## List of Abstracts

## Multiplicative Markoff-Lagrange spectrum and symbolic dynamics

## Shigeki Akiyama

University of Tsukuba
Markoff-Lagrange spectrum is a discrete phenomenon that appeared in classical Diophantine approximation and correlates badly approximable numbers and Sturmian sequences. Since many problems in number theory are related to the study of fractional parts of exponential growth sequences, it is interesting if we can observe this spectrum phenomena in multiplicative setting. I will talk on recent development on the multiplicative Markoff-Lagrange spectrum. The key formula is to intertwine this problem into the symbolic dynamical setting and its inverse. Then I will explain the information on spectra obtained from this basic formula, together with some proofs to illustrate our results. The employed techniques are widespread from combinatorics on words, number theory, and fractal geometry.

This is joint work with H. Kaneko and T. Kamae.

## On the dimensions of random statistically self-affine Baransky carpets and sponges

## Julien Barral

Université Sorbonne Paris Nord
We will present some results on the dimension theory of random statistically self-affine Baransky carpets and sponges, and the inhomogeneous Mandelbrot measures they support.

## Fractal percolation on statistically self-similar and self-affine sets

## Kenneth Falconer

University of St Andrews
Originally introduced by Benoit Mandelbrot, fractal percolation is a statistically self-similar process based on a hierarchy of square grids leading to a random set $F$. With each square selected independently with probability $p$, Mandelbrot suggested that there was a critical probability $p_{c}$ such that $F$ undergoes a topological phase transition, changing as $p$ increases through $p_{c}$ from being totally disconnected to having non-trivial connected components. This was confirmed by Chayes, Chayes and Durrett who derived further properties of $F$, as did Dekking, Meester and others.

We will give an overview of fractal percolation and consider differences and similarities with the analogous process based on a rectangular grid leading to a statistically self-affine set.

## Stationary random fields and Trigonometric multiplicative chaos

## Ai-Hua Fan <br> Université de Picardie \& Central China Normal University

We introduce a class of stationary random fields on compact Abelian groups and there are many unsolved problems on these fields. In 1930's, Paley, Zygmund and Wiener studied three types (Rademacher, Steinhaus, Gauss) of random trigonometric series. One of series of Gauss type defines the Brownian motion. The series of Steinhaus type, which involve naturally the group structure of the cercle $\mathbb{T}=\mathbb{R} / \mathbb{Z}$, are stationary fields on the group $\mathbb{T}$. In 1960's, Kahane studied general random trigonometric series under the condition that the coefficients of the series is square summable. Important improvements and developments are then followed (Billard, Marcus-Pisier, Talagrand et al). See Kahane's book (Some Random Series of Functions, 1985, Cambridge Press). If the coefficients of a random trigonometric series is not square-summable, the series does not define a function neither a measure, but a distribution. What happens about the series about its partial sums? With Yves Meyer, we construct a class of trigonometric chaotic measures in the setting of Multiplicative Chaos of Kahane (1987) to study the random stationary distributions on $\mathbb{T}$, as well as on $\mathbb{T}^{d}$. It is proved that the behavior of the partial sums are strongly multifractal. We give a full study of the associated chaotic operators, by describing their kernels and images, and consequently we have computed the Hausdorff dimensions of the chaotic measures. Our trigonometric chaos is very similar to the Gaussian Multiplicative Chaos, which is related to the Gaussian Free Field.

## Tails of heat kernels for jump processes

## Alexander Grigor'yan

This talk is based on a series of joint papers with Eryan Hu and Jiaxin Hu.
We prove upper bounds of the heat kernel $p_{t}(x, y)$ of a jump type Dirichlet form on a doubling metric measure space ( $M, d, \mu$ ), where the off-diagonal term depends on a certain $L^{q}$ tail estimate of the jump kernel $J(x, y)$.
If the measure $\mu$ is $\alpha$-regular then the said tail estimate is as follows:

$$
\|J(x, \cdot)\|_{L^{q}\left(B^{c}(x, r)\right)} \leq \frac{\text { const }}{r^{\gamma}}
$$

where $B(x, r)$ denotes metric balls. We prove that if $q \in[2, \infty]$ and

$$
\gamma=\frac{\alpha}{q^{\prime}}+\beta
$$

where $\beta>0$ and $q^{\prime}$ is the Hölder conjugate of $q$ then (), together with the Faber-Krahn inequality and the generalized capacity condition with parameter $\beta$, is equivalent to the following upper bound of the tail of the heat kernel:

$$
\left\|p_{t}(x, \cdot)\right\|_{L^{q}\left(B^{c}(x, r)\right)} \leq \frac{\text { const }}{t^{\alpha /\left(\beta q^{\prime}\right)}}\left(1+\frac{r}{t^{1 / \beta}}\right)^{-\gamma} .
$$

It follows from () that the heat kernel satisfies the following pointwise upper estimate:

$$
p_{t}(x, y) \leq \frac{\text { const }}{t^{\alpha / \beta}}\left(1+\frac{d(x, y)}{t^{1 / \beta}}\right)^{-\gamma}
$$

Important ingredients of the proof are the elliptic and parabolic mean value inequalities.
The case $q=\infty$ (and, hence, $q^{\prime}=1$ ) amounts to the previously known results of AG, J.Hu, K.-S.Lau Trans.AMS (2014) and Z.-Q.Chen, T.Kumagai, J.Wang, Mem.AMS (2021), while the case $q<\infty$ is entirely new.

## The weak and strong elliptic Harnack inequalities

## Jiaxin Hu

Tsinghua University
In this talk, we consider the regular resurrected Dirichlet form on the metric space equipped with a doubling measure. We show that the heat kernel estimate is equivalent to the weak elliptic Harnack inequality, the mean exit time estimate, plus the jump kernel upper bound. If further the upper jumping smoothness holds, we obtain a sharper assertion, that is, the strong elliptic Harnack inequality also comes into the stage. In particular, for the strongly local Dirichlet form where the jump vanishes (so that both the jump kernel upper bound and the upper jumping smoothness are trivially satisfied), our assertion coincides with the one achieved by Grigor'yan, Hu and Lau (2015 JMS Japan). This talk is based on the joint work with Zhenyu Yu.

# Density of minimal points and Bergelson-Hindman question 

## Wen Huang University of Science and Technology of China

Furstenberg's multiply recurrent theorem states that any dynamical system has multiply recurrent points, and points out that this result is equivalent to the van der Waerden theorem. An equivalent form of van der Waerden's theorem is that any piecewise syndetic subset of a natural number contains any arbitrarily long arithmetic progressions. In this talk, we discuss the correlation between multiple recurrence and piecewise syndetic set, and provide some applications in combinatorial number theory.

In 1998 Furstenberg and Glasner proved that the set composed of the first term and common difference of all arithmetic progressions of length $k$ appearing in the piecewise syndetic subset of natural numbers is also piecewise syndetic subsets in $\mathbb{Z}^{2}$. In 2001 Bergelson and Hindman raised the question of whether the polynomial version of this result holds. We will answer the Bergelson-Hindman question by showing the density of minimal points of a dynamical system of $\mathbb{Z}^{2}$ action associated with the piecewise syndetic set and the polynomials. This based on joint works with Professors Shao and Ye.

## Some examples of random covering sets

## Esa Järvenpää

University of Oulu
We consider dimensions of random covering sets generated by balls and driven by general measures. We improve the general lower bound given by Ekström and Persson and prove their conjecture concerning the exact value of dimension in a special case. We also give various examples demonstrating the complexity of dimension for generating balls with arbitrary sequences of radii. This is joint work with Maarit Järvenpää, Markus Myllyoja and Örjan Stenflo.

## On the fibres of planar self-similar sets with dense rotations

## Xiong Jin

University of Manchester
In this talk we will look at the size of fibres of planar self-similar sets with dense rotations. We shall first review some existing examples for which the size of fibres are known. Then we will look at sufficient conditions under which one may deduce a lower bound of the dimension of fibres. These investigations are built on the application of Mandelbrot percolations on self-similar sets. They are also connected to the problem of finding interior points in the radial projections of self-similar sets and in the arithmetic sum of Cantor sets.

## Yet another construction of "Sobolev spaces" on metric spaces

## Jun Kigami

Kyoto University
The counterpart of "Sobolev space" on metric spaces has been intensively studied for the last 20 years after the pioneering works by Cheeger, Hajlasz, and Shanmugalingam. The mainstream of the ideas is to use the local Lipschitz constant of a function as a suitable substitute for its gradient. However, a recent study by Kajino and Murugan on the conformal walk dimension revealed that the Dirichlet form associated with the Brownian motion on the Sierpinski carpet can not be a Sobolev space in this sense. In this talk, we will propose a new way of constructing "Sobolev spaces" on compact metric spaces including the Sierpinski carpet.

## Birth-death type random walks on hyperbolic graphs

Shi-Lei Kong
Sichuan University
As a natural generalization of the classic birth-death chains on nonnegative integers, we study a class of reversible random walks of birth-death type on hyperbolic graphs, and analyze the quadratic forms of induced energy on the boundaries. The result provides a discretization of certain non-local regular Dirichlet forms on doubling metric measure spaces. In addition, we show that a hyperbolic graph carries such birth-death type random walks if and only if it is roughly starlike and has bounded degree.

## On Erdős similarity problem and its variants

## Chun-Kit Lai

San Francisco State University
Erdős similarity conjecture asserted that patterns of infinite cardinality can be avoided by a set of positive Lebesgue measure in the sense that the set does not contain affine copies of the given pattern. The conjecture is currently open and fast decaying sequences like $2^{-n}$ has been a bottleneck in resolving the conjecture. In this talk, we will report on two recent progresses of this conjecture. First, we will consider the pattern being Cantor sets. Second, we will consider bi-Lipschitz copies instead of affine copies. Interesting and sharp results will be presented in both considerations. These are joint works with De-Jun Feng, Ying Xiong, and some of my students.

## Spectral analysis for some periodic quantum graphs

## Chun-Kong Law

National Sun Yat-sen University
We shall derive and analyze the dispersion relations of some periodic quantum graphs associated with Archimedean tilings, where the potentials are even, or non-even. Furthermore, we study the existence of Dirac points, which are points where different sheets of dispersion surface touch to form a conical singularity. We prove there exist infinitely many Dirac points located at the periodic eigenvalues. We shall also see that this occurs when the potential function has a special form.

This is joint work with E.O. Jatulan of University of the Philippines Los Baños.

## Local times of anisotropic Gaussian random fields and stochastic heat equation

## Cheuk Yin Lee The Chinese University of Hong Kong (Shenzhen)

In this talk, we discuss the local times of a class of anisotropic Gaussian random fields and related fractal properties. We present some moment estimates and regularity results for the local times. Our key estimates rely on geometric properties of Voronoi partitions with respect to an anisotropic metric and the use of Besicovitch's covering theorem. Our results can be applied to the solutions to systems of stochastic heat equations with additive Gaussian noise. As a consequence, we determine the exact gauge function for the parabolic Hausdorff measure of the level sets of the solutions. This talk is based on joint work with Yimin Xiao.

## The products of calibrated sets and paired calibrated sets in Plateau's problem

## Xiangyu Liang

Beihang University
Plateau's problem is a main interest in geometric measure theory. It aims at understanding the behavior of physical objects that admit certain minimizing property, such as soap films. Physical soap films are probably more accurately modeled by Almgren's minimal sets, but the lack of algebraic coherence makes it difficult to prove minimality. The theory of calibrated geometry is a powerful tool to study minimizing manifold (possibly with singularities). It was introduced by Harvey-Lawson in the 80 's, and builds a bridge between classical theory of manifolds and geometric measure theory. On the other hand, it cannot be applied directly to Plateau's problem. Then in the 90 's, K.Brakke, G, Lawlor \& F. Morgan introduced the method of paired calibration to prove various minimality of sets satisfying a given separation condition. It is very often used in the classification of singularities for codimension 1 minimal sets in Plateau's problem. Compared to the above ordinary calibration methods, a major advantage of paired calibrations is that it ignores algebraic multiplicities, which corresponds to the spirit of Plateau's problem. However, in general we do not know a generalisation to codimension larger than one, and, at first glance, the minimality of the products of calibrated sets or paired calibrated sets is unknown. In this talk, we will first give very simple examples to show how to use calibration and paired calibratoin method to prove various minimalities for sets, and explain the main different of these two theories. Then we introduce the background and definitions for Almgren minimal sets, classification of singularities for Plateau's problem, and how the theories of calibration and paired calibration applies. Finally, we will discuss the minimality of the products of these two kinds of sets in codimension 2.

## Uniform approximation problems of expanding Markov maps

## Lingmin Liao

Wuhan University
Let $T:[0,1] \rightarrow[0,1]$ be an expanding Markov map with a finite partition. Any Hölder continuous potential $\phi$ produces an invariant Gibbs measure $\mu_{\phi}$. For $\kappa>0$, we investigate $\mu_{\phi}$-almost surely the size of the uniform approximation set

$$
\mathcal{U}^{\kappa}(x):=\left\{y \in[0,1]: \forall N \gg 1, \exists n \leq N \text {, such that }\left|T^{n} x-y\right|<N^{-\kappa}\right\} .
$$

The critical value of $\kappa$ such that the Hausdorff dimension of $\mathcal{U}^{\kappa}(x)$ equals to 1 for $\mu_{\phi}$-a.e. $x$ is proven to be $1 / \alpha_{\max }$, where $\alpha_{\max }=-\int \phi d \mu_{\max } / \int \log \left|T^{\prime}\right| d \mu_{\max }$ and $\mu_{\text {max }}$ is the Gibbs measure associated with the potential $-\log \left|T^{\prime}\right|$. Moreover, when $\kappa>1 / \alpha_{\max }$, we show that for $\mu_{\phi}$-a.e. $x$, the Hausdorff dimension of $\mathcal{U}^{\kappa}(x)$ as a function of $1 / \kappa$ agrees with the multifractal spectrum of $\mu_{\phi}$. This is a joint work with Yubin He.

## $L^{p}$ estimates of orthogonal projections, dual Furstenberg problem, and discretized sum-product

## Bochen Liu

Southern University of Science and Technology
$L^{2}$ estimates of orthogonal projections are classical in geometric measure theory. In this talk we shall discuss about recent progress on $L^{p}$ estimates. Then we come up with a dual version of the Furstenberg problem and introduce some partial results. We also find that, compared with general sets, Cartesian products have better $L^{p}$-behavior. This leads to improvement on some discretized sum-product estimates. This is joint work with Longhui Li.

## Products of random matrices

## Quansheng Liu

Université de Bretagne-Sud
Some recent progress on limit theorems for products of random matrices will be presented. We focus on large deviations and Gaussian approximation, and we also consider the multifractal spectrum of Lyapunov's exponent for random matrices on regular trees. This talk is mainly based on joint works with De-Jun Feng, Ion Grama and Hui Xiao.

## On Bi-accessibility Dimension of Quadratic Julia Sets

## Jun Luo

Sun Yat-Sen University
The classical core entropy $h_{\text {core }}(f)$ for post critically finite (PCF) polynomials with degree $\geq 2$ is defined to be the topological entropy of $f$ restricted to its Hubbard tree. We fully generalize this notion to a new quantity $\mathcal{E}_{\text {core }}(f)$, called the core entropy of $f$, which is well defined if only $f$ has a connected Julia set. It has four properties. First, $\mathcal{E}_{\text {core }}(f)=h_{\text {core }}(f)$ when $f$ is PCF. Second, $\mathcal{E}_{\text {core }}\left(f^{n}\right)=n \mathcal{E}_{\text {core }}(f)$ for all $n \geq 2$. Third, $\mathcal{E}_{\text {core }}(f)=\mathcal{E}_{\text {core }}(g)$ whenever $f$ and $g$ are $J$ equivalent. Finally, if $f$ has no irrationally neutral cycle there is a compact set $B^{*}(f) \subset \mathbb{S}^{1}$ invariant under $\sigma_{d}(w)=w^{d}$ such that $\operatorname{dim}_{H} B^{*}(f)=\frac{\mathcal{E}_{\text {core }}(f)}{\log d}$. In such a case, we further relate to $f$ a set $K_{\text {biac }}(f) \subset \mathbb{S}^{1}$. The Hausdorff dimension of this set will be called the bi-accessibility dimension of $f$. Restricting to quadratic polynomials. we set $h(c)=\mathcal{E}_{\text {core }}\left(f_{c}\right)$ and $g(c)=\operatorname{dim} K_{\text {biacc }}\left(f_{c}\right)$ for $f_{c}(z)=z^{2}+c$, where $c$ runs through the whole Mandelbrot set $\mathcal{M}$. Then $h: \mathcal{M} \rightarrow[0, \log 2]$ is continuous nowhere, although its restriction to the subset $\left\{c \in \mathcal{M}: f_{c}\right.$ is PCF $\}$ has a continuous extension $\bar{h}: \mathcal{M} \rightarrow[0, \log 2]$. It is known that $g(c)=h(c)$ whenever $f_{c}$ is PCF or $c$ is NOT a tip parameter. The following issues remains open: Is it true that $g(c)$ is continuous over the whole Mandelbrot set $\mathcal{M}$ ?

## Hausdorff dimension of intersections

## Pertti Mattila

University of Helsinki
Let $A$ and $B$ be Borel subsets of $\mathbb{R}^{n}$. What can we say about the Hausdorff dimensions of the intersections of $A$ and typical rigid motions of $B$ ? More precisely, of $\operatorname{dim} A \cap(g(B)+z)$ for almost all rotations $g \in O(n)$ and for translations $z \in \mathbb{R}^{n}$ in a set of positive Lebesgue measure. Optimally one could hope that this dimension is at least $\operatorname{dim} A+\operatorname{dim} B-n$, which happens when smooth surfaces meet in a general position. This is open, but I shall discuss some new partial results.

## Dimensions of Non-autonamous self-affine sets

## Jun-Jie Miao

East China Normal University
In the talk, we define a class of Iterated function systems named "Non-autonamous self-affine Iterated function system", and we call its invariant set Non-autonamous self-affine set or self-affine Moran set which is the generalization of classic Moran sets and self-affine sets. Simply to say, we apply different IFS at each step in the iteration.

To study the dimensions of self-affine Moran sets, we define two critical values $s^{*}$ and $s_{A}$, and the upper box-counting dimensions and Hausdorff dimensions of self-affine Moran sets are bounded above by $s^{*}$ and $s_{A}$, respectively. Unlike self-affine fractals where $s^{*}=s_{A}$, we have that $s^{*} \geq s_{A}$, and the inequality may hold strictly.

Under certain conditions, we obtain that the upper box-counting dimensions and Hausdorff dimensions of self-affine Moran sets may equal to $s^{*}$ and $s_{A}$, respectively. In particular, we study self-affine Moran sets with random translations, and the Hausdorff dimensions of such sets equal to $s_{A}$ almost surely.

## Reflected diffusion on uniform domains

## Mathav Murugan

University of British Columbia
I report recent progress on heat kernel estimates for reflected diffusion on uniform domains where the underlying space admits sub-Gaussian heat kernel bounds. A key novelty of our work is the use of an extension operator that extends functions from the domain of the Dirichlet form for the reflected diffusion to that of the diffusion in the ambient space. If time permits, I will discuss heat kernel estimates for trace of reflected diffusion on the boundary of a uniform domain. The results on the trace jump process are based on a joint work with Naotaka Kajino.

## Spectral properties of Kreĭn-Feller operators

Sze-Man Ngai

Hunan Normal University \& Georgia Southern University
A Kreĭn-Feller operator we study is a Laplacian defined on a domain by a measure. The spectral dimension of a Kreǐn-Feller operator is a fundamental quantity that plays an important role in studying the analytic properties of the operator. We report some results concerning the spectral dimension of Kreĭn-Feller operators defined by fractal measures, focusing on self-similar measures with overlaps. We discuss some applications, including heat kernel estimates and wave propagation speed. We also discuss the extension of such Laplacians to Riemannian manifolds. This talk is based on joint work with Qingsong Gu, Jiaxin Hu, Lei Ouyang, Wei Tang, and Yuanyuan Xie.

## Almost sure dimensional properties for the spectrum and the density of states of Sturmian Hamiltonians

## Yanhui Qu

Tsinghua University
We find a full Lebesgue measure set of frequencies $\check{\mathbb{I}} \subset[0,1] \backslash \mathbb{Q}$ such that for any $(\alpha, \lambda) \in$ $\check{\mathbb{I}} \times[24, \infty)$, the Hausdorff and box dimensions of the spectrum of the Sturmian Hamiltonian $H_{\alpha, \lambda, \theta}$ coincide and are independent of $\alpha$. Denote the common value by $D(\lambda)$, we show that $D(\lambda)$ satisfies a Bowen type formula, and is locally Lipschitz. We obtain the exact asymptotic behavior of $D(\lambda)$ as $\lambda$ tends to $\infty$. This considerably improves the result of Damanik and Gorodetski (Comm. Math. Phys. 337, 2015). We also show that for any $(\alpha, \lambda) \in \check{\mathbb{I}} \times[24, \infty)$, the density of states measure of $H_{\alpha, \lambda, \theta}$ is exact-dimensional; its Hausdorff and packing dimensions coincide and are independent of $\alpha$. Denote the common value by $d(\lambda)$, we show that $d(\lambda)$ satisfies a Young type formula, and is Lipschitz. We obtain the exact asymptotic behavior of $d(\lambda)$ as $\lambda$ tends to $\infty$. During the course of study, we also answer several questions in the same paper of Damanik and Gorodetski. This is a joint work with Jie CAO.

## Loosely Bernoulli nonhyperbolic ergodic measures

Michat Rams
Polish Academy of Sciences
Given a finite collection of matrices $A=\left\{A_{1}, \ldots, A_{k}\right\} \subset S L(2, \mathbb{R})$, the matrix cocycle generated by $A$ is the dynamical system $F: S L(2, \mathbb{R}) \times\{1, \ldots, k\}^{\mathbb{Z}} \rightarrow S L(2, \mathbb{R}) \times\{1, \ldots, k\}^{\mathbb{Z}}$ defined by

$$
F(B, \omega)=\left(A_{\omega_{0}} B, \sigma \omega\right)
$$

For a given point $\omega \in\{1, \ldots, k\}^{\mathbb{Z}}$ we define the Lyapunov exponent of $\omega$ the following way:

$$
\chi(\omega)=\lim _{n \rightarrow \infty} \frac{1}{n} \log \left\|\pi_{1} \circ F^{n}(B)\right\|
$$

where $\pi_{1}$ is the projection to the first coordinate. Clearly, the limit (if it exists) does not depend on the choice of $B$. By the Oseledets Theorem, one can define the Lyapunov exponent of any ergodic measure as the value the Lyapunov exponent takes at almost every point with respect to this measure.

The classical result of Furstenberg states that, except for a meager set of matrix cocycles, every Bernoulli measure has positive Lyapunov exponent (his result is actually much more general, I'm just presenting it in the simplest case), that is every Bernoulli measure is hyperbolic. It was further generalized by Vircer and by Goldsheid to the class of Markov measures.

I will present the result, joint with Katrin Gelfert and Lorenzo Diaz, in which we prove that the Furstenberg Theorem does not work for loosely Bernoulli measures. Namely, we prove that for an open class of $\mathrm{SL}(2, \mathbb{R})$ matrix cocycles there exist loosely Bernoulli nonhyperbolic measures. Moreover, for those matrix cocycles the nonhyperbolic loosely Bernoulli measures are dense in the class of all nonhyperbolic ergodic measures (in the weak*+entropy topology), and their metric entropies take all possible values in $\left[0, h_{0}\right)$, where $h_{0}$ is the topological entropy of the set of points with Lyapunov exponent 0 .

## Conformal dimension of p.c.f. self-similar sets

## Hui Rao

Central China Normal University
Conformal dimension of a metric space is the infimum of the Hausdorff dimensions of the quasisymmetric images of the space. By constructing new metrics on a p.c.f. self-similar set, we show that a large class of p.c.f. self-similar sets have conformal dimension 1 . This considerably generalize a result of J. Tyson and J.M. Wu in 2006.

## Measures, annuli and dimensions

## Stéphane Seuret

Université Paris Est Créteil
In this talk, we investigate the possibility that probability measures charge very thin annuli infinitely often around a given point. This problem is related to return times of dynamical systems. The answer depends on the measure, the thinness of the annuli, and the norm chosen to define the annuli. This is a joint work with Z. Buczolich.

## "Entropies" in negatively curved spaces

## Lin Shu

Peking University
For dynamical systems in negatively curved spaces, "entropies" are important quantities to characterize their dynamical complexities. In this talk, we will explain briefly how to use these dynamical "entropies" to understand the interactions between dynamical systems and the geometry of the underlying spaces.

## BV functions and fractional Laplacians on Dirichlet spaces

## Alexander Teplyaev

University of Connecticut
The talk will present bounded variation (BV) and fractional Sobolev functional spaces, Besov critical exponents, and isoperimetric and Sobolev inequalities associated with fractional Laplacians on metric measure spaces. The main tool is the theory of heat semigroup-based Besov classes in Dirichlet Metric Measure Spaces that uses a Korevaar-Schoen space approach in a general framework of strongly local Dirichlet spaces with a heat kernel satisfying sub-Gaussian estimates. Under a weak Bakry-Emery curvature type condition, which is new in this setting, this BV class is identified with a heat semigroup-based Besov class. As a consequence of this identification, properties of BV functions and associated BV measures can be studied in detail. In particular, we prove co-area formulas, global Sobolev embeddings, and isoperimetric inequalities. It is shown that for nested fractals or their direct products, the BV class we define is dense in $L^{1}$. The examples of the unbounded Vicsek set, unbounded Sierpinski gasket, and unbounded Sierpinski carpet are discussed. This is a joint work with Patricia Alonso-Ruiz, Fabrice Baudoin, Li Chen, Luke Rogers, and Nageswari Shanmugalingam.

## Measure and dimension theory for limsup sets generated by rectangles

## Baowei Wang

Huazhong University of Science and Technology
Dirichlet's theorem and Minkowski's theorem on the distribution of rational numbers/vectors are the two most fundamental theories in Diophantine approximation which leads to the study on the measure and dimension of limsup sets generated by balls and rectangles. Starting from Khintchine, Jarník, developed by Baker \& Schmidt, Dodson and the most celebrated mass transference principle presented by Beresnevich \& Velani, the metric theory of limsup sets generated by balls has been well established. In this talk, I will speak of the metric theory for limsup sets generated by rectangles.

## Dimension of higher dimensional irreducible self-affine measures

## Meng Wu <br> University of Oulu

I will present some results regarding the dimension of self-affine measures in $\mathbb{R}^{d}(d \geq 3)$ under certain irreducibility and proximality assumptions. These rely on a projection theorem for self-affine measures.

