

# International Conference on Fractal Geometry & Related Topics

## 11-15 December, 2023



LT2, Lady Shaw Building Department of Mathematics The Chinese University of Hong Kong

## Sponsors

- Department of Mathematics, The Chinese University of Hong Kong
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## About

### **International Conference on Fractal Geometry and Related Topics**

The aim of this conference is to bring together scientists to discuss and exchange ideas on the cutting edge research on the area of fractals. The topics will include, for example, analysis on fractals, ergodic theory and dynamical systems, geometric measure theory, multifractal analysis, fractal tiling. It will also be an opportunity to honor the late Professor Ka-Sing Lau for his many pioneering and deep contributions to fractal geometry and related areas.

### **Scientific Committee**

- Kenneth Falconer (University of St Andrews)
- Ai-Hua Fan (Université de Picardie Jules Verne)
- De-Jun Feng (The Chinese University of Hong Kong)
- Alexander Grigor'yan (Universität Bielefeld)
- Palle Jorgensen (University of Iowa)
- Yang Wang (Hong Kong University of Science and Technology)
- Zhi-Ying Wen (Tsinghua University)

### **Organizing Committee**

- Xin-Han Dong (Hunan Normal University)
- De-Jun Feng (The Chinese University of Hong Kong)
- Alexander Grigor'yan (Universität Bielefeld)
- Chi-Wai Leung (The Chinese University of Hong Kong)
- Zhi-Ying Wen (Tsinghua University)
- Jun Wu (Huazhong University of Sciences and Technology)

### Enquiry

Email:	fgrt2023@math.cuhk.edu.hk
Website:	https://www.math.cuhk.edu.hk/conference/fgrt2023
Address:	Department of Mathematics
	The Chinese University of Hong Kong
	Shatin, Hong Kong

### **Conference Venue**

## All lectures are held in LT2, Lady Shaw Building (LSB), CUHK.

## 11 December 2023, Monday

9:00-9:40	Registration		
9:40-9:50	Opening		
	Chair: Ai-Hua Fan		
9.50_10.35	Fractal percolation on statistically self-similar and self-affine sets		
7.50 10.05	Kenneth J. Falconer, University of St Andrews		
	Coffee Break/Registration		
10.55 11.40	Hausdorff dimension of intersections		
10.55-11.40	Pertti Mattila, University of Helsinski		
11:45-12:30	Multiplicative Markoff-Lagrange spectrum and symbolic dynamics		
	Shigeki Akiyama, University of Tsukuba		
	Lunch		
	Chair: Jun Wu		
14.00-14.45	Some examples of random covering sets		
14:00-14:45	Esa Järvenpää, University of Oulu		
14.50-15.25	Measures, annuli and dimensions		
14.30-13.33	Stéphane Seuret, Université Paris-Est Créteil		
	Coffee Break		
15.55-16.40	Measure and dimension theory for limsup sets generated by rectangles		
13.33-10.40	Baowei Wang, Huazhong University of Science and Technology		

## 12 December 2023, Tuesday

	Chair: Kenneth Falconer			
9:00-9:45	On the dimensions of random statistically self-affine Baransky carpets and sponges			
	Julien Barral, Université Sorbonne Paris Nord			
9.50-10.35	Conformal dimension of p.c.f. self-similar sets			
7.50 10.05	Hui Rao, Central China Normal University			
	Coffee Break			
10:55-11:40	Dimension of planar non-conformal attractors with triangular derivative matrices			
	Balázs Bárány, Budapest University of Technology and Economics			
11.45-12.30	Dimension of higher dimensional irreducible self-affine measures			
11.13 12.00	Meng Wu, University of Oulu			
	Lunch			
	Chair: Xiong Jin			
14.00-14.30	Spectral analysis for some periodic quantum graphs			
14.00 14.00	Chun-Kong Law, National Sun Yat-sen University			
14.35-15.05	Birth-death type random walks on hyperbolic graphs			
11.00 15.00	Shi-Lei Kong, Sichuan University			
15.10-15.40	Bi-accessibility dimension of quadratic Julia sets			
13.10 13.10	Jun Luo, Sun Yat-Sen University			
	Coffee Break			
16.00 1/- 45	On Erdős similarity problem and its variants			
10.00 10.45	Chun-Kit Lai, San Francisco State University			
16.50-17.35	The products of calibrated sets and paired calibrated sets in Plateau's problem			
10.50-17.55	Xiangyu Liang, Beihang University			

## 13 December 2023, Wednesday

	Chair: Meng Wu			
0.00-0.45	On the fibres of planar self-similar sets with dense rotations			
9:00-9:45	Xiong Jin, University of Manchester			
0.50 10.25	Loosely Bernoulli nonhyperbolic ergodic measures			
9:50-10:55	Michał Rams, Polish Academy of Sciences			
	Coffee Break			
10.55_11.40	Density of minimal points and Bergelson-Hindman question			
10.55-11.40	Wen Huang, University of Science and Technology of China			
11.45 12.20	"Entropies" in negatively curved spaces			
11.45-12.50	Lin Shu, Peking University			
	Lunch			
	Chair: Balázs Bárány			
14.00.14.00	Uniform approximation problems of expanding Markov maps			
14.00-14.00	Lingmin Liao, Wuhan University			
14:35-15:05	$L^p$ estimates of orthogonal projections, dual Furstenberg problem, and discretized sum-product			
	Bochen Liu, Southern University of Science and Technology			
15.10-15.40	Dimensions of non-autonamous self-affine sets			
15.10-15.40	Jun-Jie Miao, East China Normal University			
	Coffee Break/Group Photo			
	Chair: Sze-Man Ngai			
16.10-16.40	Life and pictures of Ka-Sing Lau			
10.10-10.40	Eveline Young, Pittsburgh			
16:40-17:10	Audience recollections of Ka-Sing Lau			
17:10-17:15	Award presentation of Professor Ka-Sing Lau Scholarship for Mathematics 2023–24			
19:00-22:00	Banquet			

## 14 December 2023, Thursday

	Chair: Alexander Grigor'yan		
0.00-0.45	Yet another construction of "Sobolev spaces" on metric spaces		
7.00-7.45	Jun Kigami, Kyoto University		
0.50 10.05	Reflected diffusion on uniform domains		
7.50-10.55	Mathav Murugan, University of British Columbia		
	Coffee Break		
10.55 11.40	BV functions and fractional Laplacians on Dirichlet spaces		
10.55 11.40	Alexander Teplyaev, University of Connecticut		
11.45-12.30	Spectral properties of Krein-Feller operators		
11.45-12.50	Sze-Man Ngai, Hunan Normal University and Georgia Southern University		
	Lunch		
	Half Day Free		

## 15 December 2023, Friday

	Chair: Julien Barral			
0.00 0.45	Tails of heat kernels for jump processes			
7.00-7.43	Alexander Grigor'yan, Universität Bielefeld			
9.50-10.35	The weak and strong elliptic Harnack inequalities			
7.50-10.55	Jiaxin Hu, Tsinghua University			
	Coffee Break			
10:55-11:40	Stationary random fields and Trigonometric multiplicative chaos			
	Ai-Hua Fan, Université de Picardie & Central China Normal University			
11.45-12.30	Products of random matrices			
11.45-12.50	Quansheng Liu, Université de Bretagne-Sud			
	Lunch			
	Chair: Lin Shu			
14:00-14:30	Almost sure dimensional properties for the spectrum and the density of states of Sturmian Hamiltonians			
	Yanhui Qu, Tsinghua University			
14.25 15.05	Local times of anisotropic Gaussian random fields and stochastic heat equation			
17.05 15.05	<b>Cheuk Yin Lee</b> , The Chinese University of Hong Kong (Shenzhen)			

## List of Abstracts

### Multiplicative Markoff-Lagrange spectrum and symbolic dynamics

### Shigeki Akiyama

Markoff-Lagrange spectrum is a discrete phenomenon that appeared in classical Diophantine approximation and correlates badly approximable numbers and Sturmian sequences. Since many problems in number theory are related to the study of fractional parts of exponential growth sequences, it is interesting if we can observe this spectrum phenomena in multiplicative setting. I will talk on recent development on the multiplicative Markoff-Lagrange spectrum. The key formula is to intertwine this problem into the symbolic dynamical setting and its inverse. Then I will explain the information on spectra obtained from this basic formula, together with some proofs to illustrate our results. The employed techniques are widespread from combinatorics on words, number theory, and fractal geometry.

This is joint work with H. Kaneko and T. Kamae.

### On the dimensions of random statistically self-affine Baransky carpets and sponges

#### Julien Barral

We will present some results on the dimension theory of random statistically self-affine Baransky carpets and sponges, and the inhomogeneous Mandelbrot measures they support.

### Fractal percolation on statistically self-similar and self-affine sets

#### **Kenneth Falconer**

Originally introduced by Benoit Mandelbrot, fractal percolation is a statistically self-similar process based on a hierarchy of square grids leading to a random set F. With each square selected independently with probability p, Mandelbrot suggested that there was a critical probability  $p_c$ such that F undergoes a topological phase transition, changing as p increases through  $p_c$  from being totally disconnected to having non-trivial connected components. This was confirmed by Chayes, Chayes and Durrett who derived further properties of F, as did Dekking, Meester and others.

We will give an overview of fractal percolation and consider differences and similarities with the analogous process based on a rectangular grid leading to a statistically self-affine set.

University of Tsukuba

University of St Andrews

Université Sorbonne Paris Nord

### Stationary random fields and Trigonometric multiplicative chaos

### Ai-Hua Fan

Université de Picardie & Central China Normal University

We introduce a class of stationary random fields on compact Abelian groups and there are many unsolved problems on these fields. In 1930's, Paley, Zygmund and Wiener studied three types (Rademacher, Steinhaus, Gauss) of random trigonometric series. One of series of Gauss type defines the Brownian motion. The series of Steinhaus type, which involve naturally the group structure of the cercle  $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ , are stationary fields on the group  $\mathbb{T}$ . In 1960's, Kahane studied general random trigonometric series under the condition that the coefficients of the series is square summable. Important improvements and developments are then followed (Billard, Marcus-Pisier, Talagrand et al). See Kahane's book (Some Random Series of Functions, 1985, Cambridge Press). If the coefficients of a random trigonometric series is not square-summable, the series does not define a function neither a measure, but a distribution. What happens about the series about its partial sums? With Yves Meyer, we construct a class of trigonometric chaotic measures in the setting of Multiplicative Chaos of Kahane (1987) to study the random stationary distributions on  $\mathbb{T}$ , as well as on  $\mathbb{T}^d$ . It is proved that the behavior of the partial sums are strongly multifractal. We give a full study of the associated chaotic operators, by describing their kernels and images, and consequently we have computed the Hausdorff dimensions of the chaotic measures. Our trigonometric chaos is very similar to the Gaussian Multiplicative Chaos, which is related to the Gaussian Free Field.

### Tails of heat kernels for jump processes

#### Alexander Grigor'yan

This talk is based on a series of joint papers with Eryan Hu and Jiaxin Hu.

We prove upper bounds of the heat kernel  $p_t(x, y)$  of a jump type Dirichlet form on a doubling metric measure space  $(M, d, \mu)$ , where the off-diagonal term depends on a certain  $L^q$  tail estimate of the jump kernel J(x, y).

If the measure  $\mu$  is  $\alpha$ -regular then the said tail estimate is as follows:

$$\|J(x,\cdot)\|_{L^q(B^c(x,r))} \le \frac{\text{const}}{r^{\gamma}}$$

where B(x,r) denotes metric balls. We prove that if  $q \in [2,\infty]$  and

$$\gamma = \frac{\alpha}{q'} + \beta$$

where  $\beta > 0$  and q' is the Hölder conjugate of q then (), together with the *Faber-Krahn inequality* and the *generalized capacity condition* with parameter  $\beta$ , is equivalent to the following upper bound of the tail of the heat kernel:

$$\left\|p_t\left(x,\cdot\right)\right\|_{L^q\left(B^c\left(x,r\right)\right)} \leq \frac{\operatorname{const}}{t^{\alpha/(\beta q')}} \left(1 + \frac{r}{t^{1/\beta}}\right)^{-\gamma}.$$

It follows from () that the heat kernel satisfies the following pointwise upper estimate:

$$p_t(x,y) \le \frac{\operatorname{const}}{t^{\alpha/\beta}} \left(1 + \frac{d(x,y)}{t^{1/\beta}}\right)^{-\gamma}.$$

Important ingredients of the proof are the elliptic and parabolic mean value inequalities.

The case  $q = \infty$  (and, hence, q' = 1) amounts to the previously known results of AG, J.Hu, K.-S.Lau Trans.AMS (2014) and Z.-Q.Chen, T.Kumagai, J.Wang, Mem.AMS (2021), while the case  $q < \infty$  is entirely new.

### The weak and strong elliptic Harnack inequalities

#### Jiaxin Hu

Tsinghua University

In this talk, we consider the regular resurrected Dirichlet form on the metric space equipped with a doubling measure. We show that the heat kernel estimate is equivalent to the weak elliptic Harnack inequality, the mean exit time estimate, plus the jump kernel upper bound. If further the upper jumping smoothness holds, we obtain a sharper assertion, that is, the strong elliptic Harnack inequality also comes into the stage. In particular, for the strongly local Dirichlet form where the jump vanishes (so that both the jump kernel upper bound and the upper jumping smoothness are trivially satisfied), our assertion coincides with the one achieved by Grigor'yan, Hu and Lau (2015 JMS Japan). This talk is based on the joint work with Zhenyu Yu.

Universität Bielefeld

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### Density of minimal points and Bergelson-Hindman question

### Wen Huang

University of Science and Technology of China

Furstenberg's multiply recurrent theorem states that any dynamical system has multiply recurrent points, and points out that this result is equivalent to the van der Waerden theorem. An equivalent form of van der Waerden's theorem is that any piecewise syndetic subset of a natural number contains any arbitrarily long arithmetic progressions. In this talk, we discuss the correlation between multiple recurrence and piecewise syndetic set, and provide some applications in combinatorial number theory.

In 1998 Furstenberg and Glasner proved that the set composed of the first term and common difference of all arithmetic progressions of length k appearing in the piecewise syndetic subset of natural numbers is also piecewise syndetic subsets in  $\mathbb{Z}^2$ . In 2001 Bergelson and Hindman raised the question of whether the polynomial version of this result holds. We will answer the Bergelson-Hindman question by showing the density of minimal points of a dynamical system of  $\mathbb{Z}^2$  action associated with the piecewise syndetic set and the polynomials. This based on joint works with Professors Shao and Ye.

### Some examples of random covering sets

### Esa Järvenpää

We consider dimensions of random covering sets generated by balls and driven by general measures. We improve the general lower bound given by Ekström and Persson and prove their conjecture concerning the exact value of dimension in a special case. We also give various examples demonstrating the complexity of dimension for generating balls with arbitrary sequences of radii. This is joint work with Maarit Järvenpää, Markus Myllyoja and Örjan Stenflo.

### On the fibres of planar self-similar sets with dense rotations

### Xiong Jin

In this talk we will look at the size of fibres of planar self-similar sets with dense rotations. We shall first review some existing examples for which the size of fibres are known. Then we will look at sufficient conditions under which one may deduce a lower bound of the dimension of fibres. These investigations are built on the application of Mandelbrot percolations on self-similar sets. They are also connected to the problem of finding interior points in the radial projections of self-similar sets and in the arithmetic sum of Cantor sets.

University of Manchester

University of Oulu

### Yet another construction of "Sobolev spaces" on metric spaces

### Jun Kigami

The counterpart of "Sobolev space" on metric spaces has been intensively studied for the last 20 years after the pioneering works by Cheeger, Hajlasz, and Shanmugalingam. The mainstream of the ideas is to use the local Lipschitz constant of a function as a suitable substitute for its gradient. However, a recent study by Kajino and Murugan on the conformal walk dimension revealed that the Dirichlet form associated with the Brownian motion on the Sierpinski carpet can not be a Sobolev space in this sense. In this talk, we will propose a new way of constructing "Sobolev spaces" on compact metric spaces including the Sierpinski carpet.

### Birth-death type random walks on hyperbolic graphs

### Shi-Lei Kong

As a natural generalization of the classic birth-death chains on nonnegative integers, we study a class of reversible random walks of birth-death type on hyperbolic graphs, and analyze the quadratic forms of induced energy on the boundaries. The result provides a discretization of certain non-local regular Dirichlet forms on doubling metric measure spaces. In addition, we show that a hyperbolic graph carries such birth-death type random walks if and only if it is roughly starlike and has bounded degree.

### On Erdős similarity problem and its variants

### Chun-Kit Lai

Erdős similarity conjecture asserted that patterns of infinite cardinality can be avoided by a set of positive Lebesgue measure in the sense that the set does not contain affine copies of the given pattern. The conjecture is currently open and fast decaying sequences like  $2^{-n}$  has been a bottleneck in resolving the conjecture. In this talk, we will report on two recent progresses of this conjecture. First, we will consider the pattern being Cantor sets. Second, we will consider bi-Lipschitz copies instead of affine copies. Interesting and sharp results will be presented in both considerations. These are joint works with De-Jun Feng, Ying Xiong, and some of my students.

### Spectral analysis for some periodic quantum graphs

#### Chun-Kong Law

We shall derive and analyze the dispersion relations of some periodic quantum graphs associated with Archimedean tilings, where the potentials are even, or non-even. Furthermore, we study the existence of Dirac points, which are points where different sheets of dispersion surface touch to form a conical singularity. We prove there exist infinitely many Dirac points located at the periodic eigenvalues. We shall also see that this occurs when the potential function has a special form.

This is joint work with E.O. Jatulan of University of the Philippines Los Baños.

Kyoto University

### Sichuan University

San Francisco State University

#### National Sun Yat-sen University

### Local times of anisotropic Gaussian random fields and stochastic heat equation

### Cheuk Yin Lee

The Chinese University of Hong Kong (Shenzhen)

In this talk, we discuss the local times of a class of anisotropic Gaussian random fields and related fractal properties. We present some moment estimates and regularity results for the local times. Our key estimates rely on geometric properties of Voronoi partitions with respect to an anisotropic metric and the use of Besicovitch's covering theorem. Our results can be applied to the solutions to systems of stochastic heat equations with additive Gaussian noise. As a consequence, we determine the exact gauge function for the parabolic Hausdorff measure of the level sets of the solutions. This talk is based on joint work with Yimin Xiao.

### The products of calibrated sets and paired calibrated sets in Plateau's problem

### Xiangyu Liang

**Beihang University** 

Plateau's problem is a main interest in geometric measure theory. It aims at understanding the behavior of physical objects that admit certain minimizing property, such as soap films. Physical soap films are probably more accurately modeled by Almgren's minimal sets, but the lack of algebraic coherence makes it difficult to prove minimality. The theory of calibrated geometry is a powerful tool to study minimizing manifold (possibly with singularities). It was introduced by Harvey-Lawson in the 80's, and builds a bridge between classical theory of manifolds and geometric measure theory. On the other hand, it cannot be applied directly to Plateau's problem. Then in the 90's, K.Brakke, G, Lawlor & F. Morgan introduced the method of paired calibration to prove various minimality of sets satisfying a given separation condition. It is very often used in the classification of singularities for codimension 1 minimal sets in Plateau's problem. Compared to the above ordinary calibration methods, a major advantage of paired calibrations is that it ignores algebraic multiplicities, which corresponds to the spirit of Plateau's problem. However, in general we do not know a generalisation to codimension larger than one, and, at first glance, the minimality of the products of calibrated sets or paired calibrated sets is unknown. In this talk, we will first give very simple examples to show how to use calibration and paired calibratoin method to prove various minimalities for sets, and explain the main different of these two theories. Then we introduce the background and definitions for Almgren minimal sets, classification of singularities for Plateau's problem, and how the theories of calibration and paired calibration applies. Finally, we will discuss the minimality of the products of these two kinds of sets in codimension 2.

### Uniform approximation problems of expanding Markov maps

### Lingmin Liao

Wuhan University

Let  $T: [0,1] \rightarrow [0,1]$  be an expanding Markov map with a finite partition. Any Hölder continuous potential  $\phi$  produces an invariant Gibbs measure  $\mu_{\phi}$ . For  $\kappa > 0$ , we investigate  $\mu_{\phi}$ -almost surely the size of the uniform approximation set

$$\mathcal{U}^{\kappa}(x) := \{ y \in [0,1] : \forall N \gg 1, \exists n \le N, \text{ such that } |T^n x - y| < N^{-\kappa} \}.$$

The critical value of  $\kappa$  such that the Hausdorff dimension of  $\mathcal{U}^{\kappa}(x)$  equals to 1 for  $\mu_{\phi}$ -a.e. x is proven to be  $1/\alpha_{\max}$ , where  $\alpha_{\max} = -\int \phi \, d\mu_{\max} / \int \log |T'| \, d\mu_{\max}$  and  $\mu_{\max}$  is the Gibbs measure associated with the potential  $-\log |T'|$ . Moreover, when  $\kappa > 1/\alpha_{\max}$ , we show that for  $\mu_{\phi}$ -a.e. x, the Hausdorff dimension of  $\mathcal{U}^{\kappa}(x)$  as a function of  $1/\kappa$  agrees with the multifractal spectrum of  $\mu_{\phi}$ . This is a joint work with Yubin He.

## $L^p\ {\rm estimates}$ of orthogonal projections, dual Furstenberg problem, and discretized sum-product

#### Bochen Liu

Southern University of Science and Technology

 $L^2$  estimates of orthogonal projections are classical in geometric measure theory. In this talk we shall discuss about recent progress on  $L^p$  estimates. Then we come up with a dual version of the Furstenberg problem and introduce some partial results. We also find that, compared with general sets, Cartesian products have better  $L^p$ -behavior. This leads to improvement on some discretized sum-product estimates. This is joint work with Longhui Li.

### **Products of random matrices**

#### Quansheng Liu

Some recent progress on limit theorems for products of random matrices will be presented. We focus on large deviations and Gaussian approximation, and we also consider the multifractal spectrum of Lyapunov's exponent for random matrices on regular trees. This talk is mainly based on joint works with De-Jun Feng, Ion Grama and Hui Xiao.

Université de Bretagne-Sud

### **On Bi-accessibility Dimension of Quadratic Julia Sets**

### Jun Luo

#### Sun Yat-Sen University

The classical core entropy  $h_{core}(f)$  for post critically finite (PCF) polynomials with degree  $\geq 2$  is defined to be the topological entropy of f restricted to its Hubbard tree. We fully generalize this notion to a new quantity  $\mathcal{E}_{core}(f)$ , called the **core entropy** of f, which is well defined if only f has a connected Julia set. It has four properties. First,  $\mathcal{E}_{core}(f) = h_{core}(f)$  when f is PCF. Second,  $\mathcal{E}_{core}(f^n) = n\mathcal{E}_{core}(f)$  for all  $n \geq 2$ . Third,  $\mathcal{E}_{core}(f) = \mathcal{E}_{core}(g)$  whenever f and g are J-equivalent. Finally, if f has no irrationally neutral cycle there is a compact set  $B^*(f) \subset \mathbb{S}^1$  invariant under  $\sigma_d(w) = w^d$  such that  $\dim_H B^*(f) = \frac{\mathcal{E}_{core}(f)}{\log d}$ . In such a case, we further relate to f a set  $K_{\text{biac}}(f) \subset \mathbb{S}^1$ . The Hausdorff dimension of this set will be called the **bi-accessibility dimension** of f. Restricting to quadratic polynomials. we set  $h(c) = \mathcal{E}_{core}(f_c)$  and  $g(c) = \dim K_{\text{biacc}}(f_c)$  for  $f_c(z) = z^2 + c$ , where c runs through the whole Mandelbrot set  $\mathcal{M}$ . Then  $h : \mathcal{M} \to [0, \log 2]$  is continuous nowhere, although its restriction to the subset  $\{c \in \mathcal{M} : f_c \text{ is PCF}\}$  has a continuous extension  $\overline{h} : \mathcal{M} \to [0, \log 2]$ . It is known that g(c) = h(c) whenever  $f_c$  is PCF or c is NOT a tip parameter. The following issues remains open: Is it true that g(c) is continuous over the whole Mandelbrot set  $\mathcal{M}$ ?

### Hausdorff dimension of intersections

### Pertti Mattila

Let A and B be Borel subsets of  $\mathbb{R}^n$ . What can we say about the Hausdorff dimensions of the intersections of A and typical rigid motions of B? More precisely, of  $\dim A \cap (g(B) + z)$  for almost all rotations  $g \in O(n)$  and for translations  $z \in \mathbb{R}^n$  in a set of positive Lebesgue measure. Optimally one could hope that this dimension is at least  $\dim A + \dim B - n$ , which happens when smooth surfaces meet in a general position. This is open, but I shall discuss some new partial results.

### Dimensions of Non-autonamous self-affine sets

#### Jun-Jie Miao

In the talk, we define a class of Iterated function systems named "Non-autonamous self-affine Iterated function system", and we call its invariant set *Non-autonamous self-affine set or self-affine Moran set* which is the generalization of classic Moran sets and self-affine sets. Simply to say, we apply different IFS at each step in the iteration.

To study the dimensions of self-affine Moran sets, we define two critical values  $s^*$  and  $s_A$ , and the upper box-counting dimensions and Hausdorff dimensions of self-affine Moran sets are bounded above by  $s^*$  and  $s_A$ , respectively. Unlike self-affine fractals where  $s^* = s_A$ , we have that  $s^* \ge s_A$ , and the inequality may hold strictly.

Under certain conditions, we obtain that the upper box-counting dimensions and Hausdorff dimensions of self-affine Moran sets may equal to  $s^*$  and  $s_A$ , respectively. In particular, we study self-affine Moran sets with random translations, and the Hausdorff dimensions of such sets equal to  $s_A$  almost surely.

University of Helsinki

East China Normal University

### **Reflected diffusion on uniform domains**

### Mathav Murugan

I report recent progress on heat kernel estimates for reflected diffusion on uniform domains where the underlying space admits sub-Gaussian heat kernel bounds. A key novelty of our work is the use of an extension operator that extends functions from the domain of the Dirichlet form for the reflected diffusion to that of the diffusion in the ambient space. If time permits, I will discuss heat kernel estimates for trace of reflected diffusion on the boundary of a uniform domain. The results on the trace jump process are based on a joint work with Naotaka Kajino.

### Spectral properties of Krein-Feller operators

### Sze-Man Ngai

Hunan Normal University & Georgia Southern University

A Krein-Feller operator we study is a Laplacian defined on a domain by a measure. The spectral dimension of a Krein-Feller operator is a fundamental quantity that plays an important role in studying the analytic properties of the operator. We report some results concerning the spectral dimension of Krein-Feller operators defined by fractal measures, focusing on self-similar measures with overlaps. We discuss some applications, including heat kernel estimates and wave propagation speed. We also discuss the extension of such Laplacians to Riemannian manifolds. This talk is based on joint work with Qingsong Gu, Jiaxin Hu, Lei Ouyang, Wei Tang, and Yuanyuan Xie.

# Almost sure dimensional properties for the spectrum and the density of states of Sturmian Hamiltonians

### Yanhui Qu

Tsinghua University

We find a full Lebesgue measure set of frequencies  $\mathbb{I} \subset [0,1] \setminus \mathbb{Q}$  such that for any  $(\alpha, \lambda) \in \mathbb{I} \times [24, \infty)$ , the Hausdorff and box dimensions of the spectrum of the Sturmian Hamiltonian  $H_{\alpha,\lambda,\theta}$  coincide and are independent of  $\alpha$ . Denote the common value by  $D(\lambda)$ , we show that  $D(\lambda)$  satisfies a Bowen type formula, and is locally Lipschitz. We obtain the exact asymptotic behavior of  $D(\lambda)$  as  $\lambda$  tends to  $\infty$ . This considerably improves the result of Damanik and Gorodetski (Comm. Math. Phys. 337, 2015). We also show that for any  $(\alpha, \lambda) \in \mathbb{I} \times [24, \infty)$ , the density of states measure of  $H_{\alpha,\lambda,\theta}$  is exact-dimensional; its Hausdorff and packing dimensions coincide and are independent of  $\alpha$ . Denote the common value by  $d(\lambda)$ , we show that  $d(\lambda)$  satisfies a Young type formula, and is Lipschitz. We obtain the exact asymptotic behavior of  $d(\lambda)$  as  $\lambda$  tends to  $\infty$ . During the course of study, we also answer several questions in the same paper of Damanik and Gorodetski. This is a joint work with Jie CAO.

### Loosely Bernoulli nonhyperbolic ergodic measures

### Michał Rams

Polish Academy of Sciences

Given a finite collection of matrices  $A = \{A_1, \ldots, A_k\} \subset SL(2, \mathbb{R})$ , the matrix cocycle generated by A is the dynamical system  $F : SL(2, \mathbb{R}) \times \{1, \ldots, k\}^{\mathbb{Z}} \to SL(2, \mathbb{R}) \times \{1, \ldots, k\}^{\mathbb{Z}}$  defined by

$$F(B,\omega) = (A_{\omega_0}B, \sigma\omega).$$

For a given point  $\omega \in \{1, \ldots, k\}^{\mathbb{Z}}$  we define the Lyapunov exponent of  $\omega$  the following way:

$$\chi(\omega) = \lim_{n \to \infty} \frac{1}{n} \log ||\pi_1 \circ F^n(B)||,$$

where  $\pi_1$  is the projection to the first coordinate. Clearly, the limit (if it exists) does not depend on the choice of *B*. By the Oseledets Theorem, one can define the Lyapunov exponent of any ergodic measure as the value the Lyapunov exponent takes at almost every point with respect to this measure.

The classical result of Furstenberg states that, except for a meager set of matrix cocycles, every Bernoulli measure has positive Lyapunov exponent (his result is actually much more general, I'm just presenting it in the simplest case), that is every Bernoulli measure is hyperbolic. It was further generalized by Vircer and by Goldsheid to the class of Markov measures.

I will present the result, joint with Katrin Gelfert and Lorenzo Diaz, in which we prove that the Furstenberg Theorem does not work for loosely Bernoulli measures. Namely, we prove that for an open class of  $SL(2, \mathbb{R})$  matrix cocycles there exist loosely Bernoulli nonhyperbolic measures. Moreover, for those matrix cocycles the nonhyperbolic loosely Bernoulli measures are dense in the class of all nonhyperbolic ergodic measures (in the weak\*+entropy topology), and their metric entropies take all possible values in  $[0, h_0)$ , where  $h_0$  is the topological entropy of the set of points with Lyapunov exponent 0.

### Conformal dimension of p.c.f. self-similar sets

#### Hui Rao

### Central China Normal University

Université Paris Est Créteil

Conformal dimension of a metric space is the infimum of the Hausdorff dimensions of the quasisymmetric images of the space. By constructing new metrics on a p.c.f. self-similar set, we show that a large class of p.c.f. self-similar sets have conformal dimension 1. This considerably generalize a result of J. Tyson and J.M. Wu in 2006.

### Measures, annuli and dimensions

### Stéphane Seuret

In this talk, we investigate the possibility that probability measures charge very thin annuli infinitely often around a given point. This problem is related to return times of dynamical systems. The answer depends on the measure, the thinness of the annuli, and the norm chosen to define the annuli. This is a joint work with Z. Buczolich.

### "Entropies" in negatively curved spaces

### Lin Shu

For dynamical systems in negatively curved spaces, "entropies" are important quantities to characterize their dynamical complexities. In this talk, we will explain briefly how to use these dynamical "entropies" to understand the interactions between dynamical systems and the geometry of the underlying spaces.

### BV functions and fractional Laplacians on Dirichlet spaces

### Alexander Teplyaev

The talk will present bounded variation (BV) and fractional Sobolev functional spaces, Besov critical exponents, and isoperimetric and Sobolev inequalities associated with fractional Laplacians on metric measure spaces. The main tool is the theory of heat semigroup-based Besov classes in Dirichlet Metric Measure Spaces that uses a Korevaar-Schoen space approach in a general framework of strongly local Dirichlet spaces with a heat kernel satisfying sub-Gaussian estimates. Under a weak Bakry-Emery curvature type condition, which is new in this setting, this BV class is identified with a heat semigroup-based Besov class. As a consequence of this identification, properties of BV functions and associated BV measures can be studied in detail. In particular, we prove co-area formulas, global Sobolev embeddings, and isoperimetric inequalities. It is shown that for nested fractals or their direct products, the BV class we define is dense in  $L^1$ . The examples of the unbounded Vicsek set, unbounded Sierpinski gasket, and unbounded Sierpinski carpet are discussed. This is a joint work with Patricia Alonso-Ruiz, Fabrice Baudoin, Li Chen, Luke Rogers, and Nageswari Shanmugalingam.

### Measure and dimension theory for limsup sets generated by rectangles

### Baowei Wang

Huazhong University of Science and Technology

Dirichlet's theorem and Minkowski's theorem on the distribution of rational numbers/vectors are the two most fundamental theories in Diophantine approximation which leads to the study on the measure and dimension of limsup sets generated by balls and rectangles. Starting from Khintchine, Jarník, developed by Baker & Schmidt, Dodson and the most celebrated mass transference principle presented by Beresnevich & Velani, the metric theory of limsup sets generated by balls has been well established. In this talk, I will speak of the metric theory for limsup sets generated by rectangles.

### Dimension of higher dimensional irreducible self-affine measures

### Meng Wu

I will present some results regarding the dimension of self-affine measures in  $\mathbb{R}^d (d \ge 3)$  under certain irreducibility and proximality assumptions. These rely on a projection theorem for self-affine measures.

Peking University

University of Connecticut

University of Oulu

## **List of Participants**

Shigeki Akiyama	Tsukuba University
Li-Xiang An	Central China Normal University
Balázs Bárány	Budapest University of Technology and Economics
Julien Barral	Université Sorbonne Paris Nord
Changhao Chen	Anhui University
Haipeng Chen	Shenzhen Technology University
Edouard Daviaud	The Chinese University of Hong Kong
Qirong Deng	Fujian Normal University
Xinhan Dong	Hunan Normal University
Kenneth J. Falconer	University of St Andrews
Ai-Hua Fan	Université de Picardie & Central China Normal University
De-Jun Feng	The Chinese University of Hong Kong
Zhou Feng	The Chinese University of Hong Kong
Xiaoye Fu	Central China Normal University
Alexander Grigor'yan	Universität Bielefeld
Alexander Grigor'yan Qingsong Gu	Universität Bielefeld Nanjing University
Alexander Grigor'yan Qingsong Gu Xinggang He	Universität Bielefeld Nanjing University Central China Normal University
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho	Universität Bielefeld Nanjing University Central China Normal University Academia Sinica
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu	Universität Bielefeld Nanjing University Central China Normal University Academia Sinica Tsinghua University
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu Wen Huang	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of China
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu Wen Huang Esa Järvenpää	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of Oulu
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu Wen Huang Esa Järvenpää	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of Manchester
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu Wen Huang Esa Järvenpää Xiong Jin Jun Kigami	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of ManchesterKyoto University
Alexander Grigor'yanQingsong GuXinggang HeChing Wei HoJiaxin HuWen HuangEsa JärvenpääXiong JinJun KigamiDerong Kong	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of ManchesterKyoto UniversityChongqing University
Alexander Grigor'yan Qingsong Gu Xinggang He Ching Wei Ho Jiaxin Hu Wen Huang Esa Järvenpää Siong Jin Jun Kigami Derong Kong	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of ManchesterKyoto UniversityChongqing UniversitySichuan University
Alexander Grigor'yanQingsong GuXinggang HeChing Wei HoJiaxin HuWen HuangEsa JärvenpääXiong JinJun KigamiDerong KongShilei KongChun-Kit Lai	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of ManchesterKyoto UniversityChongqing UniversitySichuan UniversitySan Francisco State University
Alexander Grigor'yanQingsong GuXinggang HeChing Wei HoJiaxin HuWen HuangEsa JärvenpääXiong JinJun KigamiDerong KongShilei KongChun-Kit LaiChun-Kong Law	Universität BielefeldNanjing UniversityCentral China Normal UniversityAcademia SinicaTsinghua UniversityUniversity of Science and Technology of ChinaUniversity of OuluUniversity of ManchesterKyoto UniversityChongqing UniversitySichuan UniversitySan Francisco State UniversityNational Sun Yat-Sen University

Chi-Wai Leung	The Chinese University of Hong Kong
King Shun Leung	The Hong Kong Institute of Education
Bing Li	South China University of Technology
Xiangyu Liang	Beihang University
Lingmin Liao	Wuhan University
Bochen Liu	Southern University of Science and Technology
Jingcheng Liu	Hunan Normal University
Quansheng Liu	Université de Bretagne-Sud
Jun Jason Luo	Chongqing University
Jun Luo	Sun Yat-sen University
Fan Lü	Sichuan Normal University & The Chinese University of Hong Kong
Caiyun Ma	The Chinese University of Hong Kong
Pertti Mattila	University of Helsinki
Junjie Miao	East China Normal University
Mathav Murugan	University of British Columbia
Sze-Man Ngai	Georgia Southern University
Yuka Ota	Kyoto University
Hua Qiu	Nanjing University
Yanhui Qu	Tsinghua University
Michal Rams	Polish Academy of Sciences
Hui Rao	Central China Normal University
Huojun Ruan	Zhejiang University
Stéphane Seuret	Université Paris-Est Créteil
Narn-Rueih Shieh	National Taiwan University
Lin Shu	Peking University
Bo Tan	Huazhong University of Science and Technology
Alexander Teplyaev	University of Connecticut
Baowei Wang	Huazhong University of Science and Technology
Xiangyang Wang	Sun Yat-Sen University
Yang Wang	The Hong Kong University of Science and Technology
Zhiqiang Wang	East China Normal University
Zhi-Ying Wen	Tsinghua University

Shengyou Wen	Hubei University
Jun Wu	Huazhong University of Science and Technology
Haihua Wu	Changsha University of Science and Technology
Meng Wu	University of Oulu
Wen Wu	South China University of Technology
Jianci Xiao	The Chinese University of Hong Kong
Yuhao Xie	The Chinese University of Hong Kong
Ying Xiong	South China University of Technology
Jian Xu	Huazhong University of Science and Technology
Yuanling Ye	South China Normal University
Tianhan Yi	The Chinese University of Hong Kong
Eveline Young	Pittsburgh
Zu-Guo Yu	Xiangtan University
Pengfei Zhang	Southwest Jiaotong University

## **Useful Information**

### Location of Lady Shaw Building

Please consult the Campus Map for the location of Lady Shaw Building, where all the talks will be held in LT2, Lady Shaw Building, CUHK.

### Coaches

Each morning, we have organized a dedicated coach service from Regal Riverside Hotel to Sir Run Run Shaw Hall, conveniently located near Lady Shaw Building.

Regal Riverside Hotel $ ightarrow$ Sir Run Run Shaw Hall				
11 Dec 2023	12 Dec 2023	13 Dec 2023	14 Dec 2023	15 Dec 2023
8:30am	8:30am	8:30am	8:30am	8:30am
Sir Run Run Shaw Hall $ ightarrow$ Regal Riverside Hotel				
5:30pm, 13 Dec 2023				

### Coach Timetable

If you happen to miss the coach, taxis are available for hire to reach the University (Lady Shaw Building). The fare is approximately 70HKD, and the journey takes about 10 minutes.

### Meals

**Lunch:** Reserved tables are available at SHHO College Student Canteen, and the associated cost is covered by the Conference.

**Dinner:** Besides the canteens and restaurants in the University, there are a lot of restaurants in New Town Plaza (e.g. at 6/F, 7/F) and Shatin Centre (3/F) near Shatin MTR station.

### **Conference Banquet**

**Time:** 7:00pm, 13 December 2023, Wednesday. **Venue:** The Forum, 1/F, Regal Riverside Hotel.

A coach will departure from Car Park, Sir Run Run Shaw Hall in the University to Regal Riverside Hotel at 5:30pm.

### Half Day Free

There will be a half day free on 14 December 2023, Thursday.

### **Other information**

### **Useful Telephone numbers (+852)**

Emergency Services (Police, Fire and Ambulance)	999
Hong Kong International Airport (24 hours)	2181 8888
Hong Kong Tourism Board, Visitor Hotline	2508 1234
Hong Kong Weather (Dial-a-weather)	1878 200
Regal Riverside Hotel	2649 7878
Department of Mathematics, CUHK	3943 7965
Conference Coordinator: De-Jun Feng	6737 2146

### **Facilities at Campus**

Bank, canteen, supermarket, barber shop and souvenir counter are located on campus. Please refer to the Campus Map.

Facilities	Address	Opening hours
Hang Seng Bank	1/F, John Fulton Centre	9:00am-4:00pm (Mon-Fri)
Benjamin Franklin Centre	G/F, Benjamin Franklin Centre	7:30am-7:30pm
Coffee/Fast food/Restaurant		
Fusion Supermarket	LG, John Fulton Centre	8:30am-10:00pm (Mon-Fri)
		8:30am-9:00pm (Sat, Sun, PH)
Barber shop	G/F, John Fulton Centre	10:00am-8:00pm (Mon-Fri)
Souvenir Counter	G/F, John Fulton Centre	9:00am-5:30pm (Mon-Fri)

### Other ways from Regal Riverside Hotel to University

- **Taxi:** If you have 3–4 people, taking taxi is the most convenient way to travel between the Regal Riverside Hotel and the campus or University MTR station.
- MTR (Subway): From Regal Riverside Hotel to University MTR Station:
  - 1. Take a bus/minibus to Shatin MTR Station; Or walk to Shatin MTR Station (about 15 min).
  - 2. Take MTR at Shatin MTR Station to University MTR Station. Due to the current policy of Campus Access Control, visitors are required to register their travel documents like passport at campus entry points in order to gain entry into campus.

## **Campus Map**

From University MTR Station to Lady Shaw Building (the conference venue):

- School Bus: Take school buses with Routes 1A/1B/3 at the stops next to University MTR station. Get off at the stop *Sir Run Run Shaw Hall* (the 2nd stop for Routes 1A/3; the 3rd stop for Route 1B). You will see the roof garden of Lady Shaw Building right after getting off the bus.
- Walk: Walking uphill is another option.



View the Campus Map in Google Map:





Eastern Campus

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Heang Sang Hall
 Chard Churn Ha Hostel
 Chard Churn Ha Hostel
 Adam Schall Residence
 HT-C. Cheng Bidg
 Sitvan Amphtheatre
 Sethichem Hall
 Chung Lhuk Shan Amenities Bidg
 Win Chung Library
 Trang Shu Tim Bidg
 Trang Shu Tim Bidg
 U.C. Staff Residence

G Posigraduate Heil Nos. 4 – 6 For International Houses 3 – 5 Journersity Residence Nos. 3 – 4 Journersity Residence Nos. 10 – 11 Junversity Residence No. 12 – 13 Z University Residence No. 14 Junversity Residence No. 15 Junversity Residence No. 16 – 17

**United College** 

10

Leung Hung Kee Bldg

Xuesi Hall Pavilion of Harmony Grace Tien Hall Daisy Li Hall Chih Hsing Hall Chih Hsing Hall Staff Student Centre – L

48 Lo Kwee-Seong Integrated Biomedical Sciences Bldg

58 Jockey Club Postgraduate Halls 2 - 3

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Maurice R. Greenberg Bldg Morningside College Student Hostel (High Block)

S.H. Ho College

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Wu Yee Sun College Activity Centre Wu Yee Sun College Student Hostel West Block Wu Yee Sun College Student Hostel East Block

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Marina Tse Chu Bldg C.W. Chu College Student Hostel Ina Ho Chan Un Chan Bldg

C.W. Chu College

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Ho Tim Hall Lee Quo Wei Hall Chan Chun Ha Hall

Lee Woo Sing College North Block
 Dorothy and Ti-Hua KOO Bldg

Lee Woo Sing College

Area 39

52 Shaqiyai Fatemiry Association Research Services Centre Research Services Centre Service And Service Science Science Service Reservicy Hong Kong, Sha Tin (Houb Facilities of the Teaching Horel of The Chinese University of Hong Kong)

Wen Lan Tang Ya Qun Lodge Huen Wing Ming Bldg Shaw College Lecture Theatre

Shaw College

Student Hostel II (Low Block) Yat Sen Hall Kuo Mou Hall (Low Block) Kuo Mou Hall (High Block) HERBSnSENSES Chinese Medicinal Plants Garden

10

10 Leung Kau Kui Bidg
 Fung Knp Hey Bidg
 L. Dak Sum Bidg
 L. Loak Sun Vac Bidg
 L. Dak Sun Yac Bidg

Central Campus

**Chung Chi College** 

Pentecostal Mission Hall Complex (Low Block) Pentecostal Mission Hall Complex (High Block)

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- Hua Lien Tang
- Chung Chi Tang Ming Hua Tang

Lady Shaw Bldg Charles Kuen Kao Bldg

Ma Lin Bldg Run Run Shaw Science Bldg Science Centre

- Science Centre East Block Choh-Ming Li Basic Medical Sciences Bldg Lady Ho Tung Hall Mong Man Wai Bldg
- Chiang's Bldg Chan Kwan Tung Inter-university Hall

36 University Sports Centre 77 Sir Philip Haddon - Cave Sports Field 18 Ho Sin-Hang Engineering Bldg 19 William M-Wing Engineering Bldg 0 International House 1

1054301

Humanities Bldg Friendship Lodge Cheng Ming Bldg Ch'ien Mu Library

University Health Centre Jockey Club Postgraduate Hall 1 Kwok Sports Bidg Yali Guest House

International House 2 Alumni Garden Swimming Pool Academic Bldg No. 1

- 27 28 29 30

Lingnan Stadium

Asia Col

- 2 Yrigʻin Tangʻ 3 Lee Silu Puirtal 4 Madam S.H. Ho Hall 5 Wen Chih Tang 4 Saunobi hiernational Academic Park 4 Saunobi Nen Bidg 7 Wu Ho Man Yuen Bidg 3 Lee Shau Kee Architecture Bidg 5 Fong Yun Vah Hall 5 Fong Yun Vah Hall 9 Fong Shu Chuen Bidg