The Second HKSIAM Biennial Meeting

August 28 - September 1, 2023

The Chinese University of Hong Kong

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Introduction

Hong Kong Society for Industrial and Applied Mathematics (HKSIAM) is based in Hong Kong and connecting the international community. We are committed to advancing the application of mathematics and computational science to engineering, industry, science, and society; to promoting research that will lead to effective new mathematical and computational methods and techniques for science, engineering, industry, and society; and to providing media for the exchange of information and ideas among mathematicians, engineers, and scientists.

The purpose of the HKSIAM Biennial conference is to provide a forum to expose the mathematical community to the most updated advances in the field of industrial and applied mathematics, which further promotes research on various mathematical problems arising from economic development and technological advancement. A key feature of the conference is our intention to invite some world-renowned mathematicians who are working at the frontiers of industrial and applied mathematics to give distinguished lectures. Another aim of the event is to bring together different groups of mathematicians working on industrial and applied mathematics in Hong Kong and to stimulate new mathematical ideas, initiate and enhance the existing collaborations between the researchers in Hong Kong and the international community to produce first-rate results in this area. This first HKSIAM Biennial conference was successfully held in January 11-15, 2021. For more information, please see: IAS Conference on Industrial and Applied Mathematics (Jan 11-15, 2021).

Scientific Background

Modern manufacturing and service industries have changed drastically due to the explosion in the knowledge economy. Fast and inexpensive computing, office products, the development and utilization of large databases, have necessitated sophisticated methods to meet new demands. Industrial and applied mathematics is the enabling factor in realizing and implementing these methods. In recent years, the mathematical community has responded to this growing need for the mathematically proved solution in the industry. Organizations such as the Fields Institute, Mitacs, and PIMS in Canada, IPAM, SIAM and the NSF in the United States, and Newton Institutes in the United Kingdom have been promoting the interaction of mathematics with industry and continuing to research on various mathematical problems arisen from economic development and technological advancement.

Industrial and applied mathematics is an inherently interdisciplinary field. In addition to mathematics, it includes subjects from fields outside mathematics such as business, computer science, medical sciences and engineering. An industrial mathematician has strong analytical and problem-solving skills built upon a background of computing, mathematics, statistics, and basic science. Numerous reports and studies carried out by professional organizations show that there is a growing demand in the workforce for mathematics scholars with the practical skills to work with managers, engineers, etc.

Industrial and applied mathematics is a well-established field within the mathematical sciences community. Every four years there is an International Congress on Industrial and Applied Mathematics. Industrial and applied mathematics focuses on problems which come from industry and aims for solutions which are relevant to the industry, including finding the most efficient (i.e. cost-effective) way to solve the problem. With the increasing complexity and sophistication of modern industry, individuals who are able to understand technical issues, formulate precise and accurate mathematical models, implement solutions using the latest computer techniques, and convey these ideas to their co-workers who may be managers or engineers, are becoming a necessary part of many organizations and companies. Examples of areas in the industry that industrial and applied mathematics plays a key role are signal and image processing, computer graphics, computer vision, risk management, system reliability, software testing, and verification, database systems, production line optimization, and marketing research.

Conference Venue and Wifi

Wifi: Wifi is available for conference participants in the CYT Building, please connect Wi-Fi "CUguest", then select "Conference Guests (@conference.cuhk.edu.hk)", login with ID "hksiam2023" and password "siam.0828".

Otherwise, Eduroam is also available in the CYT Building and other places on campus of CUHK.

Conference Venue: The conference will be held in Cheng Yu Tung (CYT) Building [Point 1]. It is located next to the Hotel Hyatt Regency Hong Kong Shatin [Point 2]. It takes approximately 2 minutes to walk from University MTR Station (Exit B) [Point 3].

From Hotel Hyatt Regency Hong Kong Shatin to the conference (CYT Building)

The CYT Building [Point 1] is located next to the Hotel Hyatt Regency Hong Kong Shatin [Point 2], the walk distance is about 2 minutes.

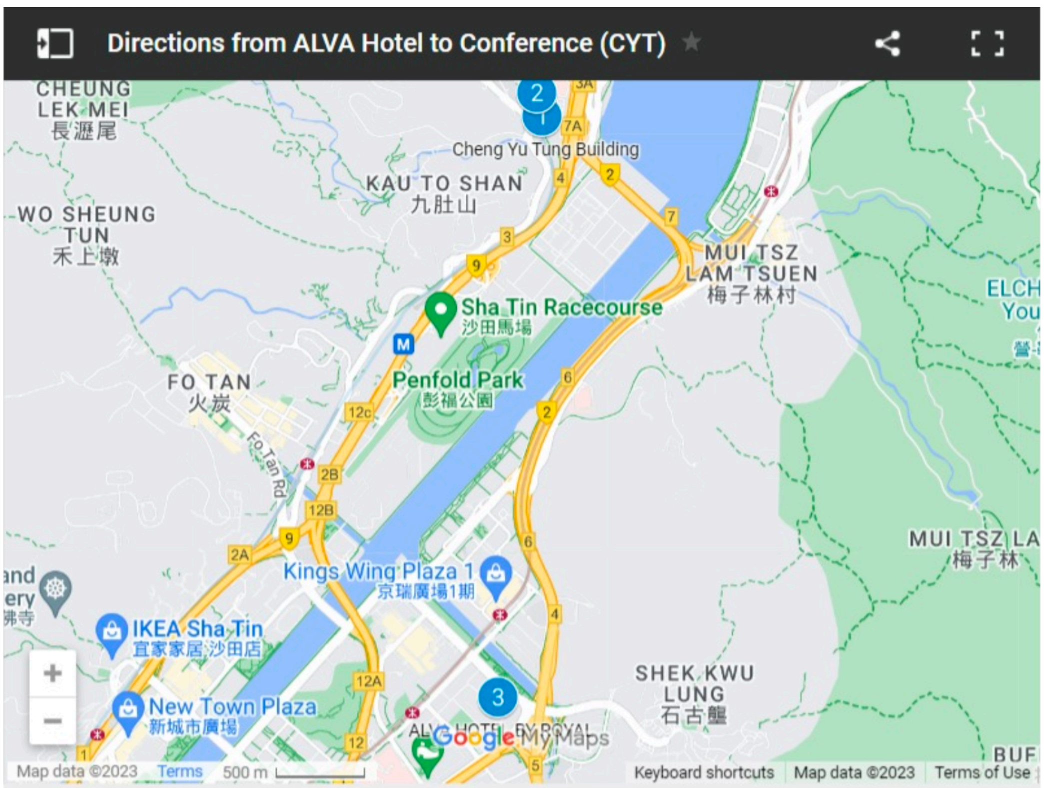
Point 1: Cheng Yu Tung (CYT) Building

Point 2: Hotel Hyatt Regency Hong Kong Shatin

From ALVA Hotel to the conference (CYT Building)

From ALVA Hotel [Point 3] to the conference (CYT Building) [Point 1], one can take MTR (Mass Transit Railway).

1. Go to the station “Shek Mun” or “City One”, take the “Tuen Ma Line” in the direction “Tuen Mun”.
2. Get off at the station “Tai Wai” to change.
3. At “Tai Wai”, take the “East Rail Line” in the direction “Lo Wo” or “Lok Ma Chau”.
4. Get off at the station “University”, the CYT Building is near the “Exit B”.



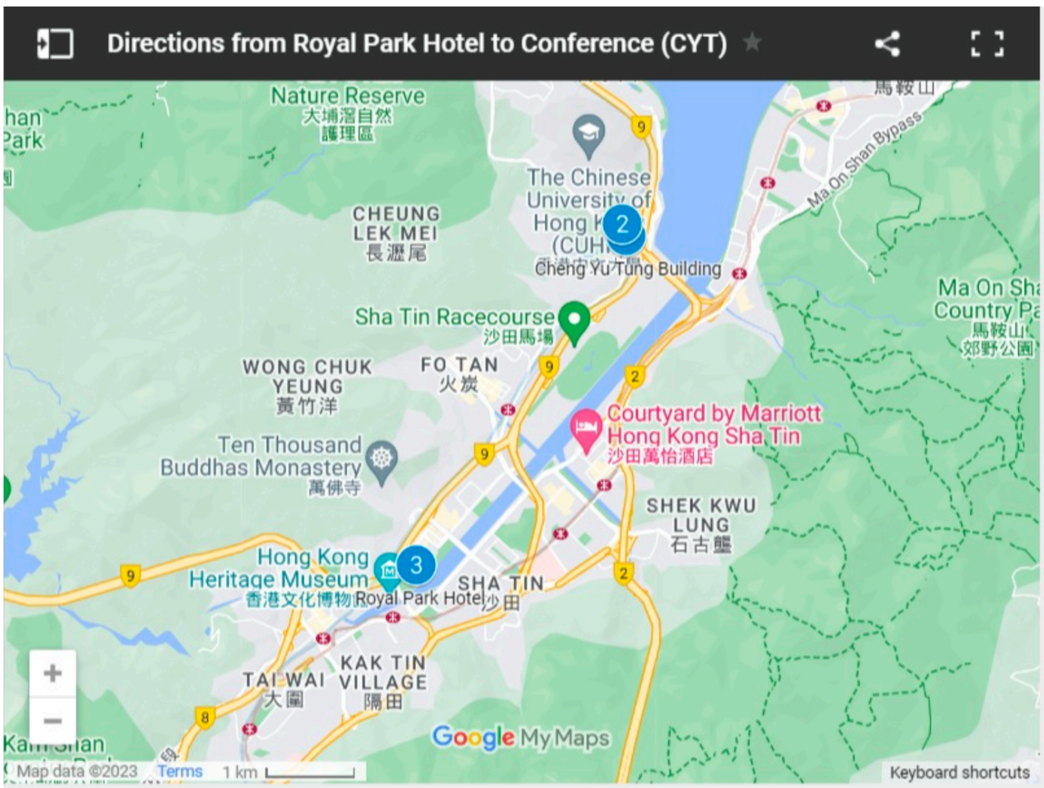
Point 1: Conference Venue: Cheng Yu Tung (CYT) Building

Point 2: University Station (Exit B)

Point 3: ALVA Hotel

From Royal Park Hotel to Conference (CYT)

From Royal Park Hotel [Point 3] to the conference [Point 1], one can take MTR "East Rail Line" at the station "Shatin". Take the train in the direction "Lo Wo" or "Lok Ma Chau", get off at the station "University", the CYT Building is near the "Exit B".



Point 1: Conference Venue: Cheng Yu Tung (CYT) Building

Point 2: University Station (Exit B)

Point 3: Royal Park Hotel

Lunch and Banquet arrangement

Lunch: Participants will be provided with Lunch Tickets, on which one of the following two options is specified: either the "Stage Restaurant" at Level 3 of the Conference Venue (CYT Building), or the "Chung Chi College Staff Club Clubhouse" on campus (see the map below). Please attend the lunch at the venue specified on your lunch ticket.

Banquet: The Conference Banquet will be held on Wednesday 19:00 - 21:00 at the restaurant ClubONE on the PARK: 12, Science Park W Ave, Science Park, Hong Kong.

Shuttle Bus will pick up guests at 18:15 at the roundabout between CYT building and Hyatt Hotel. For return, shuttle bus will depart from ClubONE restaurant at 21:30.

Conference Schedule

Overall Schedule

Monday	Tuesday	Wednesday	Thursday	Friday
8:30 – 8:50 Registration				
9:00 – 9:50 Bjorn Engquist	9:00 – 9:50 Barbara Wohlmuth	9:00 – 9:50 Qiang Du	9:00 – 9:50 Chi-Wang Shu	9:00 – 9:50 Jan Hesthaven
9:50 – 10:40 Masahiro Yamamoto	9:50 – 10:40 Zuowei Shen	9:50 – 10:40 Mete Soner	Coffee Break	Closing Ceremony + Coffee
Coffee Break	Coffee Break	Group Photo + Coffee	10:20 – 12:20 MS01(III)/03(II)/04(IV)/09(II)/10(II) (4 talks session)	10:20 – 12:20 MS08(II)/09(III)/05(II)/15(II)/18(I) (4 talks session)
11:00 – 12:30 MS01(I)/11(I)/12(I)/14(I)/17(I) (3 talks session)	11:00 – 12:30 MS02(I)/06(I)/12(II)/14(II)/15(I) (3 talks session)	11:00 – 12:30 MS01(II)/02(II)/04(III)/11(III)/12(III)/16(I) (3 talks session)		
12:30 – 14:30 Lunch	12:30 – 14:30 Lunch	12:30 – 14:30 Lunch	12:30 – 14:30 Lunch	12:30 – 14:30 Lunch
14:30 – 15:20 Jong-Shi Pang	14:30 – 15:20 Habib Ammari	Free Afternoon + 15:30-17:30 HKSIAM Bi-Annual Meeting	14:30 – 15:20 Kai King Chan	14:30 – 16:30 MS06(II)/14(III)/16(III)/18(II)/19 (4 talks session)
Coffee Break	Coffee Break		Coffee Break	
15:50 – 17:50 MS03(I)/04(I)/07(I)/11(II)/13 (4 talks session)	15:50 – 17:50 MS04(II)/07(II)/09(I)/10(I)/17(II) (4 talks session)		15:50 – 17:50 MS03(III)/05(I)/08(I)/10(III)/16(II) (4 talks session)	
		19:00 – 21:00 Banquet		

The Second HKSIAM Biennial Meeting

Monday, 28th August 2023

8:30 - 9:00	Registration and Welcome Talk				
9:00 - 10:40	Plenary Session LT1 Chair: Jun Zou	Bjorn Engquist Global convergence in unconstrained stochastic optimization			
		Masahiro Yamamoto Recent results on uniqueness and stability for inverse problems for parabolic, Schrödinger and hyperbolic equations			
10:40 - 11:00	Coffee Break				
11:00 - 12:30	MS01, Room 201	MS12, Room 202	MS14, Room 203	MS17, Room 209A	MS11, Room 209B
	Yayan Lu	Xiaochuan Tian	Huibin Chang	Lina Zhao	David Gu
	Yi Zhu	Xiaobo Yin	Faming Fang	Kyungkeun Kang	Na Lei
	Guochuan Thiang	Yufeng Nie	Jun Liu	Seungchan Ko	Xiaoqun Zhang
12:30 - 14:30	Lunch Break				
14:30 - 15:20	Plenary Session LT1 Chair: Bjorn Engquist	Jong-Shi Pang On the minimization of piecewise functions: pseudo stationarity			
15:20 - 15:50	Coffee Break				
15:50 - 17:50	MS03, Room 201	MS04, Room 202	MS07, Room 203	MS11, Room 209A	MS13, Room 209B
	Jing Zhang	Zhi Zhou	Jiang YANG	Min Zhang	Andrej Brojatsch
	Yiqing Lin	Dong Wang	Beiping Duan	Gunay Dogan	Andreas Hauptmann
	Leonard Wong	Junxiong Jia	Seungchan Ko	Emil Saucan	Zeljko Kereta
	Shuoqing Deng	Marco Inglesia	Zhi Zhou		Remo Kretschmann

The Second HKSIAM Biennial Meeting

Tuesday, 29th August 2023

9:00 - 10:40	Plenary Session LT1 Chair: Qiang Du	Barbara Wohlmuth Multi-physics models with mixed dimensions: Bio-medical and seismic applications			
		Zuowei Shen Deep Approximation via Deep Learning			
10:40 - 11:00	Coffee Break				
11:00 - 12:30	MS02, Room 201	MS06, Room 202	MS12, Room 203	MS14, Room 209A	MS15, Room 209B
	Atsushi Kawamoto	Shubin Fu	James Scott	Qiyu Jin	Xue Jiang
	Zhiyuan Li	Changqing Ye	Jieqiong Zhang	Zhi LI	Xiaokai Yuan
		Shan Jiang	Kuang Huang	Meng Ding	Maohui Lyu
12:30 - 14:30	Lunch Break				
14:30 - 15:20	Plenary Session LT1 Chair: Barbara Wohlmuth	Habib Ammari Mathematical foundations of subwavelength physics			
15:20 - 15:50	Coffee Break				
15:50 - 17:50	MS04, Room 201	MS07, Room 202	MS17, Room 203	MS09, Room 209A	MS10, Room 209B
	Tim Jahn	Kai Zhang	Yong Yu	In-Jee Jeong	Min Tang
	Sui Tang	Yonglin Li	Shu Ma	Donghyun Lee	Shuai Su
	Lingyun Qiu	Wei Gong	Hantaek Bae	Jinmyoung Seok	Chunmei Su
	Tianhao Hu	Shengfeng Zhu	Qiqi Rao	Young-Pil Choi	Jie Du

The Second HKSIAM Biennial Meeting

Wednesday, 30th August 2023

9:00 - 10:40	Plenary Session LT1 Chair: Habib Ammari	Qiang Du Nonlocal models on bounded domains: formulation, analysis and computation Mete Soner Eikonal Equations on Wasserstein Spaces				
10:40 - 11:00	Group Photo + Coffee Break					
11:00 - 12:30	MS01, LT1	MS02, 201	MS04, 202	MS11, 203	MS12, 209A	MS16, 209B
	Sanghyeon Yu	Michel Cristofol	Ye Zhang	Haixia Liu	Jiwei Zhang	Shuting GU
	Bryn Davies	Shumin Li	Yuling Jiao	Xiaohao Cai	Zuoqiang Shi	Li Shan
	Jiayu Qiu		Fengru Wang	Daoping Zhang		Shuonan Wu
12:30 - 14:30	Lunch Break					
14:30 - 19:00	Free Afternoon HKSIAM Bi-Annual Meeting (15:30-17:30 LT1A)					
19:00 - 21:00	Banquet					

The Second HKSIAM Biennial Meeting
Thursday, 31st August 2023

9:00 - 9:50	Plenary Session LT1 Chair: Jan Hesthaven	Chi-Wang Shu Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions			
9:50 - 10:20	Coffee Break				
10:20 - 12:20	MS01, Room 201	MS03, Room 202	MS04, Room 203	MS09, Room 209A	MS10, Room 209B
	Ludovic Tangpi	Jun Lai	Ruchi Guo	Jin Woo Jang	Kailiang Wu
	Chao Zhou	Tao Yin	Wenbin Li	Gyounghun Ko	Yong Liu
	Camilo Hernandez	Wangtao Lu	Xiliang Lu	Tak Kwong Wong	Qian Huang
		Erik Orvehed Hiltunen	Guozhi Dong	Seok-Bae Yun	Liwei Lu
12:30 - 14:30	Lunch Break				
14:30 - 15:20	Plenary Session LT1 Chair: Mete Soner	Kai King Chan Application of Mathematical Geometric Analysis on Pavement Defect Detection			
15:20 - 15:50	Coffee Break				
15:50 - 17:50	MS03, Room 201	MS05, Room 202	MS16, Room 203	MS10, Room 209A	MS08, Room 209B
	Alex Tse	Bastian von Harrach	Xingding Chen	Haijin Wang	Tao Xiong
	Yufei Zhang	Yifeng Xu	Shubin Fu	Qi Tao	Chang Yang
	Yunzhang Li	Yi-Hsuan Lin	Ying Yang	Hui Yu	Shengtong Liang
	Chenchen Mou	Siyu Cen	Haibiao Zheng	Jiwei Li	Yiwen Lin

The Second HKSIAM Biennial Meeting
Friday, 1st September 2023

9:00 - 9:50	Plenary Session LT1 Chair: Chi-Wang Shu	Jan Hesthaven Neural Networks as Closure Models			
9:50 - 10:20	Closing Ceremony + Coffee Break				
10:20 - 12:20	MS08, Room 201	MS09, Room 202	MS05, Room 203	MS15, Room 209A	MS18, Room 209B
	Kunlun Qi	Gayoung An	Kwancheol Shin	Lei Zhang	Chenglong Bao
	Xiaoqian Xu	Sungbin Park	Daijun Jiang	Kristoffer Linder-Steinlein	Huangxin Chen
	Xuda Ye	Gi-Chan Bae	Jiahua Jiang	Jose Pinto	Hao Luo
		Beomjun Choi	Chang Min Hyun	Bo Wang	Sam Reynolds
12:30 - 14:30	Lunch Break				
14:30 - 16:30	MS06, Room 201	MS14, Room 202	MS16, Room 203	MS18, Room 209A	MS19, Room 209B
	Yat Tin Chow	Zhi-Feng Pang	Chen Cui	Bin Shi	Jinqiao Duan
	Yi Yu	Youwei Wen	Jie Peng	Jingrong Wei	Ting Gao
	Fuchen Chen	Tingting Wu	Feng Wang	Yuan Yao	Lihu Xu
	Sai-Mang Pun	Ke Yin	Junxian Wang	Xinyuan Zhao	Xiaopeng Chen

Plenary Talks

Habib Ammari

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Mathematical foundations of subwavelength physics

The ability to manipulate and control waves at scales much smaller than their wavelengths is revolutionizing nanotechnology. The speaker will present a mathematical and numerical framework for this emerging field of physics and elucidate its duality with condensed matter theory.

Kai King Chan

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Application of Mathematical Geometric Analysis on Pavement Defect Detection

Traditional visual inspection of pavement defects is inefficient. Some manufacturers used AI techniques from 2D pavement photos collected by a surveying vehicle but the result are still not satisfactory as outdoor environment is complicated. With the high-resolution 3D pavement point cloud acquired by Freetech's scanning system, a new detection approach, mathematical geometric analysis, is implemented.

Qiang Du

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Nonlocal models on bounded domains: formulation, analysis and computation

There has been much interest in nonlocal models associated with a finite horizon parameter that characterizes the effective range of nonlocal interactions. For such models defined on a bounded domain, it is an important topic to study their mathematical formulations near or at the domain boundary. This lecture will mainly offer an overview of related works. We will discuss a few possibilities for imposing suitable local or nonlocal boundary conditions to complement the nonlocal equations prescribed in the interior of the domain. We will also present recent development in the mathematical and numerical analysis of the resulting problems and their impact on practical applications.

Bjorn Engquist

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Global convergence in unconstrained stochastic optimization

A stochastic component is often added for global convergence when the objective function is not convex to escape from local minima. We propose an algorithm with adaptive state dependent variance. It can be used in an exploration phase followed by a faster converging algorithm in its basin of attraction. It can also be used as a stand-alone algorithm with provable convergence of algebraic rate. This replaces earlier results with logarithmic rate.

Jan S. Hesthaven

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Neural Networks as Closure Models

The development of closures is a classic problem in many models across continuum mechanics with perhaps the most prominent example being the long standing effort to develop turbulence models to account for unresolved dynamics in Reynolds averaged models. An even more fundamental challenge is the closure of the hierarchy of models arising when taking moments of the Boltzmann equation. Historically, such closures have relied on physical insight and, in many cases, parametric fitting of models to experimental data. However, such techniques are highly problem dependent and prone to failure when used outside their intended application areas.

For computational models, a different closure problem can be identified by seeking to account for the accumulated impact of truncation errors – a problem that is easily realized but which generally can be addressed only by theoretically and computationally studying model convergence to quantify the impact of such errors. In this talk we discuss how different types of neural networks can be used to build closure models for computational models to account for unresolved dynamics or unknown physics. We consider first the use of LSTM-networks as closure models for time-dependent parameterized problems, modeled using reduced basis methods. The Mori-Zwansig formulation offers insight into the closure problem and the appropriate design of the LSTM network and we demonstrate the benefits on a number of problems of increasing complexity. We highlight how this approach is particularly natural for reduced order modeling for dynamic problems and illustrate the enhanced predictive accuracy.

As a second example we discuss the use of a feed-forward network to enhance the accuracy of the modeling of dynamic stall of a drone (UAV). We presents an aerodynamic force prediction model based on a small number of pressure sensors located on the surface of UAV. The model uses a reduced basis method to make a rough prediction and a nonlinear correction term for accuracy improvement by using sensor information. The model is tested on numerical and experimental dynamic stall data and shown to be able to make fast and accurate predictions of the aerodynamic forces using just a few pressure sensors.

We conclude with a few perspectives on the use of machine learning techniques to enhance the accuracy of complex simulations by allowing to account for the impact of unresolved scales or unknown physics by fusing the model with additional information, be it computational or experimental. This work is done in collaboration with J. Duan (EPFL, CH), N. Ripamonti (EPFL, CH), and Q. Wang (Beijing CSRC, PRC)

Jong-Shi Pang

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On the minimization of piecewise functions: pseudo stationarity

We discuss the minimization of a class of (possibly discontinuous) piecewise functions that has many applied sources. We define the concept of a pseudo stationary solution and present two approaches for its computation: an epigraphical approach and a noniffier approach; the latter is the nonsmooth generalization of the mollifier approach of Norkin and Wets that lies at the foundation of smooth approximation of discontinuous minimization.

Joint work with Ying Cui (University of Minnesota) and Junyi Liu (Tsinghua University).

Zuowei Shen

National University of Singapore, Singapore
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Deep Approximation via Deep Learning

The primary task of many applications is approximating/estimating a function through samples drawn from a probability distribution on the input space. The deep approximation is to approximate a function by compositions of many layers of simple functions, that can be viewed as a series of nested feature extractors. The key idea of deep learning network is to convert layers of compositions to layers of tuneable parameters that can be adjusted through a learning process, so that it achieves a good approximation with respect to the input data. In this talk, we shall discuss mathematical theory behind this new approach and approximation rate of deep network; we will also show how this new approach differs from the classic approximation theory, and how this new theory can be used to understand and design deep learning networks.

Chi-Wang Shu

Brown University, USA
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Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions

When solving partial differential equations, finite difference methods have the advantage of simplicity, however they are usually only designed on Cartesian meshes. In this talk, we will discuss a class of high order finite difference numerical boundary condition for solving hyperbolic Hamilton-Jacobi equations, hyperbolic conservation laws, and convection-diffusion equations on complex geometry using a Cartesian mesh. The challenge results from the wide stencil of the interior high order scheme and the fact that the boundary may not be aligned with the mesh. Our method is based on an inverse Lax-Wendroff procedure for the inflow boundary conditions coupled with traditional extrapolation or weighted essentially non-oscillatory (WENO) extrapolation for outflow boundary conditions. The schemes are shown to be high order and stable, under the standard CFL condition for the inner schemes, regardless of the distance of the first grid point to the physical boundary, that is, the "cut-cell" difficulty is overcome by this procedure. Recent progress in nonlinear conservation laws with sonic points, and a conservative version of the method, will be discussed. Numerical examples are provided to illustrate the good performance of our method.

Mete Soner

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Eikonal Equations on Wasserstein Spaces

Mean-field or McKean-Vlasov type optimal control is closely related to the exciting program of mean-field games as initiated by Larry and Lions. Dynamic programming approach to these control problems result in nonlinear partial differential equations on the space of probability measures. These equations not only require the solution to be differentiable but impose further regularity on the derivatives which are being on the dual of the set of measures are also functions themselves. Despite these difficulties, several approaches to characterize the value function of the control problems as the unique appropriate weak solutions have been developed. In this talk, I will first introduce the mean field games through an interesting example of Kuramoto type synchronization. Then, I will extend this example to a general setting and prove uniqueness of for a class of equations that are analogous to classical Eikonal equations. This talk is based on joint works with Rene Carmona and Qinxin Yan of Princeton, and Quentin Cormier of INRIA.

Barbara Wohlmuth

Technical University of Munich, Germany
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Multi-physics models with mixed dimensions: Bio-medical and seismic applications

In this talk, we consider bi-directionally and one-directionally coupled mixed dimensional problems. The resulting systems result from a dimension reduction. Of special interest are bi-directionally coupled problems of co-dimension two. Due to the lack of suitable trace results the coupling is performed via a suitable lifting. As example, we consider the well-posedness analysis of a highly non-linear time dependent 1D-3D PDE system and track the influence of the coupling parameter. The mathematical model is then enriched by stochastic components mimicking the dynamic growth of capillary systems in case of angiogenesis. The 1D flow and transport process is given by nonlinear and linear hyperbolic PDEs suitable for larger arteries with highly elastic vessel walls and smaller ones with stiffer vessel walls, respectively. Numerical results illustrate the effect of the random growth algorithm but also how the system can be closed by 0D models if the flow in the arterioles and the peripheral circulation is not fully resolved. Additionally, we consider two examples of one-directionally coupled 1D-3D problems. The conceptual difference between the two settings is which dimension affects the solution of the other one. The first example shows how the ground-motion triggers the vibration of 1D structures. The ground-motion itself is realized by a 3D elasto-acoustic coupled wave-propagation model, discretized in space by a spectral multi-patch DG approach. The motion is initiated by a seismic source given by a fault plane calibrated from the magnitude Izmir 2020 earthquake. A challenging task is to resolve the multiple length scales that are present due to the huge differences in size between the close by dam building structure and the area of interest. The second example illustrates the influence of a 1D folded coil within an aneurysm on the 3D non-Newtonian blood flow discretized by a Lattice Boltzmann scheme. Alternatively to the fully resolved 2D flow one can use a one domain approach in which homogenized porosity and permeability enters into a porous media approach.

This talk is partly joint work with J.T. Oden and P. Jha (University of Texas at Austin), I. Mazzieri (Politecnico di Milano), M. Stupazzini (Munich RE), V. Nicolić (Radboud University) and M. Fritz, G. Gutierrez, T. Köppl, M. Muhr, N. Nebulishvili, A. Peiraviminaei, A. Wagner (TU Munich).

Masahiro Yamamoto

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Recent results on uniqueness and stability for inverse problems for parabolic, Schrödinger and hyperbolic equations

As a principal inverse problem, we can refer to the determination of spatially varying coefficients for evolutionary partial differential equations by a single observation on subboundary. The mathematical issues are the uniqueness and the stability, and since Bukhgeim and Klibanov [1], such researches have been developed and many results are available. Here we refer only to Bellassouend and Yamamoto [2], Isakov [3], Klibanov and Timonov [4], Yamamoto [5], [6].

However, the uniqueness and the stability are open in several important cases. We show some answers to such open problems.

Let $\Omega \subset \mathbb{R}^d$ be a bounded smooth domain, $x = (x_1, ..., x_d) \in \mathbb{R}^d$, ν be the unit outward normal vector to $\partial\Omega$. Moreover let $\gamma \subset \partial\Omega$ be an arbitrarily chosen subboundary, $0 \le t_0 \le T$ be arbitrarily fixed.

(I) Inverse parabolic problems
For $\partial_t u(x,t) = \Delta u(x,t) + p(x)u(x,t)$ in $\Omega \times (0,T)$, we consider the determination of $p(x)$, $x \in \Omega$ by data

$$(u|_{\gamma\times(0,T)},\ \nabla u|_{\gamma\times(0,T)},\ u(\cdot,t_0)|_\Omega).$$

Only for the case of $0 < t_0 < T$, the uniqueness and the stability are proved (e.g., [3], [5], [6]). The problems are not solved for $t_0 = 0$ and $t_0 = T$.

We obtained

- the uniqueness for the one-dimensional case $\Omega := (0,\ell)$, that is, $p(x)$, $0 < x < \ell$ is uniquely determined by $(u(0,t),\ \partial_x u(0,t),\ u(x,0))$ with $0 < t < T$ and $x \in (0,\ell)$.
- the uniqueness by data $(u|_{\gamma\times(0,T)},\ \nabla u|_{\partial\Omega\times(0,T)},\ u(\cdot,0)|_\Omega)$, provided that the initial value $u(\cdot,0)$ is sufficiently smooth.
- the Lipschitz stability by data $(u|_{\gamma\times(0,T)},\ \nabla u_{\partial\Omega\times(0,T)},\ u(\cdot,T)|_\Omega)$.

(II) Unique continuation for the Schrödinger equation
Let $\gamma \subset \partial\Omega$ and $T > 0$ be arbitrarily chosen. Then, for $\sqrt{-1}\partial_t u + \Delta u = p(x)u$ in $\Omega \times (0,T)$, we show that if $u = \partial_\nu u = 0$ on $\gamma \times (0,T)$, then $u = 0$ in $\Omega \times (0,T)$. Moreover we apply it to inverse source problems.

(III) Inverse hyperbolic problems.
We consider

$$\begin{cases} \partial_t^2 u(x,t) = \Delta u(x,t) - p(x)u(x,t) + \delta(x_1)\delta'(t), \\ \qquad\qquad\qquad\qquad x_1 > 0,\ (x_2,...,x_d) \in \mathbb{R}^{d-1},\ t > 0, \\ u|_{t<0} = 0, \end{cases}$$

where $x = (x_1, ..., x_d)$, $d \geq 2$, δ, δ' denote the Dirac delta function and its derivative. Then we discuss the Lipschitz stability in determining $p(x)$ by data on $\partial\Omega \times (0, T)$.

If we assume that $|u(\cdot, 0)|$ is strictly positive, then we already have proved the Lipschitz stability (the proof is found in e.g., [2]). In the case of the zero initial values, assuming an impulsive input $\delta(x_1)\delta'(t)$ concentrating on the hyperplane $x_1 = 0$ at $t = 0$, we can obtain the stability.

The talk is based on joint articles with Professor Oleg Y. Imanuvilov (Colorado State University).

[1] A.L. Bukhgeim and M.V. Klibanov, *Global uniqueness of a class of multidimensional inverse problems*, Sov. Math.- Dokl. **24** (1981) 244-247.

[2] M. Bellassoued and M. Yamamoto, *Carleman Estimates and Applications to Inverse Problems for Hyperbolic Systems*, Springer Japan, Tokyo, 2017.

[3] V. Isakov, *Inverse Source Problems*, American Mathematical Society, Providence, RI, 1990.

[4] M.V. Klibanov and A.A., Timonov, *Carleman Estimates for Coefficient Inverse Problems and Numerical Applications*, VSP, Utrecht, 2004.

[5] M. Yamamoto, *Carleman estimates for parabolic equations and applications*, Inverse Problems **25** (2009) 123013.

[6] M. Yamamoto, *Introduction to Inverse Problems for Evolution Equations: Stability and Uniqueness*, Lecture Note at University of Rome Tor Vergata, 2021.

Minisymposium Talks

MS 01: Mathematical theory and computational methods for wave propagation in structured media

Organizer:

Hai Zhang
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Session 1 (Aug 28 11:00-12:30)

Ya Yan Lu
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Propagating Bound states in the continuum in periodically perturbed slabs

In a periodic dielectric slab that is invariant in x, periodic in y, perpendicular to z, and surrounded by air, there are continuous families of guided modes below the light line, i.e., $k < |\beta|$, where k is the free space wavenumber and β is the Bloch wavenumber. A bound state in the continuum (BIC) is a special guided mode above the light line, i.e., $k > |\beta|$. BICs have many important applications in photonics, but the existence of propagating BICs (with a nonzero β) has not been established rigorously. In this paper, we show the existence of two types of BICs in periodically perturbed slabs with a relative permittivity given by $\epsilon(y,z) = \epsilon_{\rm sl}(z) + \delta F(y,z)$, where $\epsilon_{\rm sl}$ is the relative permittivity of a uniform slab, F is a real even function of y and z, and δ is real near zero. In the first case, BICs are found near crossing points of folded bands of the uniform slab. In the second case, BICs are located near special points on folded bands related to the perturbation profile F. Our theory is validated by numerical examples. This is a joint work with Amgad Abdrabou and Wangtao Lu

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Conically degenerate spectral points of the periodic Schrodinger operator

Conical spectral points on the dispersion bands are the origin of many novel topological phenomena including various topological phases. I will first review recent mathematical theories on these points, especially Fefferman & Weinstein's results (JAMS 2012) on 2D Dirac points which paved the way for rigorous justifications of such points. Then I will focus on our recent progress in constructing 3-fold Weyl points at which two energy bands intersect conically with an extra band sandwiched in between. We give the existence of such points in the spectrum of the 3-dimensional Schrodinger operator $H = \Delta + V(x)$ with $V(x)$ being in a large class of periodic potentials. This is the first rigorous result on the existence of 3-fold Weyl points for a broad family of 3D continuous Schrodinger equations. Our result extends Fefferman-Weinstein's pioneering work to a higher dimension and higher multiplicities. This talk is mainly based on the joint work with H. Guo and M. Zhang at Tsinghua university.

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Large-scale obstruction to atomic limits

For 2D Schrödinger operators with periodic potential, it was realized in the late 2000s that certain spectral subspaces cannot admit a periodic orthonormal basis of exponentially localized wavefunctions, and therefore have no sensible "periodic atomic/tight-binding limit". The obstruction is usually attributed to a topological Chern number of the corresponding Bloch band. I will explain that the obstruction is much more serious and fundamental — topological insulators on a general manifold do not admit "atomic limits", periodic or otherwise. Mathematically, the obstruction is a large-scale index, and is realized by quantum Hall insulators with very unusual wave propagation.

[1] Matthias, Ludewig and Guo Chuan, Thiang, *Large-scale geometry obstructs localization*, J. Math. Phys. 63, 091902, 2022

[2] Matthias, Ludewig and Guo Chuan, Thiang, *Good Wannier bases in Hilbert modules associated to topological insulators*, J. Math. Phys. 61, 061902, 2020

Session 2 (Aug 30 11:00-12:30)

Sanghyeon Yu
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A unified approach to the field concentration problem

Composite materials shows the high field concentration when the inclusions have geometric singularities in their boundaries. This phenomenon has many practical applications in imaging, spectroscopy, and meta-materials. In this talk, we discuss a new way of tackling the field concentration problem via the spectral analysis of the Neumann-Poincare operator. We focus on two kinds of important singularities: nearly touching surfaces and high curvature points.

Bryn Davies
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Periodic approximants of one-dimensional Fibonacci quasicrystals

Quasicrystals have exotic spectra that are challenging to understand and are the basis of several longstanding problems in spectral analysis. In the applied physical literature, however, there is a common tendency to approximate the spectrum of a quasicrystal with a periodic approximation, known as a supercell. In this work, we prove that supercell approximations converge in the sense that there are spectral gaps that exist for all sufficiently large supercells, in the case of Fibonacci quasicrystals. This shows that supercell approximations give accurate predictions of the main spectral gaps. This is based on characterising the growth of the underlying recursion relation and corroborates the existence of previously observed super band gaps. We demonstrate our results through applications to simple one-dimensional wave systems, including a strategy for creating localised edge modes.

[1] B. Davies and L. Morini *Super band gaps and periodic approximants of generalised Fibonacci tilings.* arXiv preprint arXiv:2302.10063, 2023.

Jiayu Qiu
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Mathematical theory for the interface mode in a waveguide bifurcated from a Dirac point

We present our resent work in which we prove the existence of a bound state in a waveguide that consists of two semi-infinite periodic structures separated by an interface. The two periodic structures are perturbed from the same periodic medium with a Dirac point and they possess a common band gap enclosing the Dirac point. The bound state, which is called interface mode here, decays exponentially away from the interface with a frequency located in the common band gap and can be viewed as a bifurcation from the Dirac point. Using the layer potential technique and asymptotic analysis, we first characterize the band gap opening for the two perturbed periodic media and derive the asymptotics of the Bloch modes near the band gap edges. By formulating the eigenvalue problem for the waveguide with two semi-infinite structures using a boundary integral equation over the interface and analyzing the characteristic values of the associated boundary integral operator, we prove the existence of the interface mode for the waveguide when the perturbation of the periodic medium is small.

[1] Jiayu Qiu, Junshan Lin, Peng Xie, and Hai Zhang, *Mathematical theory for the interface mode in a waveguide bifurcated from a dirac point.* 2023.

Session 3 (Aug 31 10:20-12:20)

Jun Lai
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Selective focusing of multiple elastic particles

Inverse elastic scattering has a wide range of applications in geophysical exploration, non-destructive testing etc. This talk is concerned with the inverse time-harmonic elastic scattering of multiple small and well-resolved particles. We apply the so-called D.O.R.T. method to this case so that selective focusing can be achieved on each particle with far field measurements. A rigorous mathematical justification for the related eigenfunctions of the time-reversal operator to the location of the particles is presented based on the asymptotic analysis of the far field operator and decaying property of oscillatory integrals. Numerical experiments are given to verify the theoretical result.

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Multiple-scattering frequency-time hybrid solver for the wave-equation problems

This talk will present our recent works on the Fourier transform based boundary integral equation method for solving the time-domain wave equation problems. A well accepted principle in wave physics indicates that solutions of the wave equation propagate at the speed of sound, and that the wave field vanishes identically before the arrival of a wavefront. For the problem exterior to multiple closed/open obstacles, relying on the Huygens-like domain-of-influence, a new multiple scattering strategy is proposed to decompose the original problem into a sequence of wave equation sub-problems, each of which is only imposed on a single closed/open obstacle. To treat the interior problem, based on a partitioning of the closed curve of the obstacle into an arbitrary number of overlapping open arcs and well-designed iterated boundary conditions, the multiple scattering strategy allows to decompose the interior problem into a sequence of exterior wave equation sub-problems. At each step of the multiple scattering, the solutions to sub-problems can be evaluated in parallel and by means of the Fourier transform, each one can be reduced to a Helmholtz frequency-domain problem for which the corresponding boundary integral equation, that are uniquely solvable for all real frequencies, can be constructed. Relying on the proposed multiple scattering strategy and a Fourier transform algorithm that delivers numerically dispersionless, spectrally-accurate time evolution, novel multiple-scattering frequency-time hybrid integral equation solvers are developed. Numerical examples will be presented to demonstrate the accuracy and efficiency of the proposed methodology.

[1] O.P. Bruno, T. Yin, *Multiple-scattering frequency-time hybrid solver for the wave equation in interior domains*, to appear in Mathematics of Computation.

[2] G. Bao, O.P. Bruno, T. Yin *Multiple-scattering frequency-time hybrid integral equation solver for the wave equation problems with bounded obstacles*, in preparation.

Wangtao Lu
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Mathematical theory for electromagnetic scattering resonances and field enhancement in a subwavelength annular gap

In this talk, we shall present a mathematical theory for electromagnetic scattering resonances in a subwavelength annular hole embedded in a metallic slab, with the annulus width $h \ll 1$. The model is representative among many 3D subwavelength hole structures, which are able to induce resonant scattering of electromagnetic wave and the so-called extraordinary optical transmission. We develop a multiscale framework for the underlying scattering problem based upon a combination of the integral equation in the exterior domain and the waveguide mode expansion inside the tiny hole. The matching of the electromagnetic field over the hole aperture leads to a sequence of decoupled infinite systems, which are used to set up the resonance conditions for the scattering problem. By performing rigorous analysis for the infinite systems and the resonance conditions, we characterize all the resonances in a bounded domain over the complex plane. It is shown that the resonances are associated with the TE and TEM waveguide modes in the annular hole, and they are close to the real axis with the imaginary parts of order $O(h)$. We also investigate the resonant scattering when an incident wave is present. It is proved that the electromagnetic field is amplified with order $O(1/h)$ at the resonant frequencies that are associated with the TE modes in the annular hole. On the other hand, one particular resonance associated with the TEM mode can not be excited by a plane wave but can be excited with a near-field electric dipole source, leading to field enhancement of order $O(1/h)$. This is based on a joint work [1] with Junshan Lin and Hai Zhang.

[1] J. Lin, W. Lu and H. Zhang, *Mathematical theory for electromagnetic scattering resonances and field enhancement in a subwavelength annular gap*, Multiscale Modeling and Simulation, 2023

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Band structure and Dirac points of real-space quantum optics in periodic media

The field of photonic crystals is almost exclusively based on a Maxwell model of light. While often an effective model, it is natural to study such systems under a quantum-mechanical photon model instead. In the real-space parametrization, interacting photon-atom systems are governed by a system of nonlocal partial differential equations. In this talk, we study resonant phenomena of such systems. Using integral equations, we phrase the resonant problem as a nonlinear eigenvalue problem. In a setting of high-contrast atom inclusions, we obtain fully explicit characterizations of resonances, band structure, and Dirac cones. Additionally, we present a strikingly simple relation between the Green's function of the nonlocal equation and that of the local (Helmholtz) equation. Based on this, we are able to achieve highly efficient numerical calculations of band structures of interacting photon-atom systems.

[1] E. O. Hiltunen, J. Kraisler, J. C. Schotland and M. I.Weinstein, *Nonlocal PDEs and Quantum Optics: Bound States and Resonances.* arxiv:2306.10431, 2023.

MS 02: Inverse problems and related topics for evolution equations

Organizer:

Shumin Li
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Session 1 (Aug 29 11:00-12:30)

Atsushi Kawamoto
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Homogenization and Inverse Problems for Fractional Diffusion Equations

We consider the homogenization for time-fractional diffusion equations in a periodic structure and we also discuss its applications to inverse problems. First, we derive the homogenized time-fractional diffusion equations. Next, we show the stability in determining a constant diffusion coefficient by minimum data. Moreover, we investigate the inverse problems of estimating the homogenized diffusion coefficient by data for the periodic structure. This talk is based on a joint work with M. Machida and M. Yamamoto.

[1] Atsushi, Kawamoto; Manabu, Machida and Masahiro, Yamamoto *Homogenization and Inverse Problems for Fractional Diffusion Equations.* arXiv:2303.01798, 2023

Zhiyuan Li
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Hopf lemma for fractional diffusion equations and application to inverse problem

In this talk, we consider an inverse problem of simultaneously determining the spatially dependent source term and the Robin boundary coefficient in a time fractional diffusion equation, with the aid of extra measurement data at a subdomain near the accessible boundary. Firstly, the spatially varying source is uniquely determined in view of the unique continuation principle and Duhamel principle for the fractional diffusion equation. The Hopf lemma for a homogeneous time-fractional diffusion equation is proved and then used to prove the uniqueness of recovering the Robin boundary coefficient.

[1] Daijun, Jiang and Zhiyuan, Li *Hopf's lemma and uniqueness of simultaneously determining source profile and Robin coefficient in a fractional diffusion equation by interior data.* preprint, 2023

Session 2 (Aug 30 11:00-12:30)

Michel Cristofol
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An inverse problem for a generalized FitzHugh-Nagumo type system

In this talk we consider the inverse problem of determining simultaneously three coefficients of a FitzHug-Nagumo type system defined in a bounded domain. Using the strategy by Carleman estimate, we establish an Holder type estimate of these coefficients by the observation of a finite number of measurements of only one component of the system.

Shumin Li
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Inverse problems for a singular parabolic equation with variable coefficients

We consider inverse problems for the equation: $\partial_t u(x,t) - \nabla \cdot (P(x)\nabla u) - \frac{\mu}{|x|^2} u(x,t) = f(x,t), \quad (x,t) \in \Omega \times (0,T)$. The main difficulty here as compared with the existing result is that there is a singular potential in the equation. A local logarithmic stability estimate for an inverse problem of the determination of the coefficient $P(x)$ is obtained using the method of Carleman estimates. Our proof relies on the Bukhgeim-Klibanov method which was originated in [1] to prove inverse source or coefficient results.

[1] A. L., Bukhgeim and M. V., Klibanov MV *Uniqueness in the large of a class of multidiimensional inverse problems.* Dokl. Akad. Nauk SSSR, 1981

[2] Xue, Qin and Shumin, Li *Local logarithmic stability of an inverse coefficient problem for a singular heat equation with an inverse-square potential.* Appl. Anal., 2023

MS 03: Optimal control and mathematical finance

Organizers:

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Session 1 (Aug 28 15:50-17:50)

Jing Zhang
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Stochastic differential games with random coefficients and stochastic Hamilton-Jacobi-Bellman-Isaacs equations

In this paper, we study a class of zero-sum two-player stochastic differential games with the controlled stochastic differential equations and the payoff/cost functionals of recursive type. As opposed to the pioneering work by Fleming and Souganidis [Indiana Univ. Math. J., 38 (1989), pp. 293–314] and the seminal work by Buckdahn and Li [SIAM J. Control Optim., 47 (2008), pp. 444–475], the involved coefficients may be random, going beyond the Markovian framework and leading to the random upper and lower value functions. We first prove the dynamic programming principle for the game, and then under the standard Lipschitz continuity assumptions on the coefficients, the upper and lower value functions are shown to be the viscosity solutions of the upper and the lower fully nonlinear stochastic Hamilton-Jacobi-Bellman-Isaacs (HJBI) equations, respectively. A stability property

of viscosity solutions is also proved. Under certain additional regularity assumptions on the diffusion coefficient, the uniqueness of the viscosity solution is addressed as well.

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BSDEs with mean reflexion driven by marked point processes

This talk introduces our study on the backward stochastic differential equations driven by marked point processes (MPPs). The compensators of MPPs are not necessarily absolutely continuous with respect to the Lebesgue measure. In this framework, we consider this type of equations with constraints on the expectation of solutions, which is corresponding to the superhedging problem with risk constraints. The discussion on the solvability of such equations are under two groups of conditions: Lipschitz generators with L^2 terminal values and quadratic growth generators with exponentially integrable terminal values.

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Bregman-Wasserstein divergence

Optimal transport has found increasing use in quantitative finance, statistics and machine learning. For example, it provides loss functions for fitting models and uncertainty sets in distributionally robust optimization. While the 2-Wasserstein distance is the most common choice, it is not appropriate in all situations. In this talk we introduce the Bregman-Wasserstein divergence which is the optimal transport cost where the underlying cost function is a Bregman divergence. We discuss some geometric and computational aspects of the Bregman-Wasserstein divergence as well as some potential applications. Based on a joint work with Cale Rankin.

Shuoqing Deng
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On optimal time-consistent equilibrium stopping under aggregation of diverse discount rates

We revisit the collective decision making for a group under diverse discount rates. In the context of optimal stopping, we propose a smooth aggregation preference to incorporate all heterogeneous discount rates and an attitude function reflecting the aggregation weight similar to the smooth ambiguity preference in Klibanoff et al. (2005) when handling the model uncertainty. The optimal stopping problem renders to be time inconsistent, for which we develop an iterative approach using consistent planning and characterize all time-consistent equilibrium stopping polices as fixed points of an operator in the setting of one-dimensional diffusion processes. More importantly, we provide some sufficient conditions on both the underlying models and the attitude function such that the smallest equilibrium attains the optimal equilibrium and the attitude function is equivalent to the linear aggregation rule as of diversity neutral. Joint work with Xiang Yu and Jiacheng Zhang.

Session 2 (Aug 31 10:20-12:20)

Ludovic Tangpi
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Forward-backward propagation of chaos via displacement monotonicity

In this talk I will present quantitative convergence results for a class of mean field games with common noise and controlled volatility. The basic strategy we employ is the one introduced recently by Laurière and Tangpi – roughly speaking, we use a synchronous coupling argument to prove a "forward-backward propagation of chaos" result for the FBSDEs which characterize the (open-loop) equilibria of the N-player and mean field games.

Unlike in earlier works which have adopted this strategy, we do not require smallness conditions, and instead rely on monotonicity. In particular, (displacement) monotonicity of the Hamiltonian and the terminal cost allow us to establish a (uniform in N) stability estimate for the N-player FBSDEs, which implies the convergence result. The arguments are relatively simple, and flexible enough to yield similar results in the setting of mean field control and infinite horizon (discounted) mean field games.

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Convergence analysis on the particle systems with centralized control

This paper deals with the optimization problem of a class of controlled N-particle systems. We establish the regularity results, which is uniform in N , on the HJB equations corresponding to the N-particle system. The uniform regularity results are obtained by the stochastic maximum principle and the analysis on a Riccati type BSDE. Using the uniform regularity results, we show the convergence of value function and optimal control as the number N of particles tends to infinity, where the convergence rates are also given. This is based on a joint work with Huafu Liao, Alpar Meszaros and Chenchen Mou.

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Propagation of chaos for Schrödinger problems with interacting particles

The mean field Schrödinger problem (MFSP) is the problem of finding the most likely path of a McKean-Vlasov type particle with constrained initial and final configurations. It was first introduced by Backhoff et al. (2020), who studied its existence and long time behavior. This talk aims to show how ideas from propagation of chaos for backward particle systems allow to derive the MFSP as the (large population) limit of a sequence of classical Schrödinger problems among finite (but interacting) particles. The method rests upon the study of suitably penalized problems using stochastic control techniques, and it further allows to derive other interesting results on the MFSP. This talk is based on a joint work with Ludovic Tangpi.

Session 3 (Aug 31 15:50-17:50)

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Optimal Investment and Consumption with Epstein-Zin Stochastic Differential Utility and Proportional Transaction Costs

We study the optimal investment-consumption problem over the infinite horizon for an agent with Epstein-Zin stochastic differential utility preferences who invests in a Black-Scholes-Merton market. We assume that purchases and sales of the risky asset are subject to proportional transaction costs. We fully characterise all parameter combinations for which the problem is well-posed (which may depend on the level of transaction costs). We also provide a full verification argument that relies on no additional technical assumptions and uses primal methods only. The relative simplicity of our approach allows us to easily analyse aspects of the problem which previously have been challenging, such as the small transaction cost regime and comparative statics. A key and novel key idea is to parameterise consumption and the value function in terms of the shadow fraction of wealth.

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Exploration-exploitation trade-off for continuous-time reinforcement learning

Recently, reinforcement learning (RL) has attracted substantial research interests. Much of the attention and success, however, has been for the discrete-time setting. Continuous-time RL, despite its natural analytical connection to stochastic controls, has been largely unexplored and with limited progress. In particular, characterising sample efficiency for continuous-time RL algorithms remains a challenging and open problem.

In this talk, we develop a framework to analyse model-based reinforcement learning in the episodic setting. We then apply it to optimise exploration-exploitation trade-off for linear-convex RL problems, and report sublinear (or even logarithmic) regret bounds for a class of learning algorithms inspired by filtering theory. The approach is probabilistic, involving analysing learning efficiency using concentration inequalities for correlated continuous-time observations, and applying stochastic control theory to quantify the performance gap between applying greedy policies derived from estimated and true models.

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Time Discretization of Partially Observable Stochastic Optimal Control Problems

In this talk, we propose a time discretization scheme for stochastic control problem under partial observation, which allows one to solve the problem in a discrete induction algorithm with finitely many steps. With the help of the compactification method and the BMO martingale theory, we prove the convergence of the time discretization. Using the dynamic programming principle for the discrete filter process, we present an implementable scheme by making the discrete signal and observation take values in a countable set. Numerical experiment with the linear quadratic structure is performed to validate the theoretical results. This work is jointed with Xiaolu Tan and Shanjian Tang.

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Master equations for extended MFG and MFG with a major player

For MFGs with a major player, the value function of the major player will depend on the major player's state and the minor player's law. The main feature here is that the measure variable differs from the law of the state process corresponding to the state variable. This is exactly in the spirit of the extend MFG proposed by Lions and Souganidis. For extended MFGs, all the existing monotonicity con- ditions fail to work. On one hand, we shall investigate the mapping of the optimization problem and explore sufficient conditions for the uniqueness of its fixed point, namely the uniqueness of the mean field equilibrium, and then the global wellposedness of the corresponding master equations. On the other hand, when the MFG admits multiple mean field equilibria, we shall provide a partial order con- dition that allows us to compare different mean field equilibria and study the global result of the corresponding master equation. At the end, we will establish three results. First, in a one dimensional toy model, we obtain the uniqueness of MFE and hence the global classical solution of the master equation when the mapping is de- creasing. The conditions seem to be new for the standard MFG. Next, we obtain some sufficient conditions, which are different from smallness conditions, for a contraction mapping and thus establish the global well-posedness of the master equations. These conditions are always violated for a standard MFG. Finally, in the case that the mapping is increasing in certain sense, we obtain the minimum and maximum MFEs, which are time consistent and then lead to the minimum and maximum solution of the master equation, in the weak-viscosity sense. This result h olds true for the standard MFG as well, and we emphasize in this case MFE is in general not unique. The talk is based on ongoing joint works with Jianfeng Zhang.

MS 04: Deep Learning Meets Inverse Problems

Organizers:

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Deep learning techniques have witnessed significant uses the inverse problems community in recent years. These techniques often facilitate faster and more accurate reconstructions. In this mini-symposium, we will bring together researchers on the topic and to discuss the latest advances in the theoretical analysis and algorithmic developments in this exciting research direction, e.g., solving PDE inverse problems.

Session 1 (Aug 28 15:50-17:50)

Zhi Zhou
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Identification of Conductivity in Elliptic equations using Deep Neural Networks

The focus of this talk is on the numerical methods used to identify the conductivity in an elliptic equation. Commonly, a regularized formulation consists of a data fidelity and a regularizer is employed, and then it is discretized using finite difference method, finite element methods or deep neural networks. One key issue is to establish a priori error estimates for the recovered conductivity distribution. In this talk, we discuss our recent findings on using deep neural networks for this class of problems, by effectively utilizing relevant stability estimates.

Dong Wang
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Adversarial Neural Networks for Constrained Optimization Problems

In this talk, we focus on integrating adversarial thought into the constrained optimization problem. For optimization problems with constraints, we first transform them into a minimax problem using the Augmented Lagrangian method. Then, we use two(or several) Deep Neural Networks(DNNs) to represent the primal and adversarial variables, respectively, and solve this saddle point problem by updating the parameters of these neural networks to obtain the solution to the original problem. The proposed method has several advantages over traditional deep learning methods for solving PDEs. Firstly, the algorithm is less sensitive to the parameter settings of the loss function, and the introduction of the Augmented Lagrangian multiplier makes it more robust. Secondly, it is a framework-based approach that can handle various types of constrained problems, such as scalar constraints, PDE constraints, inequality constraints, etc., using the same framework. In the numerical experiment, we demonstrate the effectiveness and robustness of the proposed method on different constrained optimization problems, including the Ginzburg-Landau Energy problems, Dirichlet Partition problems, Fluid-Solid Optimization problems, and Obstacle problems.

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Learning prediction policy of prior measures for statistical inverse problems of PDEs

In this talk, we view the statistical inverse problems of partial differential equations (PDEs) as PDE-constrained regression problems and focus on learning the prediction function of the prior probability measures. Under this perspective, we propose general generalization bounds for learning infinite-dimensionally defined prior measures in the style of the probability approximately correct Bayesian learning theory. The whole theoretical framework is rigorously defined on infinite-dimensional separable function space, which makes the theories intimately connected to the usual infinite-dimensional Bayesian inverse approach. Inspired by the concept of α-differential privacy, a generalized condition (containing the usual Gaussian measures employed widely in the statistical inverse problems of PDEs) has been proposed, which allows the learned prior measures able to depend on the measured data (the prediction function with measured data as input and the prior measure as output can be introduced). After illustrating the general theories, the specific settings of linear and nonlinear problems have been given, which easily casts into our general theories to obtain concrete generalization bounds. Based on the obtained generalization bounds, infinite-dimensionally well-defined practical algorithms have been formulated. Finally, numerical examples of the backward diffusion and Darcy flow problems are given that demonstrate the potential applications of the proposed approach in learning the prediction function of the prior probability measures.

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Ensemble Kalman Inversion for shape identification

In this talk I discuss recent advances in the implementation of the level-set approach to parameterise unknown interfaces and discontinuous properties with the Ensemble Kalman Inversion (EKI) framework for inverse problems. The proposed approach uses an underlying level-set function parameterised via the SPDE formulation for Whittle-Matern (WM) random fields. We compose the level-set-based WM parameterisation with the forward map and, using the capability of EKI to handle parameter-to-output maps in a blackbox fashion, we infer all relevant (hyper-)parameters of the level-set-based WM parameterisation within the EKI algorithm. We demonstrate the applicability of this approach to solve various inverse problems where the unknown is a discontinuous property arising from the presence of an anomalous material/tissue. We will present numerical examples with applications to (i) non-destructive testing of composite materials, (ii) ground penetrating radar and (iii) magnetic resonance elastography.

Session 2 (Aug 29 15:50-17:50)

Tim Jahn
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Early stopping of untrained neural networks

In recent years new regularisation methods based on neural networks have shown promising performance for the solution of ill-posed problems, e.g., in imaging science. Due to the non-linearity of the networks, these methods often lack profound theoretical justification. In this talk we rigorously discuss convergence for an untrained convolutional network. Untrained networks are particularly attractive for applications, since they do not require any training data. Its regularising property is solely based on the architecture of the network. Because of this, appropriate early stopping is essential for the success of the method. We show that the discrepancy principle is an adequate method for early stopping here, as it yields minimax optimal convergence rates.

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A Robust Data-Driven Approach for Estimating Non-Local Interaction Potential in Aggregation-Diffusion Equations from Noisy Data

Interacting particle systems are widely present in science and engineering, exhibiting various collective behaviors, including flocking of birds, milling of fish, and self-propelled particles. Aggregation-diffusion equations are often used to model large scales of interacting particle systems, offering insights into how individual behaviors generate collective behaviors. Despite recent theoretical and numerical advances, matching these models with observational data remains a challenge.

In this talk, we propose a data-driven approach for estimating non-local interaction potential in aggregation-diffusion equations from single noisy trajectory data. We propose a simple L1-regularized variational approach via Basis Pursuit, and show it can overcome the ill-posedness of the corresponding inverse problems. We demonstrate the effectiveness of our approach through numerical experiments on 1D and 2D systems with nonlinear diffusions. Additionally, we provide theoretical analysis on the estimators. This work is a joint effort with Jose Carrillo, Gissell Estrande, and Laszlo Mikolas.

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Robust Full Waveform Inversion: A Source Wavelet Manipulation Perspective

Full-waveform inversion (FWI) is a powerful tool for high-resolution subsurface parameter reconstruction. Due to the existence of local minimum traps, the success of the inversion process usually requires a good initial model. Our study primarily focuses on understanding the impact of source wavelets on the landscape of the corresponding optimization problem. We thus introduce a decomposition scheme that divides the inverse problem into two parts. The first step transforms the measured data into data associated with the desired source wavelet. Here, we consider inversions with known and unknown sources to mimic real scenarios. The second sub-problem is the conventional full waveform inversion, which is much less dependent on an accurate initial model since the previous step improves the misfit landscape. A regularized deconvolution method and a convolutional neural network are employed to solve the source transformation problem. Numerical experiments on the benchmark models demonstrate that our approach improves the gradient's quality in the subsequent FWI and provides a better inversion performance.

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Solving elliptic problems with singular solutions using splitting technique

In this work, we develop an efficient solver based on neural networks for second-order elliptic equations with variable coefficients and singular sources. This class of problems covers general point sources, line sources and the combination of point-line sources, and has a broad range of practical applications. The proposed approach is based on decomposing the true solution into a singular part that is known analytically using the fundamental solution of the Laplace equation and a regular part that satisfies a suitable modified elliptic PDE with a smoother source, and then solving for the regular part using the deep Ritz method. A path-following strategy is suggested to select the penalty parameter for enforcing the Dirichlet boundary condition. Extensive numerical experiments in two- and multi-dimensional spaces with point sources, line sources or their combinations are presented to illustrate the efficiency of the proposed approach, and a comparative study with several existing approaches based on neural networks is also given, which shows clearly its competitiveness for the specific class of problems. In addition, we briefly discuss the error analysis of the approach.

Session 3 (Aug 30 11:00-12:30)

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Estimating adsorption isotherm parameters in chromatography via a virtual injection promoting double feed-forward neural network

The means to obtain the adsorption isotherms is a fundamental open problem in competitive chromatography. A modern technique of estimating adsorption isotherms is to solve a nonlinear inverse problem in a partial differential equation so that the simulated batch separation coincides with actual experimental results. However, this identification process is usually ill-posed in the sense that the uniqueness of adsorption isotherms cannot be guaranteed, and moreover, the small noise in the measured response can lead to a large fluctuation in the traditional estimation of adsorption isotherms. The conventional mathematical method of solving this problem is the variational regularization, which is formulated as a non-convex minimization problem with a regularized objective functional. However, in this method, the choice of regularization parameter and the design of a convergent solution algorithm are quite difficult in practice. Moreover, due to the restricted number of injection profiles in experiments, the types of measured data are extremely limited, which may lead to a biased estimation. In order to overcome these difficulties, we develop a new inversion method - the virtual injection promoting double feed-forward neural network (VIP-DFNN). In this approach, the training data contain various types of artificial injections and synthetic noisy measurement at outlet, generated by a conventional physics model - a time-dependent convection-diffusion system. Numerical experiments with both artificial and real data fromlaboratory experiments show that the proposed VIP-DFNN is an efficient and robust algorithm.

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Inversion of drift in SDE with deep learning

In this talk, I will talk about inversion of drift and the invariant distribution of ergodic Itô diffusion processes from a discrete-time dependent data. Firstly, we employ deep neural networks to estimate the drift term of the diffusion process by solving appropriate supervised learning tasks. We establish the convergence of the proposed scheme in the framework of nonparametric estimation with dependent data.

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Deep Ritz Method for Optimization with Elliptical Constrains in High Dimensional Space

In this article, we propose a novel approach to address optimization problems with elliptical constraints in high-dimensional spaces using neural networks. Our method is based on the augmented lagrange algorithm and the Deep Ritz Method. To demonstrate the effectiveness of our approach, we consider two numerical examples: the inverse source problem and boundary control problem, both of which are challenging tasks for finite element methods (FEM) in high dimensional space due to the curse of dimensionality. Furthermore, we provide a convergence analysis of our method to enhance its practicality.

Session 4 (Aug 31 10:20-12:20)

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Transformer meets boundary value inverse problems

A Transformer-based deep direct sampling method is proposed for solving a class of boundary value inverse problem. We try to answer a critical question: whether and how one can benefit from the theoretical structure of a mathematical problem to develop task-oriented and structure-conforming deep neural network? Inspired by direct sampling methods for inverse problems, the 1D boundary data are preprocessed by a partial differential equation-based feature map to yield 2D harmonic extensions in different frequency input channels. Then, by introducing learnable non-local kernel, the approximation of direct sampling is recast to a modified attention mechanism. The proposed method is then applied to electrical impedance tomography, a well-known severely ill-posed nonlinear inverse problem. The new method achieves superior accuracy over its predecessors and contemporary operator learners, as well as shows robustness with respect to noise. This research shall strengthen the insights that the attention mechanism, despite being invented for natural language processing tasks, offers great flexibility to be modified in conformity with the a priori mathematical knowledge, which ultimately leads to the design of more physics-compatible neural architectures.

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Convex neural networks for inverse problems in imaging

We propose learning approaches using convex neural networks for inverse problems in imaging. Given a general neural network architecture, we prescribe sufficient conditions to achieve a trained neural network which is component-wise convex or uniformly convex; we build modern architecture embracing state-of-the-art neural networks, and provide concrete examples of realizing convexity and uniform convexity in the U-net architecture. With the tools of convex neural networks, we propose learning algorithms for the solution of inverse problems in imaging: (1) an iterated network Tikhonov (iNETT) method is developed to solve ill-posed inverse problems, which is successfully applied to computerized tomography; (2) an end-to-end learning approach is developed to solve the inverse problem in image deblurring.

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Current density impedance imaging with PINNs

In this talk, we propose a computationally efficient method called CDII-PINNs for solving CDII (current density impedance imaging) problems within the framework of Tikhonov regularization. CDII-PINNs combines physics-informed neural networks (PINNs) with a regularized least-squares output functional and a differential equation that describes the relationship between conductivity and voltage. By constructing a physics-informed loss function, we couple neural networks representing conductivity and voltage, and minimize the loss function to obtain a reconstruction. We provide a rigorous theoretical guarantee and perform an error analysis for CDII-PINNs, establishing a convergence rate based on pre-selected neural network parameters and the number of samples. Numerical simulations demonstrate that CDII-PINNs are efficient, accurate, and robust against noise levels ranging from 1% to 20%.

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Optimal control of ReLU neural network informed partial differential equations

Machine learning methods for learning physical models, in particular partial differential equations (PDEs), has been an emerging topic nowadays. We consider optimal control of such learning-informed PDEs with ReLU neural networks. Analytical and numerical aspects on solutions of the optimal control problems will be discussed.

MS 05: Recent Advances in Inverse Problems

Organizers:

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Inverse problems represent an extremely important class of practical problems. Their effective numerical solution poses significant challenges due to the ill-posed nature of the problems. In this mini-symposium, we aim at bringing together to discuss the recent exciting developments on the topic.

Session 1 (Aug 31 15:50-17:50)

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Towards global convergence for inverse coefficient problems

Several applications in medical imaging and non-destructive material testing lead to inverse elliptic coefficient problems, where an unknown coefficient function in an elliptic PDE is to be determined from partial knowledge of its solutions. This is usually a highly non-linear ill-posed inverse problem, for which unique reconstructability results, stability estimates and global convergence of numerical methods are very hard to achieve.

In this talk we will consider an inverse coefficient problem with finitely many measurements and a finitedesired resolution. We will present a criterion based on monotonicity, convexity and localized potentials arguments that allows us to explicitly estimate the number of measurements that is required to achieve the desired resolution. We also obtain an error estimate for noisy data, and overcome the problem of local minimaby rewriting the problem as an equivalent uniquely solvable convex non-linear semidefinite optimization problem.

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AFEM for Elliptic Eigenvalue Optimization in Phase-Field Setting

This talk is concerned with adaptive approximations of an elliptic eigenvalue optimization problem in a phase-field setting by a conforming finite element method. An adaptive algorithm is proposed and implemented in several examples for illustration of efficiency and accuracy. Theoretical findings include a vanishing limit of the estimators up to a subsequence by the algorithm and the convergence of the relevant subsequence of adaptively-generated solutions to a solution to the continuous optimality system. This is a joint work with Jing Li and Prof. Shengfeng Zhu, both from East China Normal University.

Yi-Hsuan Lin
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Inverse problems for semilinear reaction-diffusion equations

We investigate an inverse boundary value problem of determination of reaction-diffusion processes, which are modeled by general form semilinear parabolic equations. We determine, under additional assumptions, the semilinear term up to this symmetry in a time-dependent anisotropic case modeled on the Euclidean space.

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Recovery of Multiple Parameters in Subdiffusion from One Lateral Boundary Measurement

This talk is concerned with numerically recovering multiple parameters in a partly unknown subdiffusion model from one lateral measurement on the boundary. We prove that the boundary measurement uniquely determines the fractional order and the polygonal support of the diffusion coefficient, without knowing either the initial condition or the source. We present an algorithm for recovering the fractional order and diffusion coefficient which combines small-time asymptotic expansion, analytic continuation and the level set method.

Session 2 (Sep 01 10:20-12:20)

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Electrical Impedance Tomography with Deep Calderon's Method

Electrical impedance tomography (EIT) is a noninvasive medical imaging modality utilizing the current-density/voltage data measured on the surface of the subject. Calderon's method is a relatively recent EIT imaging algorithm that is non-iterative, fast, and capable of reconstructing complex-valued electric impedances. However, due to the regularization via low-pass filtering and linearization, the reconstructed images suffer from severe blurring and underestimation of the exact conductivity values. In this work, we develop an enhanced version of Calderon's method, using convolution neural networks (i.e., U-net) via a post processing step. Specifically, we learn a U-net to postprocess the EIT images generated by Calderon's method so as to have better resolutions and more accurate estimates of conductivity values. We simulate chest configurations with which we generate the current-density/voltage boundary measurements and the corresponding reconstructed images by Calderon's method. With the paired training data, we learn the neural network and evaluate its performance on real tank measurement data. The experimental results indicate that the proposed approach indeed provides a fast and direct (complex-valued) impedance tomography imaging technique, and substantially improves the capability of the standard Calderon's method.

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Convergence Rates of Tikhonov Regularizations for Elliptic and Parabolic Inverse Radiativity Problems

In this talk, we shall study the convergence rates of the Tikhonov regularized solutions for the recovery of the radiativities in elliptic and parabolic systems in general dimensional spaces. The conditional stability estimates are first derived. Due to the difficulty of the verification of the existing source conditions or nonlinearity conditions of the inverse radiativity problems in high dimensional spaces, some new variational source conditions are proposed. The conditions are rigorously verified in general dimensional spaces under the conditional stability estimates. We will also derive the reasonable convergence rates under the new source conditions, and the results reveal the explicit relation between the regularity of the radiativities and the convergence rates.

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Hybrid Projection Methods for Solution Decomposition in Large-scale Bayesian Inverse Problems

We develop hybrid projection methods for computing solutions to large-scale inverse problems, where the solution represents a sum of different stochastic components. Such scenarios arise in many imaging applications (e.g., anomaly detection in atmospheric emissions tomography) where the reconstructed solution can be represented as a combination of two or more components and each component contains different smoothness or stochastic properties. In a deterministic inversion or inverse modeling framework, these assumptions correspond to different regularization terms for each solution in the sum. Although various prior assumptions can be included in our framework, we focus on the scenario where the solution is a sum of a sparse solution and a smooth solution. For computing solution estimates, we develop hybrid projection methods for solution decomposition that are based on a combined flexible and generalized Golub-Kahan processes. This approach integrates techniques from the generalized Golub-Kahan bidiagonalization and the flexible Krylov methods. The benefits of the proposed methods are that the decomposition of the solution can be done iteratively, and the regularization terms and regularization parameters are adaptively chosen at each iteration. Numerical results from photoacoustic tomography and atmospheric inverse modeling demonstrate the potential for these methods to be used for anomaly detection.

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TBA

MS 06: Multiscale problems and their related recent methodologies

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Session 1 (Aug 29 11:00-12:30)

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Generalized multiscale finite element method for highly heterogeneous compressible flow

In this talk, we present the generalized multiscale finite element method (GMsFEM) for single phase compressible flow in highly heterogeneous porous media. We follow the major steps of the GMsFEM to construct permeability dependent offline basis for fast coarse-grid simulation. The offline coarse space is efficiently constructed only once based on the initial permeability field with parallel computing. A rigorous convergence analysis is performed for two types of snapshot spaces. The analysis indicates that the convergence rates of the proposed multiscale method depend on the coarse meshsize and the eigenvalue decay of the local spectral problem. To further increase the accuracy of multiscale method, residual driven online multiscale basis is added to the offline space. The construction of online multiscale basis is based on a carefully designed error indicator motivated by the analysis. We find that online basis is particularly important for the singular source. Rich numerical tests on typical 3D highly heterogeneous media are presented to demonstrate the impressive computational advantages of the proposed multiscale method.

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FFT-based homogenization: analysis and implementation

FFT-based discretization schemes are widely applied in computational micromechanics for deriving effective coefficients. The talk will contain two parts. In the first part, I will provide several mathematical analyses of the convergence theories of several schemes; In the second part, I will demonstrate our ongoing work which aims to develop an FFT-variant scheme to obtain effective thermal conductivity. Especially, in the second part, I will emphasize the acceleration of computing performance from utilization of GPUs.

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Parameter-uniform superconvergence of multiscale computation on layer-adapted meshes for singularly perturbed problems

A multiscale finite element method is proposed for singularly perturbed 1D and 2D problems. We present the detailed mapping behaviors among coarse scales and fine ones, and provide the enriched data from microscopic scales into macroscopic ones. A graded mesh is generated for uniform adaption to the perturbed parameter and the transient location, with capturing the boundary layers effectively. We verify the multiscale error analysis with high-order convergence in the energy norm. With the global computation reduction in the multiscale scheme, high accuracy, uniform stability and super-convergence are validated theoretically as well as experimentally, and its efficiency is prominent through scientific tests.

Session 2 (Sep 01 14:30-16:30)

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An inverse problem in mean field game from partial boundary measurement

In this work, we consider a novel inverse problem in mean-field games (MFG). We aim to recover the MFG model parameters that govern the underlying interactions among the population based on a limited set of noisy partial observations of the population dynamics under the limited aperture. Due to its severe ill-posedness, obtaining a good quality reconstruction is very difficult. Nonetheless, it is vital to recover the model parameters stably and efficiently in order to uncover the underlying causes for population dynamics for practical needs. Our work focuses on the simultaneous recovery of running cost and interaction energy in the MFG equations from a finite number of boundary measurements of population profile and boundary movement. To achieve this goal, we formalize the inverse problem as a constrained optimization problem of a least squares residual functional under suitable norms with L1 regularization. We then develop a fast and robust operator splitting algorithm to solve the optimization using techniques including harmonic extensions, three-operator splitting scheme, and primal-dual hybrid gradient method. Numerical experiments illustrate the effectiveness and robustness of the algorithm. A future direction will be to develop a technique for algorithmic speedup for inverse problems in higher dimensions with the help of bilevel optimization, machine learning techniques and neural network architecture.

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Nonoverlapping Spectral Additive Schwarz Methods (NOSAS) and Their Extension to Multiscale Discretizations

Domain decomposition methods are one of the most important techniques commonly used in parallel computation. The basic idea is to divide the solution of a large linear system into smaller problems whose solutions can be used to produce a preconditioner for the system of equations that result from discretizing the PDEs on the entire domain. Domain decomposition methods have a lot of connection with multiscale technique, especially when we consider the construction of the coarse space. This talk is about some variants of new domain decomposition methods, called Nonoverlapping Spectral Additive Schwarz Methods (NOSAS), and their extension to multiscale discretizations. NOSAS methods are designed for elliptic problems with highly heterogeneous coefficients. The methods are of the nonoverlapping type, and the subdomain interactions are obtained via the coarse space. The adaptive coarse space in NOSAS guarantees the robustness of the preconditioners for any heterogeneous coefficients and the number of subdomains. The similar idea of constructing a coarse space can similarly extend to the framwork of multiscale discretizations.

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Least squares mixed GMsFEM for elliptic and parabolic problems

In this talk, we present least-squares mixed GMsFEM for elliptic problems and parabolic problems. A mixed formulation is considered such that both pressure and velocity are approximated simultaneously. This formulation arises naturally in many applications such as flows in porous media. Due to the multiscale nature of the solutions, using model reduction is required to efficiently obtain approximate solutions. There are many multiscale approaches for elliptic problems in mixed formulation. It has been a challenging task to construct a method giving accurate representation for both pressure and velocity. The goal in this paper is to construct multiscale basis functions for both pressure and velocity. We will apply the framework of Generalized Multiscale Finite Element Method (GMsFEM), and design systematic strategies for the construction of basis functions. The mixed formulation is minimized in the sense of least-squares. The compatibility condition for multiscale finite element spaces of the pressure and velocity is not required in the least-squares mixed form. This gives more flexibility for the construction of multiscale basis functions for velocity and pressure. Convergence analysis is carried out for the least-squares mixed GMsFEM. Several numerical examples for various permeability fields are presented to show the performance of the presented method. The numerical results show that the least-squares mixed GMsFEM can give accurate approximation for both pressure and velocity using only a few basis functions per coarse element.

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Computational multi-scale method and scientific machine learning

In this talk, I will present some recent advances in computational multisscale methods and scientific machine learning. Researchers have incorporated machine learning techniques to create more effective simulation methods, addressing scientific problems that include forward and inverse problems in different domains. Specifically, we will focus on the applications of heterogeneous problems that comprise multiple scale features and the approximation of exchange-correlation functionals in density functional theory.

MS 07: Numerical PDEs and related problems

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Session 1 (Aug 28 15:50-17:50)

Jiang YANG
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Higher-Order Energy Stable Exponential Time Differencing Runge-Kutta methods for Gradient Flows

We develop a general framework to construct higher-order unconditional energy stable exponential time differencing Runge-Kutta (ETDRK) methods

for a class of gradient flows. More precisely, we identify a set of conditions that ETDRK schemes need to satisfy in order to have the energy stable property, show that the commonly used third-order and fourth-order ETDRK schemes do not satisfy these conditions, and construct, with proper stabilization, new third-order ETDRK schemes which satisfy the conditions and thus unconditionally decrease the energy. We present ample numerical experiments using the new schemes to validate their accuracy and stability, large time step behavior, long time evolution and the adaptive time stepping strategy for various gradient flows. This is the

first work to study the unconditionally energy stability of arbitrarily high-order ETDRK methods, and it is hopeful that our general framework will lead to constructions of higher than third-order unconditional energy stable ETDRK schemes.

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New artificial tangential motions for parametric finite element approximation of surface evolution

A new class of parametric finite element methods, with a new type of artificial tangential velocity constructed at the continuous level, is proposed for solving surface evolution under geometric flows. The method is constructed by coupling the normal velocity of the geometric flow with an artificial tangential velocity determined by a harmonic map from a fixed reference surface to the unknown surface, formulated at the continuous level as a system of geometric partial differential equations in terms of a Lagrange multiplier. Since the harmonic map is almost angle-preserving, the new method could preserve the mesh quality, i.e., the shapes of the triangles, as long as the mesh quality of the reference surface is good. Extensive numerical experiments and benchmark examples are presented to demonstrate the convergence of the proposed method and the advantages of the method in preserving the mesh quality of the surfaces for mean curvature flow and surface diffusion.

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Finite Element Operator Network for Solving Parametric PDEs

In this talk, I will introduce a novel approach for solving parametric PDEs using a Finite Element Operator Network (FEONet). The proposed method leverages the power of deep learning in conjunction with traditional numerical methods, specifically the finite element method, to solve parametric PDEs in the absence of any paired input-output training data. I will demonstrate the effectiveness of our approach on several benchmark problems and show that it outperforms existing state-of-the-art methods in terms of accuracy, generalization, and computational flexibility. Theoretical convergence result will also be provided.

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The energy technique for BDF methods

The energy technique is probably the easiest way to establish stability of parabolic differential equations. The application of the energy technique to numerical methods with very good stability properties, such as algebraically stable Runge-Kutta methods or A-stable multistep methods, is straightforward. The extension to other numerical methods, such as $A(\theta)$-stable methods, requires some effort and is more interesting; the main difficulty concerns suitable choices of test functions. Here we focus on the energy technique for backward difference formula (BDF) methods. In the cases of the A-stable one- and twostep BDF methods the application is trivial. The energy technique is applicable also to the $A(\theta)$-stable three-, four- and five-step BDF methods via Nevanlinna-Odeh multipliers. The main results are: (1) No Nevanlinna-Odeh multipliers exist for the six-step BDF method. (2) The energy technique is applicable under a relaxed condition on the multipliers. (3) We present multipliers that make the energy technique applicable also to the six-step BDF method.

Session 2 (Aug 29 15:50-17:50)

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Deep neural networks for inverse scattering problems.

In this presentation, we consider artificial neural networks for inverse scattering problems. As a working model, we consider the inverse problem of recovering a scattering object from the (possibly) limited-aperture radar cross section (RCS) data collected corresponding to a single incident field. This nonlinear and ill-posed inverse problem is practically important and highly challenging due to the severe lack of information. From a geometrical and physical point of view, the low-frequency data should be able to resolve the unique identifiability issue, but meanwhile lose the resolution. On the other hand, the machine learning can be used to break through the resolution limit. By combining the two perspectives, we develop a fully connected neural network (FCNN) for the inverse problem. Extensive numerical results show that the proposed method can produce stunning reconstructions. The proposed strategy can be extended to tackling other inverse scattering problems with limited measurement information.

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A new perfectly matched layer method for the Helmholtz equation in nonconvex domains

A new coupled perfectly matched layer (PML) method is proposed for the Helmholtz equation in the whole space with inhomogeneity concentrated on a nonconvex domain. Rigorous analysis is presented for the stability and convergence of the proposed coupled PML method, which shows that the PML solution converges to the solution of the original Helmholtz problem exponentially with respect to the product of the wave number and the width of the layer. An iterative algorithm and a continuous interior penalty finite element method (CIP-FEM) are also proposed for solving the system of equations associated to the coupled PML. Numerical experiments are presented to illustrate the convergence and performance of the proposed coupled PML method, as well as the iterative algorithm and the CIP-FEM.

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A new finite element method for elliptic optimal control problems with pointwise state constraints in energy spaces

In this talk we introduce a new finite element method for solving elliptic optimal control problems with pointwise state constraints, including the distributed controls and the Dirichlet or Neumann boundary controls. The main idea is to use energy space regularizations in the objective functional, while the equivalent representations of the energy space norms, i.e., the $H^{-1}(\Omega)$-norm for the distributed control, the $H^{1/2}(\Gamma)$-norm for the Dirichlet control and the $H^{-1/2}(\Gamma)$-norm for the Neumann control, enable us to transform the optimal control problem into an elliptic variational inequality involving only the state variable. The elliptic variational inequalities are second order for the three cases, and include additional equality constraints for Dirichlet or Neumann boundary control problems. Standard C^0 finite elements can be used to solve the resulted variational inequality. We provide preliminary a priori error estimates for the new algorithm for solving distributed control problems. Extensive numerical experiments are carried out to validate the accuracy of the new algorithm.

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[3] W. Liu, W. Gong and N. Yan, *A new finite element approximation of a state-constrained optimal control problem*, J. Comput. Math., 27(2009), no. 1, 97-114.

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Geometric inverse problems in time-fractional subdiffusion

We consider two types of geometric inverse problems associated with time-fractional subdiffusion: inverse source support and interface reconstruction of a discontinuous subdiffusion coefficient. We show existence, perform shape sensitivity analysis and use shape gradients to develop numerical algorithms allowing shape and topological changes. Numerical results are presented to demonstrate effectiveness of the algorithms. This is a joint work with Xindi Hu and Wei Fan.

MS 08: Recent Advances in Numerical Methods for Kinetic and Related Models

Organizer

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Session 1 (Aug 31 15:50-17:50)

Tao Xiong
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Uniformly unconditionally stable asymptotic preserving finite difference schemes for kinetic transport equations

In this work, uniformly unconditionally stable first and second order finite difference schemes are proposed for kinetic transport equations in the diffusive scaling. We first derive an approximate evolution equation for the macroscopic density, from the formal solution of the distribution function, which is then discretized by following characteristics for the transport part with a backward finite difference semi-Lagrangian approach, while the diffusive part is discretized implicitly. After the macroscopic density is available, the distribution function can be efficiently solved even with a fully implicit time discretization, since all discrete velocities are decoupled, resulting in a low-dimensional linear system from spatial discretizations at each discrete velocity. Both first and second order discretizations in space and in time are considered. The resulting schemes can be shown to be asymptotic preserving (AP) in the diffusive limit. Uniformly unconditional stabilities are verified from a Fourier analysis based on eigenvalues of corresponding amplification matrices. Numerical experiments, including high dimensional problems, have demonstrated the corresponding orders of accuracy both in space and in time, uniform stability, AP property, and good performances of our proposed approach.

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A New Collision Avoidance Model with Semi-implicit Random Batch Resolution

Research on crowd simulation has important and wide range of applications. The main difficulty is how to lead all particles with a same and simple rule, especially when particles are numerous. Firstly we propose a two-dimensional agent-based collision avoidance model, which is a N-particles Newtonian system where three interaction forces are designed to mimic the deviation, the deceleration and aligning actions of agents when avoiding collisions. Secondly, direct simulation of the N-particles Newtonian system is time-consuming, since the computational complexity is $\mathcal{O}(N^2)$, we therefore propose an efficient algorithm, called the semi-implicit random batch method (RBM) with kernel splitting (SRBMS) which reduces the computational complexity to $\mathcal{O}(N)$ and converge to the proposed collision avoidance model with a convergence rate equal to 1/2. SRBMS is a combination of the Random Batch method (Shi Jin, Lei Li, and Jian-Guo Liu. Random batch methods (RBM) for interacting particle systems. Journal of Computational Physics, 400:108877, 2020.) and the kernel splitting strategy. Finally, various tests are presented to show robustness and efficiency of the proposed model and the numerical resolution.

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An asymptotic-preserving PN method for the three temperature radiative transfer equation

We propose an asymptotic preserving (AP) numerical method for the three temperature radiative transfer equation in the framework of the PN method. A specially designed AP splitting scheme is proposed to decouple the radiation and electron system. An implicit and explicit numerical scheme

is built with the higher-order expansion coefficients of the specific intensity in PN method treated explicitly and lower-order implicitly, which leads to an implicit system that can be solved at the computational cost of an explicit scheme. Several numerical examples validate the efficiency of this scheme.

[1] Zhengyi Li and Yanli Wang, et. al. *Solving Boltzmann equation using neural sparse representation* arXiv: :2302.09233, 2023

Yiwen Lin
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Kinetic-fluid multi-phase flow system with random inputs

Consider coupled models for particulate flows, where the disperse phase is made of particles with distinct sizes. This leads to a system coupling the incompressible Navier–Stokes equations to the Vlasov–Fokker–Planck equations. For the model with random initial data near global equilibrium, we establish uniform regularity in suitable Sobolev spaces using energy estimates and demonstrate exponential decay of energy over time by hypocoercivity arguments. We also prove that the generalized polynomial chaos stochastic Galerkin (gPC-sG) method has spectral accuracy uniformly in time and the Stokes number, with exponential decay of an error over time. Additionally, we provide uniform error estimates of the bifidelity method for this coupled model with random initial inputs. An asymptotic-preserving numerical scheme is designed to simulate the behavior of multi-phase flow system, efficiently in both kinetic and hydrodynamic regimes. Numerical examples illustrate the accuracy and asymptotic behavior of the scheme.

[1] Shi Jin and Yiwen Lin, *Energy estimates and hypocoercivity analysis for a multi-phase Navier-Stokes-Vlasov-Fokker-Planck system with uncertainty.* preprint

[2] Shi Jin and Yiwen Lin, *Asymptotic-preserving schemes for kinetic-fluid modeling of mixture flows with distinct particle sizes.* preprint

Session 2 (Sep 01 10:20-12:20)

Kunlun Qi
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Stability and convergence analysis of the Fourier-Galerkin spectral method for the Boltzmann equation

Numerical approximation of the Boltzmann equation is a challenging problem due to its high-dimensional, nonlocal, and nonlinear collision integral. Over the past decade, the Fourier-Galerkin spectral method has become a popular deterministic method for solving the Boltzmann equation, manifested by its high accuracy and potential of being further accelerated by the fast Fourier transform. Albeit its practical success, the stability of the method is only recently proved by utilizing the "spreading" property of the collision operator. In this talk, we introduce a new proof based on a careful L^2 estimate of the negative part of the solution. We also discuss the applicability of the result to various initial data, including both continuous and discontinuous functions.

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Mixing flow and advection-diffusion-reaction equations

In the study of incompressible fluid, one fundamental phenomenon that arises in a wide variety of application is dissipation enhancement by so-called mixing flow. In this talk, I will give a brief introduction to the idea of mixing flow and the role it plays in the field of advection-diffusion-reaction equation, such as the famous Keller-Segel equation for chemotaxis. I will also discuss about the examples of such flows in this talk.

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Exact Calculation of Quantum Thermal Average from Continuous Loop Path Integral Molecular Dynamics

The quantum thermal average plays a central role in describing the thermodynamic properties of a quantum system. From the computational perspective, the quantum thermal average can be computed by the path integral molecular dynamics (PIMD), but the knowledge on the quantitative convergence of such approximations is lacking. We propose an alternative computational framework named the continuous loop path integral molecular dynamics (CL-PIMD), which replaces the ring polymer beads by a continuous loop in the spirit of the Feynman–Kac formula. By truncating the number of normal modes to a finite integer N, we quantify the discrepency of the statistical average of the truncated CL-PIMD from the true quantum thermal average, and prove that the truncated CL-PIMD has uniform-in-N geometric ergodicity. These results show that the CL-PIMD provides an accurate approximation to the quantum thermal average, and serves as a mathematical justification of the PIMD methodology.

MS 09: Recent progress in mathematical theory for kinetic equations

Organizer

Jin Woo Jang
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Session 1 (Aug 29 15:50-17:50)

In-Jee Jeong
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Twisting in Hamiltonian Flows

We prove that the twisting in Hamiltonian flows on annular domains, which can be quantified by the differential winding of particles around the center of the annulus, is stable to perturbations. In fact, it is possible to prove the stability of the whole of the lifted dynamics to non-autonomous perturbations (though single particle paths are generically unstable). These all-time stability facts are used to establish a number of results related to the long-time behavior of inviscid fluid flows. We also discuss applications to the Vlasov–Poisson equations.

[1] Theodore D Drivas, Tarek M Elgindi, In-Jee Jeong. *Twisting in Hamiltonian Flows and Perfect Fluids.* arXiv:2305.09582

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On the constructive coercivity of the linearized Boltzmamm operator in concentric cylinder with specular boundary condition

Coercivity estimate for the linearized Boltzmann operator is one of the central interest in the Boltzmann theory which has many applications. In particular, constructive coercivity esitmates under several types of boundary conditions are very intersting problems with some physical insights. However, constructive coercivity estimate under specular boundary condition is widely open. In this talk, we introduce a novel ζ-correction method which generates Laplace-Beltrami operator in local coordinates. In conclusion, we derive coercivity estimate in (periodic) concentric cylinder under specular boundary condition.

[1] Gyounghun, Ko., Chanwoo, Kim., and Donghyun, Lee. *Dynamical billiard and a long-time behavior of the Boltzmann equation in general 3D toroidal domains*, *preprint*

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Theory of stars in nonlinear PDEs

In nonlinear PDE theories, there are several description of gaseous stars, depending on their scopes and equation of states. In microscopic scale, quantum mechanical descriptions of stars are given by Hartree or Hartree-Fock equations while the Euler-Poisson equations replace this in macroscopic scale. In this talk, we discuss about the existence, properties such as shape or stability of these stars and interconnection between them.

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Quantified overdamped limit for Vlasov–Fokker–Planck equations with singular interaction forces

In this talk, I will discuss a quantified overdamped limit for kinetic Vlasov–Fokker–Planck equations with nonlocal interaction forces. We provide explicit bounds on the error between solutions of that kinetic equation and the limiting equation, which is known under the names of aggregation-diffusion equation or McKean–Vlasov equation. Our strategy only requires weak integrability of the interaction potentials, thus in particular it includes the quantified overdamped limit of the kinetic Vlasov–Poisson–Fokker–Planck system to the aggregation-diffusion equation with either repulsive electrostatic or attractive gravitational interactions.

Session 2 (Aug 31 10:20-12:20)

Jin Woo Jang
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Vanishing angular singularity limit to the hard-sphere Boltzmann equation

In this talk we study Boltzmann's collision kernel for inverse power law interactions $U_s(r) = 1/r^{s-1}$ for $s > 2$ in dimension $d = 3$. We prove the limit of the non-cutoff kernel to the hard-sphere kernel and give precise asymptotic formulas of the singular layer near $\theta \simeq 0$ in the limit $s \to \infty$. Consequently, we show that solutions to the homogeneous Boltzmann equation converge to the respective solutions weakly in L^1 globally in time as $s \to \infty$.

Gyounghun Ko
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Large-amplitude problem of BGK model

BGK equation is a relaxation model of the Boltzmann equation for simulation of various kinetic flow problems. In this work, we study asymptotic stability of the BGK model when the initial data is not necessarily close to global equilibrium pointwisely. Main difficulty of the BGK equation comes from the highly nonlinear structure of the relaxation operator. To overcomes this issue, we derive refined control of macroscopic fields to guarantee the system enters quadratic nonlinear regime.

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Magnetic confinement for the 2D axisymmetric relativistic Vlasov-Maxwell system in an annulus

This talk deals with the mathematical analysis of the magnetic confinement of the plasma via kinetic equations. We prove the global-in-time well-posedness of axisymmetric solutions to the relativistic Vlasov-Maxwell system in a two-dimensional annulus, provided that a huge external magnetic potential is imposed near the boundary. The external magnetic potential well that we impose remains finite within a finite time interval and from that, we prove that the plasma never touches the spatial boundary. In addition, we provide a sufficient condition on the magnitude of the external magnetic potential to guarantee that the plasma is confined in an annulus of the desired thickness which is slightly larger than the initial support. Our method uses the cylindrical coordinate forms of the relativistic Vlasov-Maxwell system.

This is a joint-work with Jin Woo Jang and Robert M. Strain.

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Title: Stationary weak solutions to the BGK model in a slab

Abstract: We consider stationary flow between two condensed phases that emerges from the evaporation and condensation process on the two phases in the framework of the stationary BGK model in a slab. Under the physically minimum conditions on the inflow functions, namely the finite mass flux, energy flux and entropy flux, the existence of stationary weak solutions is derived. The main difficulties are, among others, (1) the impossibility of truncation of the relaxation operator in the vanishing velocity region in the last limit processs, and (2) the control of the velocity distribution functions near vanishing velocity region using the macroscopic fields. This is a joint work with Stephane Brull.

Session 3 (Sep 01 10:20-12:20)

Gayoung An
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High-velocity tails of the inelastic and the multi-species mixture Boltzmann equations

We study high-velocity tails of some homogeneous Boltzmann equations. First, we consider spatially homogeneous *Inelastic* Boltzmann equation with *noncutoff* collision kernel, in the case of moderately soft potentials. We also study spatially homogeneous mixture Boltzmann equations : for both noncutoff collision kernel with moderately soft potentials and cutoff collision kernel with hard potentials. In the case of noncutoff *Inelastic* Boltzmann, we obtain

$$f(t,v) \geq a(t)e^{-b(t)|v|^p}, \quad 2 < p < 6.213$$

by extending Cancellation lemma and spreading lemma and assuming $f \in C^\infty$. For the Mixture type Boltzmann equations, we prove Maxwellian $p = 2$.

[1] C. Imbert, C. Mouhot, and L. Silvestre *Gaussian lower bounds for the Boltzmann equation without cutoff.* SIAM J. Math. Anal., 52(3):2930–2944, 2020

[2] R. Alexandre, L. Desvillettes, C. Villani, and B. Wennberg *Entropy dissipation and long-range interactions.* Arch. Ration. Mech. Anal., 152(4): 327–355, 2000.

Sungbin Park
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Linear Landau Damping in Specular Reflective Boundary Condition

In this talk, we will discuss linear Landau damping for the Vlasov-Poisson equation with specular reflective boundary condition. In specular reflective boundary condition, the conventional Fourier analytic method does not seem to be well applied in showing Landau damping. To circumvent this problem, we introduce an alternative integration by parts method. Then we consider linear Landau damping in some bounded domains other than torus. This is a joint work with Donghyun Lee.

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BGK model for two-component gases near a global Maxwellian

In this talk, we establish the existence of the unique global-in-time classical solutions to the multi-component BGK model when the initial data is a small perturbation of global equilibrium. The main difference between this model compared to the classic mixture Boltzmann equation is that the rate of interchange of momentum and energy between the two species is controllable through two free parameters. Based on the energy method, we carefully analyze the dissipative nature of the linearized multi-component relaxation operator, and observe that the partial dissipation from the intra-species and the inter-species linearized relaxation operators are combined in a complementary manner to give rise to the desired dissipation estimate of the model. Furthermore, the dissipation estimate depends on the momentum interchange rate and the energy interchange rate. So the larger interchange rate leads to stronger dissipation, and therefore, the faster convergence to the global equilibrium. This is joint work with C. Klingenberg, M. Pirner, and S.-B. Yun.

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Thom's gradient conjecture for PDEs with variational structure

We discuss the asymptotic behavior of PDEs with gradient-like structure. Examples include equations from geometric analysis, mathematical physics and modeling such as mean curvature flow, harmonic map, Ricci flow, Yang-Mills gradient flow, porous medium, Ginzburg-Landau, semilinear heat equations etc. The main result characterizes the rate and the direction of convergence for slowly converging solutions. Our result partially confirms Thom's gradient conjecture in the context of infinite-dimensional problems. This is a joint work with Pei-Ken Hung at Minnesota.

MS 10: High order numerical methods for PDE models in applied sciences

Organizers:

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Session 1 (Aug 29 15:50-17:50)

Min Tang
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Multiscale solver for radiation transport equation with strong nonlinearity

The opacity of FRTE depends on not only the material temperature but also the frequency, whose values may vary several orders of magnitude for different frequencies. The gray radiation diffusion and frequency-dependent diffusion equations are two simplified models that can approximate the solution to FRTE in the thick opacity regime. The frequency discretization for the two limit models highly affects the numerical accuracy. However, classical frequency discretization for FRTE considers only the absorbing coefficient. In this paper, we propose a new decomposed multi-group method for frequency discretization that is not only AP in both gray radiation diffusion and frequency-dependent diffusion limits, but also the frequency discretization of the limiting models can be tuned. Based on the decomposed multi-group method, a full AP scheme in frequency, time, and space is proposed. Difficulties due to the strong nonlinearity in the absorption and scattering coefficients are discussed as well.

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Positivity-preserving finite volume schemes for radiation diffusion equations on general meshes

Radiation diffusion equations arise in a wide range of physical applications such as astrophysics and inertial confinement fusion. Two key ingredients in constructing their numerical schemes are both positivity-preserving property and good accuracy for discontinuous and/or anisotropic problems on arbitrary distorted meshes. This talk will introduce our recent research progress including positivity-preserving finite volume schemes, theoretical analysis, and numerical applications.

[1] Shuai, Su and Jiming, Wu *A symmetric, coercive and extreme-preserving finite volume scheme for anisotropic diffusion equations on star-shaped polygonal meshes.* Submitted, 2023

[2] Shuai, Su and Huazhong, Tang and Jiming, Wu *An efficient positivity-preserving finite volume scheme for the nonequilibrium three-temperature radiation diffusion equations on polygonal meshes.* Communcations in Computational Physics, 2021

[3] Shuai, Su and Qiannan, Dong and Jiming, Wu *A decoupled and positivity-preserving discrete duality finite volume scheme for anisotropic diffusion problems on general polygonal meshes.* Journal of Computational Physics, 2018

[4] Xiaoping, Zhang and Shuai, Su and Jiming, Wu *A vertex-centered and positivity-preserving scheme for anisotropic diffusion problems on arbitrary polygonal grids.* Journal of Computational Physics, 2017

Chunmei Su
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A second-order in time, BGN-based parametric finite element method for geometric flows of curves

Geometric flows have recently attracted lots of attention from scientific computing communities. One of the most efficient numerical schemes for solving geometric flows is the well-known BGN scheme, which was proposed by Barrett, Garcke, and Nürnberg (J. Comput. Phys., 222 (2007), pp. 441-467). However, the BGN scheme only can attain first-order accuracy in time, and how to design a temporal high-order numerical scheme is challenging. In this paper, we propose a temporal second-order, BGN-based parametric finite element method for solving geometric flows of curves. The key idea lies in that the chosen discrete inner product over the curve Γ^m at which the time is exactly the same as the time when all quantities are the approximations with an error of $O(\tau^2)$, where τ is the time step size. Furthermore, we point out that the shape metrics (i.e., manifold distance and Hausdorff distance), instead of the function norms, should be used to measure numerical errors of the proposed schemes. Finally, ample numerical experiments demonstrate that the proposed BGN-based schemes are second-order in time in the sense of the shape metric, and much more efficient than the first-order BGN schemes.

[1] W. Jiang, C. Su and G. Zhang, *A second-order in time, BGN-based parametric finite element method for geometric flows of curves.* Preprint.

Jie Du
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High order bound preserving methods for compressible multi-species flow with chemical reactions

In this talk, we consider bound preserving problems for multispecies and multireaction chemical reactive flows. In this problem, the density and pressure are nonnegative, and the mass fraction should be between 0 and 1. The mass fraction does not satisfy a maximum principle and hence it is not easy to preserve the upper bound 1. Also, most of the bound-preserving techniques available are based on Euler forward time integration. Therefore, for problems with stiff source, the time step will be significantly limited. Some previous ODE solvers for stiff problems cannot preserve the total mass and the positivity of the numerical approximations at the same time. In this work, we will construct third order conservative bound-preserving Rugne-Kutta and multistep methods to overcome all these difficulties. Moreover, we will discuss how to control numerical oscillations. Numerical experiments will be given to demonstrate the good performance of the bound-preserving technique and the stability of the scheme for problems with stiff source terms.

Session 2 (Aug 31 10:20-12:20)

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Geometric Quasi-Linearization (GQL) for Bound-Preserving Schemes

Solutions to many partial differential equations satisfy certain bounds or constraints. For example, the density and pressure are positive for equations of fluid dynamics, and in the relativistic case the fluid velocity is upper bounded by the speed of light, etc. As widely realized, it is crucial to develop bound-preserving numerical methods that preserve such intrinsic constraints. Exploring provably bound-preserving schemes has attracted much attention and is actively studied in recent years. This is however still a challenging task for many systems especially those involving nonlinear constraints. Based on some key insights from geometry, we systematically propose a novel and general framework, referred to as geometric quasilinearization (GQL), which paves a new effective way for studying bound-preserving problems with nonlinear constraints. The essential idea of GQL is to equivalently transfer all nonlinear constraints into linear ones, through properly introducing some free auxiliary variables. We establish the fundamental principle and general theory of GQL via the geometric properties of convex regions, and propose three simple effective methods for constructing GQL. We apply the GQL approach to a variety of partial differential equations, and demonstrate its effectiveness and advantages for studying bound-preserving schemes, by diverse challenging examples and applications which cannot be easily handled by direct or traditional approaches.

[1] Kailiang, Wu and Chi-Wang, Shu *Geometric Quasilinearization (GQL) Framework for Analysis and Design of Bound-Preserving Schemes.* SIAM Review, 2023+

[2] Kailiang, Wu *Positivity-preserving analysis of numerical schemes for ideal magnetohydrodynamics.* SIAM Journal on Numerical Analysis, 2018

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An essentially oscillation-free discontinuous Galerkin method for hyperbolic conservation laws

In this talk, we propose a novel discontinuous Galerkin (DG) method to control the spurious oscillations when solving the scalar hyperbolic conservation laws. The spurious oscillations may be harmful to the numerical simulation, as it not only generates some artificial structures not belonging to the problems but also causes many overshoots and undershoots that make the numerical scheme less robust. To overcome this difficulty, we introduce a numerical damping term to control spurious oscillations based on the classic DG formulation. Compared to the classic DG method, the proposed DG method still maintains many good properties, such as the extremely local data structure, conservation, L^2-boundedness, optimal error estimates, and superconvergence. We also extend our methods to systems of hyperbolic conservation laws. Entropy inequalities are crucial to the well-posedness of hyperbolic conservation laws, which help to select the physically meaningful one among the infinite many weak solutions. By combining with quadrature-based entropy-stable DG methods, we also developed the entropy-stable OFDG method. For time discretizations, the modified exponential Runge–Kutta method can avoid additional restrictions of time step size due to the numerical damping. Extensive numerical experiments are shown to demonstrate our algorithm is robust and effective.

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Moment methods for a kinetic model of polar active matter

We propose a novel method of moments with an application to a kinetic equation associated with the celebrated Vicsek model, in which the over-damped active particles try to align with their neighbors. Our approach is analogous to a class of extended quadrature method of moments. It performs moment closure by assuming the angular distribution to be a sum of several homoscedastic kernels, each independently scaled and shifted. Efficient moment inversion algorithms are established by analyzing the well-posedness of the computational model. Then, we introduce a series of numerical tests including spatially homogeneous, one-dimensional and two-dimensional cases, which provide a solid validation of the method. This is a joint work with Yihong Chen, Wen-An Yong, Hui Yu and Ruixi Zhang at Tsinghua University.

Liwei Lu
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Weak Collocation Regression method: fast reveal hidden stochastic dynamics from high-dimensional aggregate data

Revealing hidden dynamics from the stochastic data is a challenging problem as the randomness takes part in the evolution of the data. The problem becomes exceedingly hard if the trajectories of the stochastic data are absent in many scenarios. In this work, we propose the Weak Collocation Regression (WCR) method to learn the dynamics from the stochastic data without the labels of trajectories. This method utilize the governing equation of the probability distribution function–the Fokker-Planck (FP) equation. Using its weak form and integration by parts, we move all the spacial derivatives of the distribution function to the test functions which can be computed explicitly. Since the data is a sampling of the corresponding distribution function, we can compute the integrations in the weak form, which has no spacial derivatives on the distribution functions, by simply adding the values of the integrands at the data points. We further assume the unknown drift and diffusion terms can be expanded by the base functions in a dictionary with the coefficients to be determined. Cooperating the collocation treatment and linear multi-step methods, we transfer the revealing process to a linear algebraic system. Using the sparse regression, we eventually obtain the unknown coefficients and hence the hidden stochastic dynamics. The numerical experiments show that our method is flexible and fast, which reveals the dynamics within seconds in the multi-dimensional problems and can be extended to high dimensional data. The complex tasks with variable-dependent diffusion and coupled drift can be correctly identified by WCR and the performance is robust, achieving high accuracy in the cases of noisy data. The rigorous error estimate is also included to support our various numerical experiments.

Session 3 (Aug 31 15:50-17:50)

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Stability and error estimates of LDG-IMEX-BDF methods for convection-diffusion equation

In this talk, the stability analysis and optimal error estimates are presented for a kind of fully discrete schemes for solving one-dimensional linear convection-diffusion equation with periodic boundary conditions. The fully discrete schemes are defined with local discontinuous Galerkin (LDG) spatial discretization methods coupled with implicit-explicit (IMEX) temporal discretization methods based on backward difference formulas (BDF). By combining an improved multiplier technique used in the stability analysis for multistep methods and the technique to deal with derivative and jump in LDG methods, we establish a general framework of stability analysis for the corresponding fully discrete LDG-IMEX-BDF schemes up to fifth order in time. The considered schemes are proved to be unconditionally stable, in the sense that a properly defined "discrete energy" is dissipative if the time step is upper bounded by a constant which is independent of the mesh size. Optimal orders of the L^2 norm accuracy in both space and time are also proved by energy analysis. Numerical tests are presented to validate the theoretical results.

[1] Haijin Wang and Xiaobin Shi and Qiang Zhang, *Stability and error estimates of local discontinuous Galerkin methods with implicit-explicit backward difference formulas up to fifth order for convection-diffusion equation.* Journal of Scientific Computing , 2023

Qi Tao
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Accuracy-enhancement of discontinuous Galerkin methods for PDEs containing high-order spatial derivatives

In this talk, we shall first introduce the accuracy-enhancement of discontinuous Galerkin (DG) methods for solving PDEs with high-order spatial derivatives. It is well known that there are highly oscillatory errors for finite element approximations to PDEs that contain hidden superconvergence points. To exploit this information, a Smoothness-Increasing Accuracy Conserving (SIAC) filter is used to create a superconvergence filtered solution. This is accomplished by convolving the DG approximation against a B-spline kernel. We then present theoretical error estimates in the negative-order norm for the local DG and ultra-weak local DG approximations to PDEs containing high order spatial derivatives. Numerical results will be shown to confirm the theoretical results.

[1] Q. Tao, L. Ji, J.K. Ryan and Y. Xu. *Accuracy-enhancement of discontinuous Galerkin methods for PDEs containing high order spatial derivatives*, Journal of Scientific Computing, v93 (2022), 13.

Hui Yu
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Traceability of Water Pollution: An Inversion Scheme Via Dynamic CGO Solutions

We aim to find the time-dependent source term in the diffusion equation from the boundary measurement, which allows for the possibility of tracing back the source of pollutants in the environment. Based on the idea of dynamic complex geometrical optics (CGO) solutions, we analyze a variational formulation of the inverse source problem and prove the uniqueness result. A two-step reconstruction algorithm is proposed, which first recovers the locations of the point sources, and then the Fourier components of the emission concentration functions are reconstructed. Numerical experiments on simulated data are conducted. The results demonstrate that our proposed two-step reconstruction algorithm can reliably reconstruct multiple point sources and accurately reconstruct the emission concentration functions. In addition, we decompose the algorithm into two parts: online and offline computation, with most of the work done offline. This paves the way toward real-time traceability of the pollution. The proposed method can be used in many fields, particularly those related to water pollution, to identify the source of a contaminant in the environment and can be a valuable tool in protecting the environment.

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Flow Measurement: An Inverse Problem Formulation

In this report, we propose a new mathematical formulation for flow measurement based on the inverse source problem for wave equations with partial boundary measurement. Inspired by the design of acoustic Doppler current profilers (ADCPs), we formulate an inverse source problem that can recover the flow field from the observation data on boundary receivers. To our knowledge, this is the first mathematical model of flow measurement using partial differential equations. This model is proved well-posed, and the corresponding algorithm is derived to compute the velocity field efficiently. Extensive numerical simulations are performed to demonstrate the accuracy and robustness of our model. The comparison results demonstrate that our model is ten times more accurate than ADCP. Our formulation is capable of simulating a variety of practical measurement scenarios.

MS 11: Recent advances on mathematical imaging, scientific computing and machine learning

Organizers:

Daoping Zhang
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Session 1 (Aug 28 11:00-12:30)

David Gu
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Characteristic Class of Fiber Bundle for Meshing

Structured mesh generation plays a crucial role in CAD and CAE. In this talk, we introduce the recent theoretic development of surface quad-meshing based on holomorphic line bundle theory. We show each quad-mesh can be treated as a meromorphic quartic differential and its singularities satisfy the Abel-Jacobi condition. This theorem leads to the proofs of non-existence of special structured meshes and practical algorithms.

Na Lei
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Conformal Geometric Methods for Medical Imaging Applications

Conformal geometric methods are crucial for surface registration and shape analysis. By using conformal mappings, 3D shapes can be mapped onto planar domains, and compared on the 2D plane; furthermore, the planar shapes of the images give the global geometric invariants, the conformal modules. This framework greatly improves the effeciency and accuracy for geometric analysis. For examples, in virtual colonoscopy, supine and prone colon wall surfaces are flattened onto planar regions using Ricci flow method, and registered by quasi-conformal mappings. In ossicular chain examination, the ricci curvature is used to segment the object and the conformal module is applied to detect abnormalities.

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Learnable Fourier interpolation for medical image reconstruction

Image reconstruction from down-sampled and corrupted measurements, such as fast MRI and sparse view CT problem, is mathematically ill-posed inverse problem. Deep neural network (DNN) has been becoming a prominent tool in the recent development of medical image reconstruction methods. In this talk, I will introduce our recent work on how to learn an interpolation scheme in Fourier domain for predicting Fourier coefficients for both CT and MRI reconstruction problem. The experiments on both CT and MRI experiments showed that, in comparison to existing DNN-based solutions, the proposed DNN-based Fourier interpolation method not only provided the state-of-the-art performance, but also is much more computationally efficient. The idea is further investigated for solving Electrical Impedance Tomography with D-bar methods. I will report some preliminary results.

Session 2 (Aug 28 15:50-17:50)

Min Zhang
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A Novel Extrem compression-rate Image Compression Framework Based on Optimal Transport Mapping

In this talk, I will propose a new high-fidelity image compression method using optimal transport (OT) mapping to obtain extrem compression ratio while still preserving fine details about the image. First, I will briefly introduce the theory of OT mapping and how to combine generative adversarial network (GAN) with it to build a compression system. Then I will describe our proposed method. Experimental results show that our method has better performance under objective criteria than other GAN-based methods.

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Efficient Algorithms to Compute Elastic Shape Distances between Closed Curves

Shape analysis is a fundamental area of computer vision, critical to object recognition, classification, shape retrieval. Central to shape analysis is the definition of a shape space and efficient algorithms to compute the distance between the shape representations residing in the shape space. In this work, we adopt the square-root velocity functions of Srivastava et al. (PAMI, 2011) as the shape representation, and we develop efficient optimization algorithms to compute the shape distance between two closed curves based on the SRVF representations. This shape representation is elastic, thus a monotonically-increasing reparameterization function matching the SRVF of one curve to the other needs to be computed. Moreover, the starting point on one closed curve needs to be matched to the right starting point on the other closed curve, and the curve needs to be optimally rotated as well to match the other curve. These requirements lead to a challenging optimization problem, in which the free variables are the starting point on the curve, the rotation angle, and the reparameterization function. To solve this problem, we introduce compact discretizations of curves, then develop novel optimization algorithms, by building on efficient solutions for the subproblems, i.e. FFT-based rigid alignment, linear-time dynamic programming and iterative optimization for reparameterization. We integrate these in a global optimization framework and obtain efficient algorithms that compute strong minima. We demonstrate the effectiveness of our elastic shape distance algorithms with extensive numerical experiments, and find significant improvements compared to previous approaches.

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Can we see shape from data

We suggest an approach to the shape DNA of data based on a number metric invariants intro- duced by Grove and Markvorsen that encode its essential global geometry of the given structure. Furthermore, the proposed invariants are correlated to and compared with more classical geometric measures stemming from Complex Functions Theory and employed in Graphics. First experiments on real life networks and on natural images are given to demonstrate the feasibility of this approach. This represents joint work with Vladislav Barkanass.

Session 3 (Aug 30 11:00-12:30)

Haixia Liu
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Distance Covariance-based Fair Classification

In this talk, we introduce a distance covariance-based method for fair classification tasks. We propose a suitable regularization term for fairness in machine learning, building upon distance empirical covariance between the predicted label (feature map) and sensitive attribute(s). Since the balanced parameter is difficult to set in hand, we consider it as a dual variable and update alternatingly between model parameters and the balanced parameter. In addition, we characterize theoretically the convergence of empirical distance covariance to population distance covariance in probability, and evaluate our proposed method on four datasets. Numerical results demonstrate its strong performance.

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Segmentation and Classification using Deep Learning Technologies

Deep learning technologies have revolutionised many fields including computer vision and image processing. Their success generally relies on big data. However, for the data scarcity scenarios like in medical imaging, their performance could drop significantly. Moreover, in many cases, they also lack generalisation (e.g. the cross-domain adaptation problem) and explanation (e.g. explainable AI). In this presentation, I will introduce some of our recent work on segmentation and classification targeting those challenges, such as subspace feature representations for few-shot learning, cross-domain adaptation in point clouds, multilevel explainable AI, etc.

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A Unifying Framework for n-Dimensional Quasi-Conformal Mappings

With the advancement of computer technology, there is a surge of interest in effective mapping methods for objects in higher-dimensional spaces. To establish a one-to-one correspondence between objects, higher-dimensional quasi-conformal theory can be utilized for ensuring the bijectivity of the mappings. In addition, it is often desirable for the mappings to satisfy certain prescribed geometric constraints and possess low distortion in conformality or volume. In this talk, we develop a unifying framework for computing n-dimensional quasi-conformal mappings. More specifically, we propose a variational model that integrates quasi-conformal distortion, volumetric distortion, landmark correspondence, intensity mismatch, and volume prior information to handle a large variety of deformation problems. We further prove the existence of a minimizer for the proposed model and devise efficient numerical methods to solve the optimization problem. We demonstrate the effectiveness of the proposed framework using various experiments in two and three dimensions, with applications to medical image registration, adaptive remeshing, and shape modeling.

MS 12: Recent Advances in Analysis and Computation of Nonlocal Models

Organizers:

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Session 1 (Aug 28 11:00-12:30)

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Monotone meshfree methods for linear elliptic equations in non-divergence form via nonlocal relaxation

We design a maximum principle preserving meshfree finite difference method for linear elliptic PDEs on point clouds via a nonlocal relaxation method. The key idea is a novel combination of a nonlocal integral relaxation of the PDE problem with a robust meshfree discretization on point clouds. Minimal positive stencils are obtained through a linear optimization procedure that automatically guarantees the stability and, therefore, the convergence of the meshfree discretization. A major theoretical contribution is the existence of consistent and positive stencils for a given point cloud geometry. We provide sufficient conditions for the existence of positive stencils by finding neighbors within an ellipse (2d) or ellipsoid (3d) surrounding each interior point. Our result represents a significant improvement in the stencil width estimate for positive-type finite difference methods for linear elliptic equations in the near-degenerate regime (modeling high-contrast media), compared to previously known works in this area. Numerical results are conducted in 2d and 3d, examining a range of ellipticity constants including the near-degenerate regime. Some new experiments on degenerate elliptic equations will also be discussed.

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A conforming DG method for nonlocal problem with integrable kernels and its applications

The numerical solution of nonlocal constrained value problems with integrable kernels is considered in this talk. These nonlocal problems arise in nonlocal mechanics and nonlocal diffusion. The structure of the true solution to the problem is analyzed first. The analysis leads naturally to a new kind of discontinuous Galerkin method that can more efficiently solve the problem numerically. The new method is shown to be asymptotically compatible. Moreover, it has optimal convergence rate for any dimensional case under mild assumptions. We also give some applications of this method, such as to diffusion and sub-diffusion equations.

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Several efficient collocation methods for nonlocal problem with weak singularity

After a short introduction about Peridynamics, Jacobi-Gauss Quadrature rule is used in collocation methods for one-dimension nonlocal problems with weak singularity, and some analysis conclusions and numerical results are shown in the talk. About multi-dimension nonlocal problems, just initial results are given also.

Session 2 (Aug 29 11:00-12:30)

James SCOTT
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Nonlocal Korn Inequalities and Applications

Motivated by nonlocal models in continuum mechanics, we present several nonlocal analogues of Sobolev rigidity relations for vector fields. In each setting, we show that a class of vector field spaces whose semi-norm involves the magnitude of "directional" difference quotients is in fact equivalent to the class of vector fields characterized by a semi-norm involving the full difference quotient. This equivalence can be considered a Korn-type characterization of nonlocal Sobolev spaces. We apply these inequalities to obtain quantitative statements for solutions to variational problems arising in peridynamics and dislocation models. We additionally demonstrate nonlocal analogues of a Poincaré-Korn inequality for a nonlinear peridynamic-type model characterized by a nonconvex strain energy.

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Coupling of atomistic and bond-based peridynamic models using an extended Arlequin framework

A general nonlocal coupling technique between an atomistic (AM) model and the bond-based peridynamic (PD) model is proposed, based on the Arlequin framework. This technique applies the complementary weight function and constraint conditions to transmit energies through the overlapping region between the AM and PD regions. We extend the original Arlequin framework to discrete cases by redefining constraint conditions by the peridynamic differential operator, which enables the interpolation and corresponding derivative of scattered data. Besides, the preconditioning of calibration for the PD effective micromodulus is implemented to guarantee the equilibrium of energy. One-dimensional benchmark tests investigate the coupling

effects influenced by several key factors, including the coupling length, weight function, grid size and horizon in the PD model, and constraint conditions. Two- and three-dimensional numerical examples are provided to verify the applicability and effectiveness of this coupling model. Results illustrate this AM–PD coupling model takes the mutual advantages of the computational efficiency of PD model and the accuracy of AM model, which provides a flexible extension of the Arlequin framework to couple particle methods.

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Stability analysis and robust numerical computation of a nonlocal traffic flow model for connected vehicles

Emerging connected and automated vehicle technologies present unique challenges and opportunities in the modeling of nonlocal inter-vehicle interactions and their effects on traffic flows. This talk investigates a particular nonlocal traffic flow model, revealing that asymptotic stability of traffic flow is attainable under suitable assumptions on nonlocal information utilization. Simultaneously, we present a family of numerical schemes for the model that demonstrate robustness with respect to changes in the nonlocal horizon parameter. With suitable discretization of the nonlocal integral, the convergence of numerical solutions is shown to lead towards the weak entropy solution of the respective local model as both the mesh size and the nonlocal horizon parameter approach zero. The findings may serve to inform the development of future driving algorithms for connected vehicles.

Session 3 (Aug 30 11:00-12:30)

Jiwei ZHANG
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An efficient implementation of 3D FEM for nonlocal Poisson problem with different ball approximation strategies

Nonlocality brings many challenges to the implementation of finite element methods (FEM) for nonlocal problems, such as large number of queries and invoke operations on the meshes. Besides, the interactions are usually limited to Euclidean balls, so direct numerical integrals often introduce numerical errors. The issues of interactions between the ball and finite elements have to be carefully dealt with, such as using ball approximation strategies. In this talk, an efficient representation and construction methods for approximate balls are presented based on combinatorial map, and an efficient parallel algorithm is also designed for assembly of nonlocal linear systems. Specifically, a new ball approximation method based on Monte Carlo integrals, i.e., the fullcaps method, is also proposed to compute numerical integrals over the intersection region of an element with the ball.

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Nonlocal diffusion model with Dirichlet boundary condition

In this talk, we present several nonlocal diffusion models for Poisson equation with Dirichlet boundary condition. For nonlocal model, Neumann boundary condition is easy to handle while Dirichlet boundary is more sophiscated. We will give several different approaches to approximate Dirichlet boundary in nonlocal diffusion model. Wellposedness and vanishing nonlocality convergence of the nonlocal model are analyzed.

MS 13: Inverse problems with applications in imaging science

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Session 1 (Aug 28 15:50-17:50)

Andrej Brojatsch
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On the required number of electrodes in an EIT-related problem.

Medical imaging and non-destructive testing holds the challenge of determining information of the interior of a body by taking measurements at its boundary. Motivated by impedance-based corrosion detection, we consider the problem of reconstructing an unknown Robin transmission coefficient on a known interior boundary from voltage/current measurements on electrodes attached to an outer boundary. We derive a mathematically sufficient criterion to check how many electrodes are required to achieve unique solvability of the resulting non-linear inverse problem, and we discuss globally convergent numerical reconstruction methods.

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Learned reconstruction methods and convergent regularization: linear plug-and-play denoiser

The question if a reconstruction algorithm provides a convergent regularization has been long studied in inverse problems, as it provides more than just the knowledge that a solution can be computed at a certain noise level. It tells us that stable solutions exist for all noise realizations and even more importantly that in the limit case, when noise vanishes, we obtain a solution of the underlying operator equation. In other words, we can guarantee mathematically that obtained solutions are indeed solutions to the inverse problem.

This is in contrast to some novel data-driven approaches where we may only guarantee that obtained solutions are minimizers of the empirical loss, given suitable training data. Consequently, the concept of convergent data-driven reconstructions has gained considerable interest very recently.

In this talk we shortly give an overview of learned reconstruction methods with convergence guarantees and then continue to discuss the popular framework of plug-and-play (PnP) denoising, which uses of the shelf denoisers in an iterative framework. We will specifically consider linear denoisers and propose a novel spectral filtering technique to control the strength of regularization arising from the denoiser. This allows us to show that PnP with linear denoisers leads to a convergent regularization scheme.

This is joint work with Ferdia Sherry, Subhadip Mukherjee, and Carola-Bibiane Schönlieb.

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SGD for selected inverse problems in Banach spaces

In this work we present a mathematical framework and analysis for a minibatch SGD in Banach spaces for select linear and non-linear inverse problems. Analysis in the Banach space setting presents unique challenges, requiring novel mathematical tools. This is achieved by combining insights from Hilbert space theory with approaches from modern optimisation. The developed theory and algorithms open doors for a wide range of applications, and we present some future challenges and directions.

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Bayesian hypothesis testing in statistical inverse problems

In many inverse problems, one is not primarily interested in the whole solution $u^\dagger$, but in specific features of it that can be described by a family of linear functionals of $u^\dagger$. We perform statistical inference for such features by means of hypothesis testing.

This problem has previously been treated by multiscale methods based upon unbiased estimates of those functionals [1]. Constructing hypothesis tests using unbiased estimators, however, has two severe drawbacks: Firstly, unbiased estimators only exist for sufficiently smooth linear functionals, and secondly, they suffer from a huge variance due to the ill-posedness of the problem, so that the corresponding tests have bad detection properties. We overcome both of these issues by considering the problem from a Bayesian point of view, assigning a prior distribution to $u^\dagger$, and using the resulting posterior distribution to define Bayesian maximum a posteriori (MAP) tests.

The existence of a hypothesis test with maximal power among a class of tests with prescribed level has recently been shown for all linear functionals of interest under certain a priori assumptions on $u^\dagger$ [2]. We study Bayesian MAP tests bases upon Gaussian priors both analytically and numerically for linear inverse problems and compare them with unregularized as well as optimal regularized hypothesis tests.

[1] K. Proksch, F. Werner, A. Munk *Multiscale scanning in inverse problems.* Ann. Statist 46(6B), 2018

[2] R. Kretschmann, D. Wachsmuth, F. Werner *Optimal regularized hypothesis testing in statistical inverse problems.* Preprint, arXiv: 2212:12897, 2022

MS 14: Recent Developments in Mathematical Models and Fast Algorithms of Image Processing

Organizers:

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Session 1 (Aug 28 11:00-12:30)

Huibin Chang
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A convergent splitting algorithm for the Euler Elastica model

We propose a new reformulation for the Euler Elastica (EE) model based on a bilinear deccomposition of the gradient and the variable splitting technique, effectively eliminating the singularity and mitigating the nonlinearity in the original EE model. We then develop a fast convergent alternating minimization algorithm for the reformulated, discretized EE model based on the bilinear decomposition. Under certain mild conditions, the proposed algorithm is proven to produce a globally convergent minimizing sequence.

[1] Liang-Jian Deng, Roland Glowinski, and Xue-Cheng Tai. *A new operator splitting method for the Euler elastica model for image smoothing*, SIAM Journal on Imaging Sciences, 12(2), 2019: 1190-1230.

[2] Fang He, Xiao Wang, and Xiaojun Chen, *A Penalty Relaxation Method for Image Processing Using Euler's Elastica Model*, SIAM Journal on Imaging Sciences, 14(4),2021: 389-417.

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Knowledge and Data-driven Medical Image Reconstruction

Magnetic resonance imaging (MRI) plays an important role in the diagnosis and treatment of some diseases, such as tumors. However, due to its long acquisition time, patients often feel uncomfortable, which can easily generate motion artifacts and thus affect imaging quality. Therefore, accelerating MRI acquisition is of great significance. This talk will present solutions from the following perspectives: 1) Knowledge and data-driven MRI reconstruction; 2) Multimodal assisted MRI reconstruction; 3) MR image segmentation guided reconstruction. Finally, the development trends will be discussed.

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Image deblurring and denoising with the prior setting

Image deblurring and denoising are fundamental issues in imaging science. Mathematical methods and learning neural network are effective for these tasks. I will discuss how to solve these problem with some insights by those methods.

[1] Jun Liu, Ming Yan, Tieyong Zeng, *Surface-aware Blind Image Deblurring*, IEEE Transaction on Pattern Analysis and Machine Intelligence, 2021, 43(3) : 1041–1055.

[2] Xiao-le Tang, Xi-le Zhao, Jun Liu, Jian-Li Wang, Yuchun Miao, Tieyong Zeng, *Uncertainty-Aware Unsupervised Image Deblurring with Deep Residual Prior*, IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2023, 9883–9892.

[3] Yingjie Liu, Yuxu Lu, Faming Fang, Jianing Sun, Yingxiang Xu, DGANet: Deep Learning for Despeckling with Approximation, Manuscript, 2023.

Session 2 (Aug 29 11:00-12:30)

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Non-local means theory and its extension

The Non-local Means (NLM) algorithm has demonstrated strong denoising performance and is widely employed across various domains of artificial intelligence. This report examines the convergence theory of the NLM algorithm, addressing the theoretical problem of window parameter selection to substantially enhance the algorithm's performance. Furthermore, we investigate the smoothing kernel of the NLM algorithm and develop the adaptive parameter algorithm NLM-L2 as well as the extended NLM algorithm through mean square error minimization. To enhance algorithm generalization, we transform Gaussian noise into independent and identically distributed noise with a zero mean (including Laplace noise and noise resulting from stable variance transformation of Poisson noise). Additionally, through minimization of the absolute error, we obtain the corresponding adaptive parameter algorithm NLM-L1.Theoretical analysis indicates that NLM-L1 poses greater challenges compared to NLM-L2 but demonstrates stronger robustness. Expanding on the theoretical research, the NLM algorithm is incorporated with low-rank matrix decomposition, resulting in a novel algorithm that delivers exceptional denoising performance and effectively addresses matrix completion with randomly missing entries.

[1] Q. Jin, I. Grama, C. Kervrann, Q. Liu, *Nonlocal means and optimal weights for noise removal*, SIAM Journal on Imaging Sciences, 2017, 10 (4), 1878-1920.

[2] Q. Jin, I. Grama, Q. Liu, *Convergence Theorems For The Non-Local Means Filter*, Inverse Problems and Imaging, 2018, 12(4): 853-881.

[3] Q. Jin, G. Facciolo, J.M. Morel. A Review of an Old Dilemma: Demosaicking First, or Denoising First? 2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops (CVPRW). IEEE, 2020.

[4] J. Guo, Y. Guo, Q. Jin, M.K. Ng, S. Wang, *Gaussian Patch Mixture Model Guided Low-Rank Covariance Matrix Minimization for Image Denoising*, SIAM Journal on Imaging Sciences, 2022, 15 (4), 1601-1622.

[5] Z. Jia, Q. Jin(co-first author), M. K. Ng, X-L. Zhao, *Non-Local Robust Quaternion Matrix Completion for Large-Scale Color Image and Video Inpainting*, IEEE Transactions on Image Processing, 2022,31:3868 - 3883.

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Phase retrieval from incomplete data via weighted nuclear norm minimization

In this talk, we delve into the fascinating problem of phase retrieval, specifically focusing on the recovery of an unknown object from the magnitude of its Fourier transform. We investigate a particularly challenging scenario where the observed intensity values are incomplete, further complicated by the presence of both salt-and-pepper and random-valued impulse noise. To exploit the inherent low-rank characteristics of the object image, we employ a regularization term that penalizes high weighted nuclear norm values of image patch groups. Additionally, we adopt the ℓ_{1-2} metric as the data fidelity term to effectively handle outliers. To address the optimization problem, we decompose it into smaller subproblems, utilizing techniques like weighted nuclear norm proximal mapping and ℓ_{1-2} minimization, which have closed-form solutions despite being nonconvex and nonsmooth. During this talk, we present the convergence results of our proposed method and experimental results that demonstrate the effectiveness and superiority of our approach in overcoming the challenges posed by incomplete and noisy observations during phase retrieval.

[1] Zhi Li, Ming Yan, Tieyong Zeng, Guixu Zhang, *Phase retrieval from incomplete data via weighted nuclear norm minimization*, Pattern Recognition, Volume 125, 2022, 108537.

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Hyperspectral Super-Resolution via Interpretable Block-Term Tensor Modeling

Hyperspectral super-resolution (HSR) aims at fusing a pair of hyperspectral and multispectral images to recover a super-resolution image (SRI). This work revisits coupled tensor decomposition (CTD)-based HSR. We employ an idea that models spectral images as tensors following the block-term decomposition model with multilinear rank-(Lr, Lr, 1) terms (i.e., the LL1 model) and formulates the HSR problem as a coupled LL1 tensor decomposition problem. Similar to the existing CTD approaches, recoverability of the SRI is shown under mild conditions. More importantly, the latent factors of the LL1 model can be interpreted as the key constituents of spectral images, i.e., the endmembers' spectral signatures and abundance maps. This connection allows us to incorporate prior information for performance enhancement. A flexible algorithmic framework that can work with a series of structural information is proposed to take advantages of the model interpretability. The effectiveness is showcased using simulated and real data.

[1] N. Yokoya, T. Yairi, and A. Iwasaki, *Coupled nonnegative matrix factorization unmixing for hyperspectral and multispectral data fusion*,IEEE Trans. Geosci. Remote Sens., vol. 50, no. 2, pp. 528–537, 2012.

[2] C. I.Kanatsoulis, X. Fu, N. D. Sidiropoulos, andW.-K.Ma, *Hyperspectral super-resolution: A. coupled tensor factorization approach*, IEEE Trans. Signal Process., vol. 66, no. 24, pp. 6503–6517, 2018.

[3] C. Prèvost, K. Usevich, P. Comon, and D. Brie, *Hyperspectral super-resolution with coupled tucker approximation: Recoverability and SVD-based algorithms*, IEEE Trans. Signal Process., vol. 68, pp. 931–946, 2020.

Session 3 (Sep 01 14:30-16:30)

Zhi-Feng Pang
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Image Segmentation Based on the Hybrid Bias Field Correction.

Image segmentation is the foundation for analyzing and understanding high-level images. How to effectively segment an intensity inhomogeneous image into several meaningful regions in terms of human visual perception and ensure that the segmented regions are consistent at different resolutions is still a very challenging task. In order to describe the structure information of the intensity inhomogeneous efficiently, this paper proposes a novel hybrid bias field correction model by decoupling the multiplicative bias field and the additive bias field. These kinds of bias fields are assumed to be smooth, so can employ the Sobolev space W 1,2 to feature them and use a constraint to the multiplicative bias field. Since the proposed model is a constrained optimization problem, we use the Lagrangian multiplier method to transform it into an unconstrained optimization problem, and then the alternating direction method can be used to solve it. In addition, we also discuss some mathematical properties of our proposed model and algorithm. Numerical experiments on the natural images and the medical images demonstrate performance improvement over several state-of-the-art models.

[1] ZF Pang, Z Guan, Y Li, K Chen, H Ge. *Image Segmentation Based on the Hybrid Bias Field Correction*, Applied Mathematics and Computation, 452:128050, 2023.

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Image segmentation using Bayesian inference for convex variant Mumford-Shah variational model
Joint work with Xu Xiao, Raymond Chan and Tieyong Zeng

The smoothing and thresholding (SaT) approach is a convex variant of the Mumford-Shah model to segment the image. The SaT approach separates the segmentation into two stages: first, a convex energy function is minimized to obtain a smoothed image; then, a thresholding technique is applied to segment the smoothed image. The energy function consists of three weighted terms and the weights are called the regularization parameters. Selecting appropriate regularization parameters is crucial to achieving effective segmentation results. Traditionally, the regularization parameters are chosen by trial-and-error, which is a very time-consuming procedure and is not practical in real applications. In this paper, we apply a Bayesian inference approach to infer the regularization parameters and estimate the smoothed image. Experimental results show that the proposed approach is capable of generating high-quality segmentation results.

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Image recovery under non-Gaussian noise

Cauchy noise, as a typical non-Gaussian noise, appears frequently in many important fields, such as radar, medical, and biomedical imaging. Here, we focus on image recovery under Cauchy noise. Instead of the celebrated total variation or low-rank prior, we adopt a novel deep-learning-based image denoiser prior to effectively remove Cauchy noise with blur. To preserve more detailed texture and better balance between the receptive field size and the computational cost, we apply the multi-level wavelet convolutional neural network (MWCNN) to train this denoiser. Frequently appearing in medical imaging, Rician noise leads to an interesting nonconvex optimization problem, termed as the MAP-Rician model, which is based on the Maximum a Posteriori (MAP) estimation approach. As the MAP-Rician model is deeply rooted in Bayesian analysis, we want to understand its mathematical analysis carefully. Moreover, one needs to properly select a suitable algorithm for tackling this nonconvex problem to get the best performance. Indeed, we first present a theoretical result about the existence of a minimizer for the MAP-Rician model under mild conditions. Next, we aim to adopt an efficient boosted difference of convex functions algorithm (BDCA) to handle this challenging problem. Theoretically, using the Kurdyka-Lojasiewicz (KL) property, the convergence of the numerical algorithm can be guaranteed.

[1] Tingting Wu, Xiaoyu Gu, Zeyu Li, Zhi Li, Jianwei Niu, Tieyong Zeng, *Efficient boosted DC algorithm for nonconvex image restoration with Rician noise*, SIAM Journal on Imaging Sciences, 2022, 15(2): 424-454.

[2] Tingting Wu, Wei Li, Shilong Jia, Yiqiu Dong, Tieyong Zeng, *Deep multi-level wavelet-CNN denoiser prior for restoring blurred image with Cauchy noise*, IEEE Signal Processing Letters, 2020, 27: 1635-1639.

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A smoothing technique for non-smooth optimization with applications to constrained minimax problems

The minimax problems are an important class of optimization problems arising in image processing and machine learning. Efficient algorithms for solving strongly-convex-concave type of minimax problems have been proposed. However, more general cases, in particular the constrained minimax problems are much less studied in the literature. We propose a new framework for solving constrained semi-infinite minimax problems, based on a smoothing technique for the maximum function. The convergence property and the convergence rate are presented.

MS 15: Mathematical analysis and high-performance solvers for wave scattering problems

Organizers:

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Session 1 (Aug 29 11:00-12:30)

Xue Jiang
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A PML method for signal-propagation problems in axon

This talk is focused on the modelling of signal propagations in myelinated axons. Based on reasonable assumptions on the medium properties, we derive a two-dimensional neural-signaling model in cylindrical coordinates from the time-harmonic Maxwell's equations. The well-posedness of model is established upon Dirichlet boundary conditions at the two ends of the neural structure and the radiative condition in the radial direction of the structure. Using the perfectly matched layer (PML) method, we truncate the unbounded background medium and propose an approximate problem on the truncated domain. The well-posedness of the PML problem and the exponential convergence of the approximate solution to the exact solution are established. Numerical experiments are presented to demonstrate the theoretical results and the efficiency of our methods to simulate the signal propagation in axons.

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A fast algorithm for the scattering by layered multiple cavities in 2D

This talk concerns the electromagnetic scattering by layered multiple rectangular cavities in TM or TE polarization. A transparent boundary condition on the opening aperture which involves a weakly singular integral operator is introduced to reformulated the problem into bounded cavities. A numerical quadrature rule is proposed to efficiently discretize the integral operator. Based on the Fourier series expansion, the scattering problem is reduced to ordinary differential equations for those Fourier coefficients, then a system which only contains those coefficients on the aperture is yield. After the system is solved, the total field can be immediately obtained. Some numerical experiments are presented to demonstrate the efficiency of the proposed method.

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Nodal discontinuous Galerkin methods for Maxwell's equations in nonlinear optical media

In this talk, we consider the nonlinear Maxwell equations in optical medium characterized by instantanenous linear response, linear Lorentz dispersion, nonlinear cubic Kerr and Raman effects. Using leapfrog time discretization and nodal DG spatial discretization, as well as some novel strategies to handle the nonlinearities, we develop an efficient and second order accurate DG scheme for the nonlinear Maxwell equations. Semi-discrete in space error estimates and fully discrete energy stability are established. Numerical examples are provided to demonstrate the accuracy, efficiency, energy stability and scalability of the proposed method.

Session 2 (Sep 01 10:20-12:20)

Lei Zhang
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Analysis and Calculation for Composite Scattering in Multilayered Media

In this talk, we consider the mathematical analysis and numerical method pertaining to composite scattering within a multilayered medium. The composite scattering problem encompasses the interaction of electromagnetic waves with both the interface of the medium and the obstacles present within it, particularly in the context of unbounded rough surfaces. These matters hold immense significance in various domains such as remote sensing, non-destructive testing, geophysics, national defense, military applications, and more. Our focus will be on exploring the time-harmonic and time-domain problems while highlighting the latest advancements in the field.

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Fourier method for inverse source problem using correlation of passive measurements

We consider the inverse source problem for a Cauchy wave equation with passive cross-correlation data. We propose to consider the cross-correlation as a wave equation itself and reconstruct the cross-correlation in the support of the source for the original Cauchy wave equation. Having access to the cross-correlation in the support of the source, we use a Fourier method to reconstruct the source of the original Cauchy wave equation. We show the inverse source problem is ill-posed and suffer from non-uniqueness when the mean of the source is zero, and provide a uniqueness result and stability estimate in case of non-zero mean.

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Reduce model for elastic scattering of multiples cracks

In this talk, we delve into the analysis of scattering phenomena involving multiple cracks in a two-dimensional setting. Our primary focus is on the development of a reduced model, which allows us to handle a significantly larger number of arcs when compared to conventional discretization techniques. To construct the reduced model, we exploit the solution manifold associated with a single arc problem. By making certain assumptions about the underlying perturbations, we demonstrate that we can extend this reduced basis of a single crack to the multiple cracks problem effectively. Furthermore, we explore the smoothness of the shape-to-solution map, which enables us to establish the exponential convergence of our proposed method. This analysis provides valuable insights into the accuracy and efficiency of our approach. We remark that out method is not limited to the two dimensioanl multiple cracks problem but can be extended to other scattering problems on multiples domains.

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Fast multipole method in layered media: from Helmholtz to Maxwell's equations

In this talk, a fast multipole method (FMM) for the dyadic Green's function of Maxwell's equations in layered isotropic media is presented. As in the homogeneous media, layered dyadic Green's function (LDGF) of Maxwell's equation is shown closely related to the Green's function of Helmholtz equation in layered media. Actually, there are only two essential components in the LDGF. By following the theory developed for the Green's function of Helmholtz equation, we derive multipole expansions (MEs) and local expansions (LEs) as well as the multipole-to-local translation (M2L) operators for all the reaction field components of the LDGF. Then, the FMMs for the

LDGF is implemented with the target particles and equivalent polarization sources associated with the reaction field components. Numerical results validate the fast convergence of the MEs and the O(N) complexity of the FMM for N particle problem in 3-D layered media.

MS 16: Multilevel Methods for Numerical PDE

Organizers

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Session 1 (Aug 30 11:00-12:30)

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Scalar auxiliary variable approach in iterative minimization formulation for saddle point search

Saddle points have been extensively investigated in the study of activated process in gradient flow driven by free energy. In this talk, we aim to use the iterative minimization formulation (IMF) coupled with the scalar auxiliary variable (SAV) approach to locate the transition states of activated processes in the H^{-1} gradient flow, i.e., index-1 saddle points of the corresponding energy in H^{-1} metric. In each cycle of the IMF, we introduce the SAV approach to minimize the auxiliary functional. A general principle of constructing linear, efficient and robust energy stable schemes for this approach is presented. This new SAV based IMF method improves the efficiency of saddle point search and can be implemented easily for different free energies. By conducting some numerical experiments for the Ginzburg-Landau and the Landau-Brazovskii free en- ergies, the efficient performance of the proposed method is validated.

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Fast ensemble methods for dual-porosity-Stokes model with uncertainty quantification

The coupling of dual porosity seepage flow with free flow arises in modeling fractured porous media with large conduits. Those problems are encountered in many applications,like petroleum extraction, hydrology, geothermal systems and so on. However, accurate simulations are usually not feasible due to the fact it is physically impossible to know the exact parameter values in the model of interest.In this talk, we focus on the fast simulation for a stochastic dual-porosity-Stokes system with uncertain parameters. We propose two ensemble algorithms and provide their unconditional stability and optimal error estimation. Besides, the numerical tests verify our theoretical results.

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Robust BPX preconditioner for fractional Laplacians on bounded Lipschitz domains

We propose and analyze a robust Bramble-Pasciak-Xu (BPX) preconditioner for the integral fractional Laplacian of order $s \in (0, 1)$ on bounded Lipschitz domains. Compared with the standard BPX preconditioner, an additional scaling factor $1 - \widetilde{\gamma}^s$, for some fixed $\widetilde{\gamma} \in (0, 1)$, is incorporated to the coarse levels. For either quasi-uniform grids or graded bisection grids, we show that the condition numbers of the resulting systems remain uniformly bounded with respect to both the number of levels and the fractional power. This is a joint work with J. P. Borthagaray (UDELAR), R. H. Nochetto (UMD) and J. Xu (PSU/KAUST).

Session 2 (Aug 31 15:50-17:50)

Xingding Chen
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A recycling preconditioning method with auxiliary tip subspace for elastic crack propagation simulation using XFEM

The extended finite element method (XFEM) is a powerful technique for solving fracture problems with discontinuities, singularities and localized deformations. However, these enrichment functions increase, drastically, the condition number of the stiffness matrix. In this talk, we construct a recycling preconditioning method with auxiliary tip subspace for elastic crack propagation simulation using XFEM. We propose a special domain decomposition strategy. To accelerate the convergence of the Krylov subspace method, the initial guess is built from the solution of the previous system with a local modification in the crack tip subspace. The efficiency of the proposed algorithms applied to problems with several types of cracks are validated by numerical experiments.

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An efficient multiscale preconditioner for large-scalehighly heterogeneous flow

We propose an efficient and robust multiscale preconditioner for large-scale incompressible flow in highly heterogeneous porous media. We start from the discretization of the first-order form for the single phase incompressible flow and apply a velocity elimination strategy to obtain an equation with pressure as the only unknown. Then an efficient preconditioner is designed to solve this equation. The key component of the preconditioner is adoption of a non-standard coarse space that contains the media's heterogeneity information. We solve a carefully constructed spectral problem in each coarse element to form the non-standard coarse space. Then a rigorous convergence analysis for the proposed two-grid algorithm is carried out, where the key ingredients lie in the smoothing property of ILU(0) smoother and the approximation property. In particular, our analysis shows that our preconditioner is robust and efficient thanks to this newly-constructed coarse space. Rich numerical tests with several types of large-scale three-dimensional highly heterogeneous permeability fields are presented. The experimental results show that our generalized multiscale space based preconditioner is robust with respect to the contrast, size and geometry of the permeability fields. We also successfully apply this preconditioner for multiphase flow simulation and transport problems arising from reservoir simulation.

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A virtual element method for the PNP equations on polygonal meshes

The Poisson-Nernst-Planck (PNP) equations are a kind of nonlinear coupled partial differential equations, which are widely applied to describe the transport of the charged particles in ion channels, electrochemical systems, semiconductors, etc. As a generalization of finite element method (FEM), the virtual element method (VEM) can handle very general polygonal elements including non-convex and distorted elements. Compared with some traditional numerical methods such as the finite difference method and the FEM, it has better mesh adaptability and so is more flexible. We study VEM on general polygonal meshes for the time-dependent PNP equations. The optimal H1 norm error estimates are presented for the semi-discrete VEM scheme. The Gummel iteration is used to decouple and linearize the PNP equations and the error analysis is also given for the iteration of fully discrete virtual element approximation. The numerical experiment on different polygonal meshes verifies the convergence results and shows the efficiency of the VEM. Moreover, we also present the theoretical and numerical results of the VEM for a special type of PNP– the Poisson-Boltzmann equation, which is an nonlinear interface equation with singularities.

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Non-iterative Local Parallel Algorithms for an Interface-Coupled Parabolic Problem

In this talk, we propose two novel local parallel algorithms based on the techniques of the non-iterative parallel Schwarz domain decomposition method for the interface-coupled parabolic problem. Our algorithms first divide the problem into two parallel sub-problems, then combine the idea of introducing a partition of unity which distributes the residuals reasonably into subdomains with modifying the discretized systems properly so as to maintain the exponential decay property of the residuals. The resulting schemes are fully parallel in each subdomain and no iteration is required at each time step to reach the optimal order of accuracy. Error estimates with the optimal overlapping degrees for these algorithms are presented.

Session 3 (Sep 01 14:30-16:30)

Chen Cui
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Fourier Neural Solver for Large Sparse Linear Algebraic Systems

In this talk, we propose an interpretable neural solver, the Fourier neural solver (FNS), to solve sparse linear algebraic systems. Based on deep learning and fast Fourier transformation, FNS combines a stationary iterative method and frequency space correction approach to efficiently eliminate different frequency components of the error. The local Fourier analysis indicates that FNS can detect error components within the frequency domain that cannot be eliminated effectively using stationary methods, even though the error removed by the latter is problem-dependent. Numerical experiments on several classical equations show that the FNS is more efficient and more robust than the existing neural solvers.

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An adaptive BDDC preconditioner for advection-diffusion problems with a stabilized finite element discretization

A BDDC preconditioner with adaptive coarse space for advection-diffusion problems discretized by stabilized finite element method is proposed. Since the bilinear form of the corresponding variational form is nonsymmetric and positive definite (NSPD), the adaptive BDDC preconditioner, which is always used for solving the symmetric and positive definite (SPD) problems, is extended to solve the nonsymmetric problems. By decomposing the original bilinear form to the symmetric part and the skew-symmetric part, a series of local generalized eigenvalue problems with respect to the symmetric part of the original bilinear form for the common faces/edges are designed and analyzed to form the adaptive coarse components. Numerical results are presented for model problems with various viscosities to show the performance of the proposed preconditioner.

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A two-level additive Schwarz preconditioner for the Nitsche extended finite element approximation of elliptic interface problems

In this paper, we propose a two-level additive Schwarz preconditioner for the Nitsche extended finite element discretization of elliptic interface problems. An intergrid transfer operators between the coarse and the fine spaces are constructed and a stable space decomposition is given. It is proved that the condition number of the preconditioned system is bounded by $C(1 + \frac{H}{\delta} + \frac{H}{h})$, where H and h respectively stand for the coarse and fine mesh sizes, and δ measures the size of the overlaps between subdomains. The constant C does not depend on the contrast of the coefficients, how the interface intersects with the meshes. Numerical experiments are carried out to validate theoretical results.

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A two-level preconditioner for the FE discretizations of Helmholtz equation with mixed boundery condition

For linear finite element discretizations of Helmholtz equation with mixed boundary conditions, [I. G. Graham, E. A. Spence and J. Zou, SIAM J. Numer. Anal. 58, 2515-2543, 2020] designed a symmetrized SORAS preconditioner by using the Shifted Laplacian technique and the partition of unity functions. Numerical results show the efficiency of the algorithm under specific parameters, but the performance is sensitive to the number of subregions, etc. By combining with the recent work of fast solvers developed for Helmholtz equation, we construct an appropriate coarse space through theoretical analysis and experimental observations, and design a new fast solver by combing with SORAS. Finally, we give the corresponding numerical analysis.

MS 17: Analysis and numerical computation of fluid flows

Organizer

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Session 1 (Aug 28 11:00-12:30)

Lina Zhao
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Pressure-robust staggered DG methods for the Navier-Stokes equations

In this talk, I will introduce a novel pressure-robust staggered discontinuous Galerkin method for the incompressible Navier-Stokes equations on general polygonal meshes. The devising of the method hinges on a carefully designed finite element pair and nonlinear convective term, which ensures pressure-robustness. The optimal convergence estimates for all the variables in L^2 norm are proved under a suitable smallness condition. In particular, the unique solvability and convergence error estimates are proved to be independent of the irrotational part of the source term. Numerical experiments will be presented to validate the theoretical findings and demonstrate the superior performances of the proposed method especially for problems with high Reynolds number or zero velocity.

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Local boundary regularity of weak solutions for the Stokes and Navier-Stokes equations in the half space

We study local boundary regularity of weak solutions for Stokes system. We construct weak solutions of the Stokes system whose normal derivatives are unbounded near boundary in a half space with no-slip boundary conditions. Such singular behavioes can be extended to the Navier-Stokes equations as well. This is a joint work in [1] with Dr. Tongkeun Chang.

[1] T. Chang and K. Kang *Singular weak solutions near boundaries in a half-space away from localized force for the Stokes and Navier-Stokes equations.* preprint, arXiv:2303.05746.

[2] T. Chang and K. Kang *Local regularity near boundary for the Stokes and Navier-Stokes equations.* to appear in Siam J. Math. Anal.

[3] T. Chang and K. Kang *On Caccioppoli's inequalities of Stokes equations and Navier-Stokes equations near boundary.* J. Differential Equations, **269** no. 9, 6732–6757 (2020).

[4] K. Kang *Unbounded normal derivative for the Stokes system near boundary*, Math. Ann., **331** no. 1, 87–109 (2005).

[5] K. Kang, B. Lai, C.-C. Lai, and T.-P. Tsai, *Finite energy Navier-Stokes flows with unbounded gradients induced by localized flux in the half-space*, Trans. Amer. Math. Soc., **375** no. 9, 6701–6746 (2022).

[6] K. Kang, B. Lai, C.-C. Lai, and T.-P. Tsai, *The Green tensor of the nonstationary Stokes system in the half-space*, Comm. Math. Phys. **399** no. 2, 1291–1372, (2023).

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Analysis and approximation of incompressible chemically reacting generalized Newtonian fluid

We consider a system of nonlinear partial differential equations modelling the steady motion of an incompressible non-Newtonian fluid, which is chemically reacting. The governing system consists of a steady convection-diffusion equation for the concentration and the generalized steady Navier–Stokes equations, where the viscosity coefficient is a power-law type function of the shear-rate, and the coupling between the equations results from the concentration-dependence of the power-law index. This system of nonlinear partial differential equations arises in mathematical models of the synovial fluid found in the cavities of moving joints. We construct a finite element approximation of the model and perform the mathematical analysis of the numerical method. Key technical tools include discrete counterparts of the Bogovskiĭ operator, De Giorgi's regularity theorem in two dimensions, and the Acerbi–Fusco Lipschitz truncation of Sobolev functions, in function spaces with variable integrability exponents.

Session 2 (Aug 29 15:50-17:50)

Yong Yu
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PNP and Keller Segel equation and their related topics

Poisson-Nernst-Planck equation and Keller-Segel equation are two different types of diffusive equation in physics and biology. In this talk, we will introduce their equilibrium solutions and the asymptotic behavior of the flows when time is large. The boundary layer phenomenon will also be discussed.

[1] Chia-Yu, Hsieh and Yong, Yu *Boundedness of solutions to an attraction-repulsion chemotaxis model in R2* JDE, 2022

[2] Chia-Yu, Hsieh and Yong, Yu *Existence of solutions to the Poisson-Nernst-Planck system with singular permanent charges in R2* SIMA, 2021

[3] Chia-Yu, Hsieh, Ho-Man, Tai, and Yong, Yu *Singular solutions to some semilinear elliptic equations: an approach of Born-Infeld approximation* CMS, 2021

[4] Chia-Yu, Hsieh and Yong, Yu *Debye layer in Poisson-Boltzmann model with isolated singularities* ARMA, 2019

Shu Ma
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Analysis of fully discrete finite element methods for 2D Navier–Stokes equations with critical initial data

First-order convergence in time and space is proved for a fully discrete semi-implicit finite element method for the two-dimensional Navier–Stokes equations with L^2 initial data in convex polygonal domains, without extra regularity assumptions or grid-ratio conditions. The proof utilises the smoothing properties of the Navier–Stokes equations in the analysis of the consistency errors, an appropriate duality argument, and the smallness of the numerical solution in the discrete $L^2(0, t_m; H^1)$ norm when t_m is smaller than some constant. Numerical examples are provided to support the theoretical analysis.

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Mathematical Analysis of Some MHD models

In this talk, we introduce three MHD models. The first model is Hall MHD. In this case, we reformulate it as the two and half dimensional model and we show the local well-posedness and the blow-up criterion with large initial data and global well-posedness with small initial data. As a generalized model of Hall-MHD, we introduce the second model: Lüst model. To this model, we also show the local well-posedness with large initial data and global well-posedness with small initial data. The third model is Stokes-Magneto equations with fractional Laplacian. In this case, we prove two results: strong solutions without viscous term to the equation of the magnetic field B and mild solution with viscous term to the equation of B. This is based on joint work with K. Kang (Yonsei University), J. Shin (UNIST), and H. Kwon (Brown University).

[1] Hantaek Bae, Kyungkeun Kang. *On the existence and temporal asymptotics of solutions for the two and half dimensional Hall MHD.* J. Math. Fluid Mech. **25** (2023), no. 2, Paper No. 24, 30 pp.

[2] Hantaek Bae, Jaeyong Shin. *On the local and global existence of unique solutions to the Lšt model.* Appl. Math. Lett. **137** (2023), Paper No. 108483, 7 pp.

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Convergent Evolving Finite Element Approximations of Boundary Evolution under Shape Gradient Flow

Shape gradient flow is widely used in the shape gradient descent algorithms for shape optimization problems constrained by partial differential equations. In this approach, the constraint partial differential equations could be solved by finite element methods on a domain with a solution-driven evolving boundary. Rigorous analysis for the stability and convergence of such finite element approximations is still missing from the literature due to the complex nonlinear dependence of the boundary evolution on the solution. In this article, rigorous analysis of numerical approximations to the evolution of the boundary in a prototypical shape gradient flow is addressed. First-order convergence in time and kth order convergence in space for finite elements of degree $k \geq 2$ are proved for a linearly semi-implicit evolving finite element algorithm up to a given time. The theoretical analysis is consistent with the numerical experiments, which also illustrate the effectiveness of the proposed method in simulating two- and three-dimensional boundary evolution under shape gradient flow. The extension of the formulation, algorithm and analysis to more general shape density functions and constraint partial differential equations is also discussed.

MS 18: Optimization and its Applications to PDEs

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Session 1 (Sep 01 10:20-12:20)

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On the global R-linear convergence of NAG method and beyond

The Nesterov accelerated gradient (NAG) method is an important extrapolation-based numerical algorithm that accelerates the convergence of the gradient descent method in convex optimization. When dealing with an objective function that is μ-strongly convex, selecting extrapolation coefficients dependent on μ enables global R-linear convergence. In cases μ is unknown, a commonly adopted approach is to set the extrapolation coefficient using the original NAG method, referred to as NAG-c. In this talk, we prove the global R-linear convergence of NAG-c for strongly convex problems by establishing the Q-linear convergence of certain constructed Lyapunov sequences. Finally, I will compare this result with those from the ODE perspective.

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Efficient threshold dynamics methods for topology optimization for fluids

In this talk, we will introduce an efficient threshold dynamics method for topology optimization for fluids modeled with the Stokes equation. A one-domain approach is applied to solve the problem in the whole domain and the minimization problem can be solved with an iterative scheme in which the Stokes problem is approached with a Brinkman problem. The total energy decaying property of the iterative algorithm can be obtained. The extension of applying the iterative thresholding method for topology optimization for the Navier-Stokes flow will also be discussed.

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A continuous perspective of the IC-PDPS

We provide a continuous perspective on the Inertial Corrected Primal-Dual Proximal Splitting (IC-PDPS) proposed by Valkonen (*SIAM J. Optim.*, 30(2): 1391–1420, 2020). Although it admits a tight proximal point algorithm (PPA) formulation, IC-PDPS involves many parameters and intermediate sequences that make the algorithm derivation not easy to understand. We will show that IC-PDPS actually corresponds to an implicit-explicit time discretization of a novel primal-dual dynamics, which provides an alternative interpretation from the continuous point of view. Moreover, observing the multiscale phenomenon of the practical numerical performance, we explore the fast-slow structure of the continuous dynamics. By using Fenichel's geometric singular perturbation (GSP) theory, we find the slow manifold on which the continuous trajectory is invariant and converges slowly. High order expansions of the slow manifold and the reduced flow provide more deep geometric interpretations and powerful improvement guide for the discrete algorithm.

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Punctured FEM: Incorporating multiply connected mesh cells

In the last decade, there has been growing interest in solving PDEs with finite element meshes that incorporate unconventionally shaped cells, such as generalized polytopes that have curved edges. In recent work, we extend the mesh geometry even further by allowing planar mesh cells to be multiply connected, i.e. have holes. This flexibility in mesh construction may significantly reduce the degrees of freedom involved in obtaining a finite element solution versus that of using conventional mesh geometries. We use the same function space as that used in virtual element methods (VEM), consisting of implicitly-defined solutions to Poisson problems with polynomial data. Unlike VEM, we construct and compute with basis functions directly, rather than taking projections and introducing stabilization terms. By leveraging notions from complex analysis, we efficiently obtain the H^1 semi-inner products and L^2 inner products of these basis functions by performing computations solely on cell boundaries. An additional benefit of this approach is that the values, gradient, and higher derivatives of the finite element solution can be cheaply obtained in the interior of mesh cells, and not just on the skeleton.

Session 2 (Sep 01 14:30-16:30)

Bin Shi
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Nesterov's accelerated gradient descent and high-resolution differential equation

For modern optimization and machine learning, Nesterov's accelerated gradient descent method is widely used, and revealing its acceleration mechanism is a core issue. In this report, we mainly describe how the theoretical framework of high-resolution differential equations uncover the acceleration phenomenon, and the specific mathematical techniques developed, such as phase space representation and construction of Lyapunov functions, etc. At the same time, we will also describe the latest research progress based on this perspective.

[1] Bin Shi, Simon S. Du, Michael I. Jordan and Weijie J. Su *Understanding the acceleration phenomenon via high-resolution differential equations.* Mathematical Programming, 95:79–148, 2022

[2] Bin Shi, Simon S. Du, Weijie J. Su, and Michael I. Jordan *Acceleration via symplectic discretization of high-resolution differential equations.* arXiv:1902.03694, 2019

[3] Shuo Chen, Bin Shi, and Ya-xiang Yuan *Gradient norm minimization of Nesterov acceleration: $o(1/k^3)$.* arXiv:2209.08862, 2022

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Transformed primal-dual methods for nonlinear partial differential equations

A transformed primal-dual (TPD) flow is developed for a class of nonlinear saddle point system, including convex optimization problem with affine constraints. The flow for the dual variable contains a Schur complement which is strongly convex. Exponential stability of the saddle point is obtained by showing the strong Lyapunov property. A TPD iteration is derived by time discretization of the TPD flow. Under mild assumption, the algorithm is linear convergent for smooth problems. The convergence rate depends on the relative condition number of the objective function and the Schur complement under variant metric as preconditioners. The developed algorithm is

then applied to nonlinear partial differential equations: Darcy–Forchheimer model and a nonlinear electromagnetic model. Numerical results demonstrate the efficiency of the method. This is joint work with Long Chen and Ruchi Guo.

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Towards sparse and interpretable learning via the inverse scale space method

Boosting, as the well-known gradient descent method, is often considered the 'best off-the-shelf' method in machine learning. The Inverse Scale Space (ISS) method is a Boosting-type algorithm that uses restricted gradient descent for sparsity learning, with underlying dynamics governed by differential inclusions. It is also known as Mirror Descent in optimization and (Linearized) Bregman Iterations in applied mathematics. These algorithms generate iterative regularization paths with structural sparsity, where significant features or parameters emerge earlier on the path than noise. Despite its simplicity, the ISS method may outperform popular methods like the (generalized) Lasso in both theory and experiments. Its utility and benefits are illustrated by several application cases, including sparse feature selection, learning graphical models, Alzheimer's disease detection via neuroimaging, and finding effective sparse subnet structures with improved interpretability in deep learning. Finally, we introduce a preliminary statistical theory of path consistency to explain its success.

[1] Yanwei Fu, Chen Liu, Donghao Li, Zuyuan Zhong, Xinwei Sun, Jinshan Zeng, and Yuan Yao *Exploring structural sparsity of deep networks via inverse scale spaces.* IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI), 45(2):1749-1765, 2023

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An accelerated proximal ADMM for optimal decentralized control of uncertain systems

To ensure the system stability of the H2-guaranteed cost optimal decentralized control problem (ODC), an approximate semidefinite programming (SDP) problem is formulated based on the sparsity of the gain matrix of the decentralized controller. To reduce data storage and improve computational efficiency, the SDP problem is vectorized into a conic programming (CP) problem using the Kronecker product. Then, a proximal alternating direction method of multipliers (PADMM) is proposed to solve the dual of the resulted CP. By linking the (generalized) PADMM with the (relaxed) proximal point algorithm, we are able to accelerate the proposed PADMM via the Halpern fixed-point iterative scheme. This results in a fast convergence rate for the Karush-Kuhn-Tucker (KKT) residual along the sequence generated by the accelerated algorithm. Numerical experiments further demonstrate that the accelerated PADMM outperforms both the well-known CVXOPT and SCS algorithms for solving the large-scale CP problems arising from H2-guaranteed cost ODC problems.

[1] Bo Yang, Xinyuan Zhao, Xudong Li, and Defeng Sun *An accelerated proximal alternating direction method of multipliers for optimal decentralized control of uncertain systems.* arXiv:2304.11037, 2023

MS 19: Stochastic Dynamical Systems and Their Applications

Organizers:

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Stochastic dynamical systems arise in modeling random phenomena in many research fields such as engineering, industry and science. They are usually described as stochastic (partial) differential equations under Brownian motion and/or Levy motion. This mini-symposium will focus on recent progress in the impact of non-Gaussian noise on complex dynamics, such as transition phenomena like the most probable transition pathways, data-driven techniques for examining random behaviors and extracting effective dynamics, long-term behaviors of high-dimensional stochastic systems, multi-scale stochastic dynamics, etc., with tools such as large and moderate deviations, normal and non-normal approximations by Stein's method, non-local Kramers-Moyal formula, Onsager-Muchlup action functional, stochastic optimal control, as well as various state-of-the-art machine learning techniques.

Session 1 (Sep 01 14:30-16:30)

Jinqiao Duan
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Recent Advances in Stochastic Dynamical Systems

Complex dynamical systems are often under random fluctuations. The noisy fluctuations may be Gaussian or non-Gaussian, which are usually modeled by Brownian motion or Levy motion, respectively. Stochastic differential equations are appropriate mathematical models for these random dynamical systems. The speaker will present an overview about recent advances in random dynamical systems, including stochastic bifurcation, random interacting particle dynamics, stochastic Hamiltonian dynamics, and particularly, probabilistic approaches and data science methods for investigating transition phenomena.

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Detecting Transition Pathway in Stochastic Dynamical Systems through Optimal Control and Machine Learning

Many complex real world phenomena exhibit abrupt, intermittent, or jumping behaviors, which are more suitable to be described by stochastic differential equations under non-Gaussian Lévy noise. Among these complex phenomena, the most likely transition paths between metastable states are important since these rare events may have a high impact in certain scenarios. One of the challenges to calculate the most likely transition path for stochastic dynamical systems under non-Gaussian Lévy noise is that the associated rate functional cannot be explicitly expressed by paths. For this reason, we formulate the original variational problem into an optimal control problem to obtain the optimal state as the most likely transition path. In this talk, we will present three types of efficient ways to solve this issue and the corresponding numerical analysis on the convergence and computational efficiency, including supervised learning and reinforcement learning as well as FBSDE with Pontryagin's Maximum Principle. Various stochastic dynamical systems in applications will be discussed.

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Approximation to the ergodic measure of the multiplicative SDEs driven by stable processes

We shall consider the Euler-Maruyma (EM) scheme of the multiplicative SDEs driven by stable processes, by Stein's method, we show that the ergodic measure of EM scheme will converge to that of the SDE as the step size tends to 0. The convergence rate is optimal. This is the joint work with Peng Chen, Xiaolong Zhang and Xicheng Zhang.

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Numerical Simulation of Statistical Behavior for Fractional Cox-Ingersoll-Ross Process

Cox-Ingersoll-Ross(CIR) process is an important tool to study stochastic interest rate and stochastic volatility in financial market. The statistical behavior of fractional CIR process is mainly simulated and discussed in this report. Since there is no analytical expression of the CIR process, two different functions wfbm and fbmld are used to simulate the fractional Brownian motion, and the Euler-Maruyama(EM) method is used to investigate the expectation and variance of the fractional CIR process. Because the distribution of fractional CIR process can not be expressed by the solution of Fokker- Planck equation, the empirical distribution of fractional CIR process is simulated, and the change of empirical distribution with time is obtained. In order to further verify the algorithm and compare the advantages of the two different algorithms, a backward Euler type scheme of the CIR model and the fractional Ornstein-Uhlenbeck(OU) model with analytical solution is carried out. By comparing figure and table, it is found that simulation by the function fbmld have a very high fitting precision with the theoretical analytical solution with expectation and variance.