{1 + \sin x}{1 - \sin x}}$

(c) $y = e^{\sin 3x}$

(f) $y = \ln(2 + \sin(x^2 + 1))$

(i) $y = \ln(\ln(x^4 + 1))$

5. Find $\frac{dy}{dx}$ if $y = x|\sin x|$.

6. This exercise shows that the derivative of a function may not be continuous. Let

$$f(x) = \begin{cases} x^2 \sin \left(\frac{1}{x} \right), & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

(a) Show that $f(x)$ is continuous at $x = 0$.

(b) Find $f'(x)$ for $x \neq 0$.

(c) Show that $f(x)$ is differentiable at $x = 0$ by evaluating $f'(0)$.

(d) Explain whether $f'(x)$ is continuous at $x = 0$.

End