THE 26TH INTERNATIONAL DOMAIN DECOMPOSITION CONFERENCE DD XXVI

Online, December 7–12, 2020

Hosted by the Chinese University of Hong Kong

BOOK OF ABSTRACTS

Sponsors

Contents

Useful information . . . 3
Schedule at a Glance . . . 4
Plenary Lectures . . . 6
Minisymposia . . . 11
- MS01 – Learning Algorithms, Domain Decomposition Methods, and Applications . . . 11
- MS02 – Heterogeneous Domain Decomposition Methods: Theoretical Developments and New Applications . . . 14
- MS03 – Preconditioning Methods for Frequency Domain Wave Problems . . . 18
- MS04 – Two-Grid Method and Its Applications . . . 22
- MS05 – Mini-Symposium on Advanced Numerical Methods for Electromagnetic Problems . . . 26
- MS06 – Techniques and Application of Domain Decomposition for Modelling of Environmental Flows 30
- MS07 – Domain Decomposition Methods for Isogeometric and High Order Discretizations . . . 32
- MS08 – New Developments of Domain Decomposition Methods: Non-Standard Methods and Applications . . . 35
- MS09 – Space--Time FEM/BEM: Theory and Applications . . . 40
- MS10 – Multilevel Domain Decomposition Methods and Parallel Implementations . . . 43
- MS11 – Domain Decomposition Methods in HPC: Extremely Scalable Implementations and Their Application . . . 46
- MS12 – Asynchronous Iterative Methods . . . 49
- MS13 – Domain Decomposition Methods for Optimal Control Problems . . . 51
- MS14 – Adaptive Coarse Spaces, Multipreconditioning and Reduced Basis Methods . . . 53
- MS15 – Domain Decomposition Methods for Uncertainty Quantification and Applications . . . 55
- MS16 – Polygonal Finite Elements, DG, and Related Methods . . . 58
- MS17 – Nonlinear Preconditioning and Applications . . . 61
- MS18 – Average Schwarz Method and Numerical Homogenization: Theory and Implementation . 66
- MS19 – Solution Techniques for Nonstandard Discretization Methods: Theory and Applications . 70
- MS20 – Cross Points in Domain Decomposition Methods . . . 72
- MS21 – Domain-Decomposition Methods for Coupled Problems in Fluid Dynamics . . . 77
- MS22 – Robust Solvers for Multiphysics Problems . . . 80

Contributed Sessions . . . 82

Useful information

- All talks will be conductive live via the Zoom platform. If you do not have Zoom, you may download it at `https://zoom.us/download`.
- All zoom links are posted on the attendees-only portion of the conference website at `https://www.math.cuhk.edu.hk/conference/dd26/?Attendee_Only-Attendee_Login`. All registered participants should have received an e-mail with the username and password.
- For the benefit of participants who cannot attend certain sessions due to timezone reasons, we will record all plenary, minisymposium and contributed talks, as long as we have the consent from the respective speakers. Recordings will be accessible soon after the session concludes, and will be available for viewing for up to one week after the end of the conference.
- For technical support or any inquiries, please send an e-mail to `dd26@math.cuhk.edu.hk`.

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Schedule at a Glance

All dates and times are shown in local time in Hong Kong. The following table can be used to convert between time zones.

Hong Kong	Europe (CET)	North America (EST)	North America (PST)
10:00	03:00	21:00 (−1 day)	18:00 (−1 day)
16:00	09:00	03:00	00:00
20:00	13:00	07:00	04:00
22:00	15:00	09:00	06:00

Monday Dec 7					
22:00-22:15	**Opening Remarks**				
	PT-01 Plenary Session (Chair: Jun Zou)				
22:15-23:00	Ulrich Langer				
23:00-23:45	Victorita Dolean				
Tuesday Dec 8					
10:00-12:00	**MS4-01**	**MS5-01**	**CT1**		
	Cai M.	Chen Z.	Yazdani		
	Lee P.	Cui T.	Xu K.-L.		
	Fu S.	Duan H.	Fang L.		
			Cai M.		
16:00-18:00	**MS1-01**	**MS2-01**	**MS3-01**	**MS4-02**	**MS10-01**
	Sheng H.	Vergara	Bootland	Notay	Ciaramella
	Chung	Laadhari	Ma C.	Xie M.	Papez
	Yokota	Birken	Bouziani	Wang W.	Zampini
	Lee C.-O.	Clement	Dai R.	Zheng H.	
20:00-22:00	**MS1-02**	**MS2-02**	**MS3-02**	**MS10-02**	**MS18-01**
	Weber	Vanzan	Gong S.	Sistek	Alrabeei
	Gu L.	Hennicker	Zhang H.	Hanek	Peterseim
	Kopanicakova	Kwok	Dwarka	Oh D.S.	Li G.
	Heinlein	Tang		Tomasi	Pun S. M.
	PT-02 Plenary Session (Chair: Petter Bjørstad)				
22:15-23:00	Mary Wheeler				
23:00-23:45	Adrianna Gillman				
Wednesday Dec 9					
16:00-18:00	**MS4-03**	**MS5-02**	**MS11-01**	**CT2**	
	Hou T.	Wang B.	Rheinbach	Sarkar	
	Liu F.	Wu H.	Röver	Kim H.H.	
	Wang F.	Xiang X.	Chen R.	El-Gharbi	
	Cui J.	Lu W.	Lanser	Kornhuber	
20:00-22:00	**MS5-03**	**MS6**	**MS7-01**	**MS11-02**	**MS18-02**
	Yin T.	Thery		Uran	Legoll
	Yang W.	Genseberger	Scacchi	Köhler	Marcinkowski
	Zheng C.	Ye Q.	Widlund	Heinlein	Yu Y.
	Zheng W.	Tang H.	Dohrmann	Hochmuth	Galvis
	PT-03 Plenary Session (Chair: Laurence Halpern)				
22:15-23:00	Zdenek Dostal				
23:00-23:45	Patrick Joly				

Thursday Dec 10					
	MS7-02	**MS8-01**	**MS16-01**	**MS17-01**	**CT3**
16:00-18:00	Cho	Leng W.	Ayuso de Dios	Cai X.-C.	Kinnewig
	Takacs	Laayouni	Antonietti	McCoid	Hocking
	Schneckenleitner	Van Criekingen	Zhao	Lanser	Krzyzanowski
		Kyriakis	Kim D.H.	Liu L.	
	MS12	**MS15-01**	**MS18-03**	**MS19**	**MS20-01**
19:30-20:00				Alonso Rodriguez	
20:00-22:00	Tromeur-Dervout	Tu X.	Borges	Kogler	Boubendir
	Glusa	Zhang Z.W.	Rahman	Wang K.	Claeys
	Szyld	Li K.	Faal	Li Y.	Gander
	Chaouqui		Bjørstad	Garay	Krell
	PT-04 Plenary Session (Chair: Xiao-Chuan Cai)				
22:15-23:00	Rolf Krause				
23:00-23:45	Eric Chung				
Friday Dec 11					
	MS8-02	**MS15-02**	**MS16-02**	**MS21-01**	
10:00-12:00	Chang H.	Wu Z.	Wang J.	Lee H.S.	
	Zhang Z.K.	Li M.	Kim M.-Y.	Banks	
	Lucero	Xiong J.	Park E.-H.	Hoang	
	Xu Y.	Liao Q.	Shin D.-W.	Sockwell	
	MS8-03	**MS9-01**	**MS17-02**	**CT4**	
16:00-18:00	Outrata	Zank	Klawonn	Thies	
	Song B.	Lunet	Gu Y.	Huynh	
	Quan C.	Schafelner	Hwang	Madiot	
		Pacheco	Kumbhar	Shourick	
	MS9-02	**MS13-01**	**MS20-02**	**MS21-02**	**CT5**
20:00-22:00	Zikatanov	Salomon	Modave	Wick	Arraras
	Speck	Mechelli	Nicolopoulos	Durst	Sogn
	Lenz	Delourme	Parolin	Trenchea	Mohammad
	Steinbach	Ciaramella	Pechstein	Fu G.	
	PT-05 Plenary Session (Chair: Susanne Brenner)				
22:15-23:00	Laurence Halpern				
23:00-23:45	Xuemin Tu				
Saturday Dec 12					
	MS13-02|*MS20-03*	**MS17-03**	**MS22**	**MS14**	**CT6**
20:00-22:00	Reyes-Riffo	Keyes	Boon	Knepper	Park J.H.
	Yang H.D.	Luo L.	Niu C.	Song B.	Rhofir
	Royer	Kothari	Cavanaugh	Rave	Schanen
	Lunowa	Yang H.J.	Ohm	Chaouqui	Barrata
	PT-06 Plenary Session (Chair: Martin J. Gander)				
22:15-23:00	Carola Schönlieb				
23:00-23:45	Shuonan Wu				
23:45-00:00	**Closing remarks**				

Plenary Lectures

Ulrich Langer – Adaptive Space-Time Finite Element and Isogeometric Analysis

Monday 22:15–23:00

The traditional approaches to the numerical solution of initial-boundary value problems for parabolic or hyperbolic Partial Differential Equations (PDEs) are based on the separation of the discretization in time and space leading to time-stepping methods. This separation of time and space discretizations comes along with some disadvantages with respect to parallelization and adaptivity. To overcome these disadvantages, we consider completely unstructured finite element or isogeometric (B-spline or NURBS) discretizations of the space-time cylinder and the corresponding stable space-time variational formulation of the initial-boundary value problem under consideration. Unstructured space-time discretizations considerably facilitate the parallelization and simultaneous space-time adaptivity. Moving spatial domains or interfaces can easily be treated since they are fixed in the space-time cylinder. Beside initial-boundary value problems for parabolic PDEs, we will also consider optimal control problems constraint by linear or non-linear parabolic PDEs. Here unstructured space-time methods are especially suited since the optimality system couples two parabolic equations for the state and adjoint state that are forward and backward in time, respectively. In contrast to time-stepping methods, one has to solve one big linear or non-linear system of algebraic equations. Thus, the memory requirement is an issue. In this connection, adaptivity, parallelization, and matrix-free implementations are very important techniques to overcome this bottleneck. Fast parallel solvers like domain decomposition and multigrid solvers are the most important ingredients of efficient space-time methods.
The talk is based on joint works with Christoph Hofer, Martin Neumüller, Svetlana Kyas (Matculevich), Sergey Repin, Andreas Schafelner, Rainer Schneckenleitner, Olaf Steinbach, Ioannis Toulopoulos, Fredi Tröltzsch, and Huidong Yang. This research was supported by the Austrian Science Fund (FWF) through the projects NFN S117-03 and DK W1214-04. This support is gratefully acknowledged.

Victorita Dolean – Robust Solvers for Time-Harmonic Wave Propagation Problems

Monday 23:00–23:45

Time harmonic wave propagation problems are notoriously difficult to solve especially in high frequency regime. Several reasons are at the origin of this: first of all the oscillatory nature of the solution, meaning that the number of degrees of freedom after discretisation increases drastically with the wave number (especially for lower order approximations) giving rise to complex valued large problems to solve. Secondly, the indefiniteness of the operator: its spectral properties only making it difficult to control and predict the behaviour of a Krylov type solver. Not to mention the inherent challenges when the wave propagation takes place in a heterogeneous medium. We try to answer partially to some of the questions (with strong numerical evidence) by proposing a few methods which proved to be robust with respect to the wave number for different equations such as: Helmholtz, Maxwell or elastic waves. These methods are further applied to heterogeneous physically realistic problems arising in electrical engineering and geophysics.

Mary F. Wheeler – Domain Decomposition for Modeling Two-Phase Flow in Porous Media

Tuesday 22:15–23:00

Convergence failure and slow convergence rates are among the major challenges in solving the system of nonlinear equations arising in modeling two phase flow in porous media. Although mitigated, such issues still linger when using strictly small time steps and unconditionally stable fully implicit schemes. The price that comes with restricting time steps to small scales is the enormous computational load, especially in large-scale models. To address this problem, we introduce a sequential domain decomposition local mesh refinement framework based on enhanced velocity methods (EVM). EVM is a locally conservative mixed finite element scheme that allows non-matching grids. Here we have extended EVM to treat temporal and spatial adaptivity. Two type of error estimators are introduced to estimate the spatial discretization error and the temporal discretization error separately. These estimators provide a global upper bounds on the dual norm of the residual and the non-conformity of the numerical solution for non-linear two phase flow models. The mesh refinement algorithm starts from solving the problem on the coarsest space-time mesh, then the mesh is refined sequentially based on the spatial error estimator and the temporal error estimator. After each refinement, the solution from the previous mesh is used to estimate the initial guess of unknowns on the current mesh for faster convergence. The flexibility of this framework allows for improved convergence and efficiency and and prevents convergence failure, while not restricting the whole system to small time steps, Numerical results are presented to confirm accuracy of this algorithm as compared to the uniformly fine time step and fine spatial discretization solution. We observe around 25 times speedup in the solution time by using our algorithm. This work was done in collaboration with Hanyu Li of the University of Texas at Austin and Wing Tat Leung of the University of California at Irvine.

Adrianna Gillman – An Efficient and High Order Accurate Direct Solution Technique for Variable Coefficient Elliptic Partial Differential Equations

Tuesday 23:00–23:45

For many applications in science and engineering, the ability to efficiently and accurately approximate solutions to elliptic PDEs dictates what physical phenomena can be simulated numerically. In this talk, we present a high-order accurate discretization technique for variable coefficient PDEs with smooth coefficients. The technique can be viewed as a domain decomposition method applied at the element level where the coupling conditions are applied directly. The linear system resulting from the discretization can be solved via a nested dissection inspired direct solver that scales linearly or nearly linearly with respect to the number of unknowns. The discretization is robust even for problems with highly oscillatory solutions. For example, a Helmholtz problem where the geometry is 100 wavelengths in size can be solved to 9 digits of accuracy with 3.7 million unknowns. The precomputation of the direct solver takes 6 minutes on a desktop computer and applying the computed solver takes 3 seconds. A parallel implementation of the solution technique reduces the precomputation time to roughly 30 seconds and halves the time it takes to apply the solver. Applications of the solution technique to free-space scattering and inverse scattering problems will also be presented.

Zdeněk Dostál – Improving Efficiency of Scalable TFETI/BETI Contact Solvers for Huge Problems

Wednesday 22:15–23:00

The development of scalable solvers for contact problems is a challenging task due to a priori unknown contact conditions which make the problem strongly nonlinear. In the first part of the lecture, we specify the challenges and present the tools that can be used to overcome them. We first briefly review the *TFETI/TBETI* (total finite/boundary element tearing and interconnecting) based domain decomposition methodology adapted to the solution of contact problems of elasticity discretized by matching grids. The scalability of the proposed method is based on a combination of classical estimates due to Farhat, Mandel, and Roux with the results on the rate of convergence of some special quadratic programming and QCQP (quadratic programming – quadratic constraints) algorithms. The theory guarantees that an approximate solution with prescribed relative error can be obtained in $O(1)$ matrix–vector multiplications provided the cost of the projector to the "natural coarse grid" does not dominate the computation. The results apply to the multibody frictionless problems, both static and dynamic, and to the problems with a given (Tresca) friction.

In the rest, we present three extensions starting with *mortar discretization* of the contact conditions for non-matching grids on the contact interface. The generalization is non-trivial due to the not obvious conditioning of inequality constraints. Improved numerical results are obtained by means of *adaptive augmentation* that enhances into the augmented Lagrangian algorithm the information about the current estimate of the active set of the solution.

The next improvement is the *re-orthogonalization based preconditioning* and *re-normalization based scaling* which enables to extend the scope of applications from the problems with constant coefficients to those with homogeneous subdomains. Though there is a number of results for the problems with jumping coefficients, they typically use the preconditioners transforming separable constraints into more general ones that cannot be treated by specialized algorithms.

Our final goal is to extend the scope of scalability limited by the dimension of the projector. Our tool is the *H-TFETI method*, proposed for linear problems by Klawonn and Rheinbach, and its H-TBETI counterpart. The idea is to join some subdomains by "gluing" corners, edge averages, and face averages on primal level into the clusters with a common rigid body modes. The role of clusters in H-TFETI is the same as the role of subdomains in TFETI. Though we have to avoid preconditioners, it turns out that it is still possible to get results that grant optimal complexity. We give some estimates of the conditioning of clusters and provide numerical results obtained with variants of H-TFETI.

Patrick Joly – Domain Decomposition Methods for Time Harmonic Wave Propagation Problems

Wednesday 23:00–23:45

In this talk, I will survey the research led at INRIA about non overlapping domain decomposition methods for time harmonic wave propagation problems based on generalized impedance transmission conditions. Our approach is theory driven and aims at first priority to enhance the robustness of our methods. We propose a unified framework that proves the well-posedness and the convergence of related iterative algorithms at the continuous level through PDE techniques. Within this framework, which provably follows through in the discrete setting, we shall show how it is possible to improve the speed of convergence through the design of good impedance operators. I will present the recent development on non local transmission operators which allows to achieve linear convergence, with an emphasis on the case of 3D Maxwell's equations.

Rolf Krause – Multilevel Strategies for Non-Linear Problems and Machine Learning: On Non-Linear Preconditioning, Multilevel Optimization, and Multilevel Training

Thursday 22:15–23:00

In this talk, we will discuss the main ideas of multilevel optimization techniques and their relation to classical multigrid theory. We will discuss how multilevel optimization methods for convex and non-convex minimization problems can be constructed and analyzed. We will study the significant gain in convergence speed, which can be achieved by multilevel minimization techniques.

Multilevel optimization techniques are also intimately linked to non-linear preconditioning. As it turns out, the minimization based view on non-linear problems can not only help to design efficient preconditioner, but is also is useful for the construction of globalization strategies.

In the second part of our talk we will employ multilevel optimization techniques in the context of machine learning and will discuss their benefits for the training of neural networks.

Various numerical examples from phase field models for fracture, from non-linear elasticity, and from deep learning will illustrate our findings.

Eric Chung – Local Multiscale Model Reduction and Applications

Thursday 23:00–23:45

Modeling and simulating flow and transport in porous and fracture media are important for many applications. Standard numerical methods are costly in the sense that the dimension of the resulting discrete system is large. To enhance the computational efficiency, model reduction techniques such as upscaling and multiscale methods are necessary, and the resulting systems can be solved on a coarse grid with much smaller degrees of freedoms. In this talk, we will present some recent advances in multiscale model reduction techniques including the generalized multiscale finite element method and the nonlocal multicontinua upscaling. These methodologies are applied successfully to many applications such as poroelasticity, two-phase flow and seismic wave in heterogeneous fracture media. This is a joint work with Yalchin Efendiev, Wing Tat Leung and Maria Vasilyeva. The research is partially supported by the Hong Kong RGC General Research Fund (Project numbers 14304217 and 14302018) and the CUHK Faculty of Science Direct Grant 2018-19.

Laurence Halpern – Fundamental Coarse Space Components for Schwarz Methods with Crosspoints

Friday 22:15–23:00

Two-level domain decomposition methods have been created in the 1990's to make one-level domain decomposition methods scalable. Over the last decade, new coarse space components have also been designed to treat strong heterogeneities and make domain decomposition methods robust for high contrast problems.

Our focus here is to investigate what coarse space components remain naturally when using one-level Schwarz methods to solve Laplace type problems. We start by presenting a numerical experiment which shows visually what error components a classical parallel Schwarz method can not treat effectively. We then study these components by performing a detailed analysis of the iteration map for parallel Schwarz algorithms for a rectangular domain decomposition including crosspoints. Interestingly, these coarse space components remain the same both for classical and optimized Schwarz methods, and contain discontinuous components across subdomain interfaces.

We then propose an approximation to capture these coarse space components, which can be handled by assembly subdomain by subdomain to obtain a coarse correction operator, and we show numerical experiments to illustrate its performance. We will finally also present a preliminary investigation of such coarse space components for general domain decompositions obtained by METIS.

This is a joint work with François Cuvelier (Université Paris 13) and Martin Gander (Université de Genève)

Xuemin Tu – Nonoverlapping Domain Decomposition Methods for Saddle Point Problems

Friday 23:00–23:45

FETI-DP and BDDC these two most popular nonoverlapping domain decomposition algorithms will be discussed for solving a class of saddle point problems arising from mixed finite element or hybridizable discontinuous Galerkin discretizations of partial differential equations. These algorithms reduce the original saddle point problems to symmetric positive definite problems in a special subspace and therefore the conjugate gradient methods can be used to accelerate the convergence. The condition numbers for the preconditioned systems are estimated and numerical results are provided to confirm the results. These are joint work with Drs. Jing Li, Bin Wang, and Stefano Zampini.

Carola-Bibiane Schönlieb – From Differential Equations to Deep Learning for Image Processing

Saturday 22:15–23:00

Images are a rich source of beautiful mathematical formalism and analysis. Associated mathematical problems arise in functional and non-smooth analysis, the theory and numerical analysis of partial differential equations, harmonic, stochastic and statistical analysis, and optimisation. Starting with a discussion on the intrinsic structure of images and their mathematical representation, in this talk we will learn about some of these mathematical problems, about variational models for image analysis and their connection to partial differential equations and deep learning. The talk is furnished with applications to art restoration, forest conservation and cancer research.

Shuonan Wu – General Convection-Diffusion Problems: Robust Discretizations, Fast Solvers and Applications

Saturday 23:00–23:45

In this talk, we present robust discretizations and fast solvers for $H(\text{grad})$, $H(\text{curl})$ and $H(\text{div})$ convection-diffusion PDEs discretized on unstructured simplicial grids. The derivation of these schemes makes use of some intrinsic properties of differential forms, and in particular, some crucial identities from differential geometry. The schemes are of the class of exponential fitting methods that result in special upwind schemes when the diffusion coefficient approaches to zero. Fast solvers are developed for the linear system arising from the robust discretizations of $H(\text{curl})$ and $H(\text{div})$ convection-diffusion problems, which can be viewed as a natural extension of Hiptmair-Xu preconditioners for the symmetric problems. The solvers exclusively rely on that for $H(\text{grad})$ convection-diffusion problem and the discrete commutative diagram involving the convection terms. Both theoretical analysis and numerical experiments show that the new finite element schemes provide the accurate and robust discretizations and fast solvers in many applications, and in particular, for simulation of magnetohydrodynamics systems when the magnetic Reynolds number R_m is large.

Minisymposia

MS01 – Learning Algorithms, Domain Decomposition Methods, and Applications

Organizers: *Xiao-Chuan Cai* ⋄ *Axel Klawonn*

Learning algorithms are playing an increasingly important role in data science and also in computational science and engineering. In this mini-symposium, we discuss some recent advances in machine learning, in connection with domain decomposition and multilevel methods, with applications in several areas including the processing of medical images, parameter optimization in scientific computing, and numerical solution of partial differential equations.

Tuesday 16:00–18:00

16:00–16:30 **An Overlapping Domain Decomposition Method for Solving Boundary Value Problems Using Artificial Neural Networks.**

*Hailong Sheng** ⋄ *Chao Yang*

We propose an overlapping domain decomposition method based on the artificial neural network for dealing with a class of second-order boundary-value problems. To alleviate the difficulty of global optimization, the original problem is transformed into an equivalent weak form and then be decomposed into a series of sub-problems defined on a set of overlapping sub-domains. For each sub-problem, a neural network is constructed to approximate the local solution, with a corresponding weight function to restrict its influence within the designated sub-domain. The approximate solution for the whole problem is formulated as the weighted summation of these sub-networks, which enables their training to be implemented in parallel. Compared to single network methods, the domain decomposed approach is able to not only accelerate the training process, but also achieve higher accuracy, as illustrated by a series of numerical experiments.

16:30–17:00 **Learning Multiscale Models Using Nonlocal Upscaling Techniques**

*Eric Chung** ⋄ *Maria Vasilyeva* ⋄ *Wing Tat Leung* ⋄ *Yalchin Efendiev* ⋄ *Mary Wheeler*

In this talk, we present a novel nonlocal nonlinear coarse grid approximation using a machine learning algorithm. Multiscale models for complex nonlinear systems require nonlocal multicontinua approaches. These rigorous techniques require complex local computations, which involve solving local problems in oversampled regions subject to constraints. The solutions of these local problems can be replaced by solving original problem on a coarse (oversampled) region for many input parameters (boundary and source terms) and computing effective properties derived by nonlinear nonlocal multicontinua approaches. The effective properties depend on many variables (oversampled region and the number of continua), thus their calculations require some type of machine learning techniques. We present results for two model problems in heterogeneous and fractured porous media and show that the presented method is highly accurate and provides fast coarse grid calculations. The research is partially supported by the Hong Kong RGC General Research Fund (Project numbers 14304217 and 14302018) and the CUHK Faculty of Science Direct Grant 2018-19.

17:00–17:30 **Overview of Distributed Memory Parallelism in Deep Learning**

*Rio Yokota**

There exist various partitioning strategies when training deep neural networks. The main differences between these methods and domain decomposition (DD) methods in scientific computing are as follows: 1. Scientific computing deals with meshes with local connectivity, whereas deep learning deals with neural networks that are connected globally. 2. In scientific computing the mesh is the only object that can be distributed, whereas in deep learning both the neural network and the data are objects that can be distributed. These differences strongly influence the possible partitioning strategies. This talk will cover the various distributed memory techniques that have been proposed in deep learning from a domain decomposition perspective.

17:30–18:00 **Parareal Neural Networks Emulating a Parallel-In-Time Algorithm**

Youngkyu Lee ⋄ *Jongho Park* ⋄ *Chang-Ock Lee**

As deep neural networks (DNNs) become deeper, the training time increases. In this perspective, multi-GPU parallel computing has become a key tool in accelerating the training of DNNs. In this talk, we introduce a novel methodology to construct a parallel neural network that can utilize multiple GPUs simultaneously from a given DNN. We observe that layers of the DNN can be interpreted as time steps of a time-dependent problem and can be parallelized by emulating a parallel-in-time algorithm called parareal. The parareal algorithm consists of fine structures which can be implemented in parallel and a coarse structure which gives suitable approximations to the fine structures. By emulating it, the layers of the DNN are torn to form a parallel structure, which is connected using a suitable coarse network. We report accelerated and accuracy-preserved results of the proposed methodology applied to ResNet-1001 on various datasets.

Tuesday 20:00–22:00

20:00–20:30 **Combining Machine Learning and Adaptive Coarse Spaces to Design Robust and Efficient FETI-DP Methods for Elliptic Problems in Three Dimensions**

*Janine Weber** ⋄ *Alexander Heinlein* ⋄ *Axel Klawonn* ⋄ *Martin Lanser*

The convergence rate of classical DDM in general deteriorates severly for large variations or jumps within material properties, resulting in large variations in the spectrum of the system. To retain the robustness for such highly heterogeneous problems, the coarse space can be enriched by additional coarse basis functions, which are computed from the solutions of local generalized eigenvalue problems. However, the set-up and the solution of the generalized eigenvalue problems typically takes up a significant part of the total time to solution, especially in three dimensions. In particular, for many realistic model problems, only the solution of the eigenvalue problems on a small part of the interface is necessary to design a robust coarse space. In general, it is difficult to predict a priori which of the eigenvalue problems are needed. Here, we use machine learning techniques to predict where eigenvalue problems have to be solved, often reducing its number significantly. In this talk, we extend our results for two dimensions to three dimensions which requires additional and very specific effort in the generation and preparation of the training data. We provide numerical results for linear diffusion and elasticity problems considering both, regular and graph partitioned decompositions.

20:30–21:00 **Decomposition and Composition of Deep Convolutional Neural Networks and Training Acceleration Via Sub-Network Transfer Learning**

*Linyan Gu** ⋄ *Wei Zhang* ⋄ *Jia Liu* ⋄ *Xiao-chuan Cai*

Deep convolutional neural network (DCNN) has led to significant breakthroughs in deep learning; however, larger models and larger datasets result in longer training times, which slows down the development progress of deep learning. In this paper, following the idea of domain decomposition methods, we propose and study a new method to parallelize the training of DCNNs by decomposing and composing DCNNs. First, a global network is decomposed into several sub-networks by partitioning the width of the network (i.e., along the channel dimension) while keeping the depth constant. All the sub-networks are individually trained, in parallel without any interprocessor communication, with the corresponding decomposed samples from the input data. Then, following the idea of nonlinear preconditioning, we propose a sub-network transfer learning strategy in which the weights of the trained sub-networks are recomposed to initialize the global network, which is then trained to further adapt the parameters. Some theoretical analyses are provided to show the effectiveness of the sub-network transfer learning strategy. More precisely speaking, we prove that (1) the initialized global network can extract the feature maps learned by the sub-networks; (2) the initialization of the global network can provide an upper bound and a lower bound for the cost function and the classification accuracy with the corresponding values of the trained sub-networks. Some experiments are provided to evaluate the proposed methods. The results show that the sub-network transfer learning strategy can indeed provide good initialization and accelerate the training of the global network. Additionally, after further training, the transfer learning strategy shows almost no loss of accuracy and sometimes the accuracy is higher than the network initialized randomly.

21:00–21:30 **Multilevel Training of Deep Residual Networks**

*Alena Kopaničáková** ◇ *Rolf Krause*

Deep residual networks (ResNets) are widely used for solving computer vision tasks, such as image classification, or segmentation. The ResNet architecture can be formulated as a forward Euler discretization of a nonlinear initial value problem. The training of ResNets consists of iteratively solving an optimal control problem with a given dynamical system. One of the predominant strategies for training any deep network is the stochastic gradient descent (SGD) method. Although the SGD method is simple to implement, its convergence rate deteriorates with increasing problem size. In this talk, we propose to train ResNets using a nonlinear multilevel minimization method, namely the recursive multilevel trust region (RMTR) method. The RMTR method combines the global convergence property of the trust region method and the optimality of the multilevel method. Our multilevel framework takes advantage of the time-dependent nature of the initial value problem and creates coarse-level models by discretizing the original IVP with larger time-steps. The training of the original ResNet is enhanced by training the auxiliary networks with fewer layers. Our RMTR method operates in a hybrid stochastic-deterministic regime and adaptively adjusts sample sizes during the training process. In contrast to the traditional stochastic optimization methods, the proposed method also incorporates curvature information. We approximate Hessian on each level of the multilevel hierarchy using a limited-memory secant method. In this work, we will analyze the convergence behavior of our proposed multilevel minimization method and show its robustness. We will demonstrate the significant speedup achieved by the RMTR method in comparison with the standard SGD method and single-level trust-region method using several numerical examples.

21:30–22:00 **Flow Predictions Using Convolutional Neural Networks**

*Alexander Heinlein** ◇ *Matthias Eichinger* ◇ *Axel Klawonn*

Simulations of fluid flow are generally very costly because too low grid resolutions may even lead to qualitatively incorrect solutions. In applications, however, one is often not interested in accurate approximations of the complete flow field but only in the qualitative behavior of the flow or in individual quantities (e.g., maximum velocity, pressure drop within a section of a pipe, or wall shear stresses at certain locations). In this talk, the use of Convolutional Neural Networks (CNNs) to predict fluid flow fields is investigated. Therefore, U-Net [Ronneberger et al., 2015] type convolutional neural networks, which are successfully used for image recognition and segmentation tasks, are applied in this context. As a model problem, the flow around obstacles with varying shape and size within a channel is considered. Obstacles of certain type are used as training data, and the generalization of the models to other obstacle geometries and sizes is analyzed. Even though, the training of a neural network is expensive, its evaluation is quite cheap compared to fully resolved Computational Fluid Dynamics (CFD) simulations. This results in a multitude of application possibilities for neural networks in this context, especially in time critical settings.

MS02 – Heterogeneous Domain Decomposition Methods: Theoretical Developments and New Applications

Organizers: *Martin J. Gander* ⋄ *Tommaso Vanzan*

Domain decomposition methods are the natural computational framework for heterogeneous problems. Heterogeneity can arise either because the physics is different in parts of the domain and hence different models need to be used, or because it can be convenient to use a cheaper approximation in some parts of the domain in order to save computational resources. This minisymposium will give an overview of new theoretical and computational developments in the use of domain decomposition methods for heterogeneous problems.

Tuesday 16:00–18:00

16:00–16:30 **On the Stability and Efficiency of Domain-Decomposition Algorithms for Fluid-Structure Interaction**

*Christian Vergara** ⋄ *Giacomo Gigante* ⋄ *Giulia Sambataro*

In this talk, we consider both loosely coupled/explicit and implicit algorithms for the fluid-structure interaction (FSI) problem based on a Robin interface condition for the fluid problem (Robin-Neumann (RN) scheme). We first study the dependence of the stability of the explicit-RN method on the interface parameter in the Robin condition. In particular, for a model problem we find sufficient conditions for instability and stability of the method. In the latter case, we found a stability condition relating the time discretization parameter, the interface parameter, and the added mass effect. Numerical experiments confirm the theoretical findings and highlight optimal choices of the interface parameter that guarantee accurate solutions. Then, we consider the Optimized Schwarz method designed for computational domains that feature spherical or almost spherical interfaces. In this context, we provide a convergence analysis of the implicit-RN scheme for a FSI model problem and discuss optimal choices of constant interface parameters. To validate the theorietical findings, we present 3D numerical results in almost spherical domains inspired by hemodynamic applications, in particular we consider the case of realistic abdominal aortic aneurysms.

16:30–17:00 **Fully Eulerian Fluid-Structure Interaction Methods for the Cardiovascular Modeling**

*Aymen Laadhari** ⋄ *Gabor Székely* ⋄ *Alfio Quarteroni*

The cardiovascular system has a structure of extreme complexity covering different physical scales in space and time. Although the substantial progress in the fields of mathematics and scientific computing, the simulation of the cardiovascular system as a coupled multiphysics and multiscale problem at the full level of detail remains practically impossible. In this talk, we focus on particular problems of the cardiovascular modeling involving difficult fluid-structure interaction problems. We present mathematical models and computational methods tailored for the simulation of (i) the coupled chemical fluid-structure interaction problem arising in the study of mesoscopic cardiac biomechanics [3], and (ii) the hemodynamics in both aorta and sinus of Valsalva interacting with thin and highly deformable aortic valve [4]. These problems lead to coupled systems of highly nonlinear PDEs which are tremendously challenging and entail the resolution of difficult fluid-structure interaction problems involving highly deformable thin or thick structures freely suspended in an incompressible Newtonian flow.

To model the contractility of a highly deformable cardiomyocyte surrounded by a Newtonian fluid and the mechano-chemical interactions between calcium species and active contraction generation, we develop purely Eulerian mathematical framework that helps to circumvent issues related to the large structural deformations in incompressible flows. An active strain approach is employed to account for the mechanical activation, and the deformation of the cell membrane is captured using a level set strategy [3].

The mathematical framework is adapted to model FSI problems involving extremely slender and lightweight structures in an incompressible fluid, e.g. the valve's problem. The formulation is based on the Gurtin-Murdoch surface elasticity theory and allows to compute the membrane Cauchy-stress tensor. The valve leaflets are assumed massless elastic membranes, resulting in an infinite added mass effect. The convergence and stability is a big concern and relies importantly on the techniques used to enforce the coupling conditions. Both monolithic and partitioned approaches based on operator-splitting techniques are developed [1, 2]. We will report several examples to address the relevance of the mathematical models in terms of physiological meaning and to illustrate the accuracy of the numerical method.

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17:00–17:30 **Time Adaptive Multirate DN and NN Waveform Relaxation for Heterogenous Coupled Heat Equations**

*Philipp Birken** ⋄ *Azahar Monge*

As a model for thermal fluid structure interaction (FSI), we consider the coupling of two heat equations with different material coefficients, meaning that these are discontinuous across a connecting interface. We assume that the two subproblems are solved by separate codes which exchange interface information, called a partitioned approach in FSI. Our goal is to have a fast partitioned coupling algorithm that allows the subsolvers to choose their own time step adaptively. The basis here are the recently suggested domain decomposition waveform iterations DNWR and NNWR. The Neumann-Neumann (NN) variant allows parallel exectution, whereas the Dirichlet-Neumann (DN) variant does not. To obtain a fast method, it is imperative to choose the relaxation parameter well. We derive optimal parameters for 1D model discretizations with fixed mesh width and time step, which give superlinear convergence. Then, we use this analysis to obtain fast timeadaptive multirate variants of DNWR and NNWR in 2D.

17:30–18:00 **Tools for Discrete SWR Analysis and Its Features**

*Simon Clement** ⋄ *Florian Lemarié* ⋄ *Eric Blayo*

Schwarz Waveform Relaxation (SWR) methods arise naturally when considering heterogeneous domain decomposition problems. SWR methods often need a proper optimization of their convergence rate to be efficient. When considering a discrete setting rather than the continuous differential equations, a more accurate estimation of the numerical convergence rate can be obtained. Through the study of two coupled reaction-diffusion equations with different diffusivities, this talk explores particularities of discrete and semi-discrete analyses, and compares them to the continuous setting. In particular, choices for interface discretizations are discussed and a complementary analysis based on so-called "equivalent PDE" is presented.

20:00–20:30 **How to Use Probing to Find Optimized Transmission Conditions**

*Tommaso Vanzan** ⋄ *Martin J. Gander* ⋄ *Roland Masson*

Over the last decades, several theoretical results have been obtained to establish optimized transmission conditions for many different PDEs. Despite this large effort, optimized transmission conditions are not so widely used even though they lead to a better convergence with respect to other classical domain decomposition methods. This is mainly due to the strict hypotheses used in the theoretical analysis, which are not always satisfied in practice, and they can discourage the potential user from implementing optimized transmission conditions. In this talk we will show that simply probing the Dirichlet to Neumann operator, which is the essential component of optimal transmission conditions, does surprisingly not lead to good estimates of the optimized parameters. We then propose a numerical procedure still based on probing which instead permits to find efficient optimized transmission conditions for very general domains, interfaces and coupling of PDEs.

20:30–21:00 **Error Estimates for Discrete Fracture Matrix Models**

Martin J. Gander ⋄ *Julian Hennicker** ⋄ *Roland Masson*

Models that contain thin heterogeneous layers provide particular challenges for their numerical solution (meshing issues, badly conditioned Jacobian). In view of real life simulations, one might consider reduced models, see e.g. [2], [4], [1] for Darcy flow through fractured porous media, or [3] for general advection-diffusion-reaction problems. In this work, we derive error estimates for the reduced models by domain decomposition and functional analysis techniques. We also present numerical tests to illustrate the results.

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21:00–21:30 **On the Choice of Optimized Schwarz Parameters for Elliptic Problems with Non-Conforming Heterogeneities**

*Felix Kwok** ⋄ *Yaguang Gu*

In many applications, one needs to solve the elliptic equation $\nabla \cdot (a(x)\nabla u) = 0$ where $a(x)$ is highly heterogeneous and/or exhibits large jumps, but it is impossible or inconvenient to define subdomains that conform to these discontinuities. One example is in oil reservoir simulation, where the rock may contain irregularly shaped channels within areas of varying, highly oscillatory permeability. In this talk, we propose an optimized Schwarz method (OSM) where we impose a Robin transmission condition with Robin parameter p that is non-constant along the interface. For a model problem where $a(x)$ consists of exactly two regions with diffusivity a_- and a_+ respectively, we show how to choose the Robin parameter $p = p(x)$ so that the resulting OSM has a convergence factor of $1 - Ch^{1/2}$, where C is a constant independent of the ratio a_+/a_-, and h is the mesh parameter. We also show that the choice $p = const.$ is not robust for problems with large contrasts in diffusivity, with the constant C above being proportional to $(a_-/a_+)^{1/2}$. Finally, numerical examples show that the proposed parameter choice is also robust for more general heterogeneous problems not covered by the theory.

21:30–22:00 **On Computation of Coupled Advection-Diffusion-Reaction Equations**

Hansong Tang[*]

Analysis is made on fully discretized coupled advection-diffusion-reaction equations with different coefficients in subdomains. It starts with conditions and speeds of convergence for the linear systems resulting from explicit schemes. Then, the analysis proceeds to an implicit scheme, and an optimal interface condition for Schwarz waveform relaxation is derived, which leads to 'perfect convergence', i.e., convergence within two times of iteration. Furthermore, the methods and analysis are extended to coupling of the viscous Burgers equations. Numerical experiments indicate that the conclusions, such as the 'perfect convergence', drawn in the linear situations may largely remain in computation of the Burgers equations. At last, computation of the coupling for the equations by machine learning and a numerical example will briefly be discussed.

MS03 – Preconditioning Methods for Frequency Domain Wave Problems

Organizers: *Victorita Dolean* ⋄ *Ivan G. Graham*

The efficient iterative solution of frequency domain wave problems in the mid- to high-frequency regime is a notoriously difficult problem, because (a) solutions are oscillatory and meshes have to be fine to resolve them; (b) matrices are non Hermitian, highly indefinite and generally non-normal; (c) standard intuition for preconditioning techniques born from the study of SPD elliptic problems is of limited use. Nevertheless these problems are of great practical interest and in recent years substantial progress has been made both in theory and in practice on the construction of effective preconditioners. This minisymposium will highlight recent progress in this challenging area.

Tuesday 16:00–18:00

16:00–16:30 **Analysis of Parallel Schwarz Algorithms for Time-Harmonic Problems Using Block Toeplitz Matrices**

*Niall Bootland** ⋄ *Victorita Dolean* ⋄ *Alexandros Kyriakis* ⋄ *Jennifer Pestana*

One-level Schwarz methods are not scalable in general. However, for time-harmonic wave propagation problems with absorption, it has recently been proven that when impedance transmission conditions are used, under certain assumptions, scalability can be achieved and no coarse space is required. We show here that this result is also true for the iterative version of the algorithm at the continuous level for strip-wise decompositions into subdomains that can typically be encountered when solving wave-guide problems. Our convergence proof provides a novel line of argument relying on the particular block Toeplitz structure of the global iteration matrix. Although non-Hermitian, we prove that its limiting spectrum has a near identical form to that of a Hermitian matrix of the same structure. We illustrate our results with numerical experiments.

16:30–17:00 **Coarse Spaces Based on Geneo Type Eigenvalue Problems for Indefinite Problems**

*Chupeng Ma** ⋄ *Ivan Graham* ⋄ *Robert Scheichl* ⋄ *Victorita Dolean* ⋄ *Niall Bootland*

In this talk, I will present an additive Schwarz method with a coarse space based on Geneo type eigenvalue problems for indefinite elliptic problems with rough coefficients. The convergence rate of GMRES applied to the preconditioned problem is proved. Numerical examples are given to verify the effectiveness of the method.

17:00–17:30 **A Splitting Double Sweep Method for the Helmholtz Equation**

*Nacime Bouziani** ⋄ *Henri Calandra* ⋄ *Frédéric Nataf*

Solving the Helmholtz equation numerically is a difficult task, especially when dealing with high-frequency regimes and heterogeneous media. Over the last decades a lot of effort and progress has been made in developing efficient algorithms to solve the ill-conditioned linear system resulting from the Helmholtz operator's discretization. Sweeping-type algorithms are among the most prominent domain decomposition based techniques for the Helmholtz problem, they have been made very popular due to their capability to achieve linear or nearly-linear asymptotic complexity. The sweeping algorithm was first analyzed in 1997 by F. Nataf and F. Nier [1] for convection-diffusion operators. Sweeping approach has recently seen its interest renewed, a bunch of methods have emerged using sweeping as a preconditionner to speed up the convergence of the solver : the double sweep preconditionner of Vion and Geuzaine [6, 7], where the DtN map is approximated. The PML-based sweep method of Stolk [4], and the polarized traces method of Zepeda-Núñez and Demanet [5]. There also exists sweeping-type methods that are not domain decomposition based methods, such as the sweeping PML preconditionner of Engquist and Ying [2, 3]

Considering the additive Schwarz method under the form of a substructuring method, i.e with unknowns defined on the boundaries of the subdomains, we end up with a Jacobi algorithm.

When equipping the local subproblems with exact absorbing boundary conditions (ABC), the total number of iterations is equal to the number of subdomains, this is due to the nilpotency of the iteration operator valued matrix. In practice, the exact ABC (which are also the optimal interface conditions, see [8]) procedure is tedious to implement and computationally expensive. On the other hand, for the non-exact ABC such as Robin condition, extra non-diagonal terms appear in the iteration matrix, these termes deprive it of its nilpotency property and thus deteriorate the convergence property. More precisely, these boundary conditions at the interfaces produce spurious reflected waves that significantly increase the number of iterations to converge, in particular for heterogeneous media and high frequency regimes.

We propose to precondition the substructured system by a double-sweep type algorithm modified in two ways compared to previous works:

- non-overlapping or overlapping subdomains can be used
- a novel splitting of the substructured problem prevents spurious interface reflections from hindering the convergence

Enabling overlapping subdomains enables to leverage its beneficial effect on the damping of high frequency modes (see e.g. [9]) of the error whereas the novel splitting prevents its adversary effect on the convergence of propagative modes.

This is useful since in the non-overlapping approach, the quality of the ABC is nearly the only way of impacting the convergence of the algorithm, and when dealing with more complex problem such as Maxwell equations high order ABCs are harder to handle. Also, when we tackle multiple right hand sides situations (e.g in seismic problems) the idle time induced by the sequential nature of the double sweep can be overcome by performing a pipelining with respect to the right hand sides, as it is done in [7].

We will first explain how we substructure the Helmholtz problem and formulate a modified version of the double sweep algorithm introduced in [1], then how we use this modified double sweep algorithm to build a preconditionner that efficiently speed up the convergence of the solver. Finally, we present numerical results and carry out the convergence analysis and obtain an estimation of the convergence rate.

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17:30–18:00 **Parallel Sweeping Preconditioners for Rectangular Domain Decompositions with Cross Points Applied to the Helmholtz Equation**

*Ruiyang Dai** ◇ *Axel Modave* ◇ *Jean-François Remacle* ◇ *Christophe Geuzaine*

Various wave propagation problems can be modelled in the frequency domain by the Helmholtz equation. Efficiently solving the Helmholtz equation at high frequencies (when the wavelength is small compared to the computational domain) in complex geometrical and/or material configurations is the subject of intense research, with recent promising contributions focused on high-order finite element methods coupled with optimized Schwarz Domain Decomposition (DD) schemes. Sweeping preconditioners for DD have recently gained a lot of interest because of their fast convergence, promising a number of DD iterations that is quasi independent of the number of subdomains. These preconditioners however have two major drawbacks: they rely on intrinsically sequential operations (they are related to an LU-type factorization of the underlying iteration operator) and they are naturally only suited for layered-type domain decompositions (where the layered structure allows to explicit the LU factorization as a double sweep across the subdomains). In this talk we will present two contributions aiming to improve the parallelization of sweeping preconditioners. First, we will show how for problems with multiple sources a pipelining of the sweeping preconditioners can provide parallel scaling. Second, we will present a family of generalized sweeping preconditionners where sweeps can be done, in parallel, in several directions, for rectangular "checkerboard-type" domain decompositions. Both contributions rely on the availability of accurate, high-order transmission conditions between the subdomains. In particular, we will show how an accurate treatment of cross-points is key to an accurate and efficient preconditioner.

Tuesday 20:00–21:30

20:00–20:30 **Convergence of Overlapping Domain Decomposition Methods for the Helmholtz Equation**

*Shihua Gong** ◇ *Ivan G. Graham* ◇ *Martin J. Gander* ◇ *David Lafontaine* ◇ *Euan A. Spence*

The impedance boundary condition has been widely used in designing domain decomposition methods for the Helmholtz equation. The first algorithm with a rigorous convergence theory (Bennamou and Desprès, 1997) was based on a nonoverlapping domain decomposition with general subdomains and swapped impedance data of neighbouring subdomains at each iteration. The convergence proof (but without a rate of convergence) was carried out using a "pseudo-energy" norm constructed from the sum of the L^2 norms of the impedance data on subdomain boundaries. This is a norm on the space of local solutions to the homogeneous Helmholtz equation and the convergence proof used the fact that the norms of the inward and outward impedance data are equal for solutions of the homogenous Helmholtz equation on each subdomain. In this talk, we will present some corresponding results for overlapping domain decomposition methods. We will show how convergence of the parallel Schwarz method (as an iterative method) depends on properties of the 'impedance map' which takes impedance data on the boundary to impedance data on an interior interface, via the solution of a homogeneous Helmholtz problem. Based on these properties, the convergence (and rate of convergence) of some overlapping domain decomposition methods can be theoretically guaranteed. We also present numerical results illustrating the theory.

20:30–21:00 **On the Wavenumber Robustness of Optimized Schwarz Methods for the Helmholtz Equation**

*Hui Zhang** ◇ *Martin J. Gander*

It was observed under some conditions that optimized Schwarz methods for the Helmholtz equation are robust in the wavenumber. This holds for example, if the double sweep optimized Schwarz method is equipped with sufficiently strong PMLs, or if the parallel optimized Schwarz method uses fixed overlap and subdomains to solve the free space problem. The recent work of Graham-Spence-Zou gives a rigorous proof of the robustness of a symmetrized optimized Schwarz method with low order radiation conditions. In this talk, we will present Fourier analysis results on the wavenumber robustness and we show how this is influenced by the inner and outer boundary conditions.

21:00–21:30 **Scalable Convergence Using Two-Level Deflation for the Helmholtz Equation**

*Vandana Dwarka** ⋄ *Kees Vuik*

Recent research efforts aimed at iteratively solving the Helmholtz equation has focused on incorporating deflation techniques for GMRES-convergence accelerating purposes. The two-level-deflation preconditioner combined with Complex Shifted Laplacian Preconditioner (CSLP) showed encouraging results in moderating the rate at which the eigenvalues approach the origin and cause the solver to slow down. However, for large wave numbers the initial problem resurfaces and the near-zero eigenvalues reappear. Our findings reveal that the reappearance of these near-zero eigenvalues occurs if the near-singular eigenmodes of the fine-grid operator and the coarse-grid operator are not properly aligned. This misalignment is caused by accumulating approximation errors during the inter-grid transfer operations. We propose the use of higher-order approximation schemes to construct the deflation vectors. The results from Rigorous Fourier Analysis (RFA) and numerical experiments confirm that our newly proposed scheme outperforms any deflation-based preconditioner for the Helmholtz problem. In particular, the spectrum of the adjusted preconditioned operator stays fixed near one. For the first time, the convergence properties for very large wavenumbers ($k = 10^6$ in one-dimension, $k = 1.5 \times 10^3$ in two-dimensions and $k = 100$ in three-dimensions) have been studied, and the convergence is almost wave number independent. The new scheme additionally shows very promising results for the more challenging Marmousi problem.

MS04 – Two-Grid Method and Its Applications

Organizers: *Hehu Xie* ◇ *Jinchao Xu* ◇ *Liuqiang Zhong*

Two-grid (two-level/two-space) method is one of the important tools to build fast solvers for the nonsymmetric/indefine or nonlinear problems. Since its appearance, the two-grid method have been successfully applied to solve many problems in the last two decades, such as nonlinear elliptic problems, nonlinear parabolic equations, Navier-Stokes problems, Maxwell equations, and eigenvalue problems, etc.
This minisymposium is concerned with the applications of the two-grid, two-scale, multigrid, multiscale and multilevel methods. We hope that the mini symposium is to bring together experts as well as junior researchers with common interest but with diversified backgrounds and knowledge, to discuss new types of questions at the foundation, overlap and applications of this research field.

Tuesday 10:00–11:30

10:00–10:30 **Preconditioners for Two-Fold Saddle Point and Block Tridiagonal Systems**

*Mingchao Cai** ◇ *Guoliang Ju* ◇ *Jingzhi Li*

In this work, two types of Schur complement based preconditioners are studied for two-fold saddle point and block tridiagonal systems. One is based on the nested (or recursive) Schur complement, the other is based on an additive type Schur complement after permuting the original saddle point systems. We discuss different preconditioners incorporating the exact Schur complements. It is shown that some of them will lead to better conditioned and or positive stable preconditioned systems. Our theoretical analysis is instructive for devising various exact and inexact preconditioners, as well as iterative solvers for many two-fold saddle point and block tridiagonal problems.

10:30–11:00 **Some Domain Decomposition Preconditioners for Poroelastic Model**

*Pilhwa Lee** ◇ *Mingchao Cai*

A mixed finite element method with a pressure stabilization is applied to solve the classical three-field formulation of the Biot model. In this talk, we investigate several domain decomposition preconditioners for the resulting linear system. In our first approach, the approximation of the Schur complement is derived by using Fourier analysis, and the elliptic operators are replaced by their two-level overlapping Schwarz preconditioners. In our second approach, a linear system is reduced for the Lagrange multiplier, and some FETI-DP preconditioners are designed to solve the reduced system. Numerical experiments with heterogeneous poroelasticity in substructures are provided to demonstrate the scalability and efficiency of the proposed methods. It is a foundation for preconditioners of the nonlinear poroelasticity with large deformations.

11:00–11:30 **A Two-Grid Preconditioner with an Adaptive Coarse Space for Flow Simulations in Highly Heterogeneous Media**

*Shubin Fu** ◇ *Yanfang Yang* ◇ *Eric Chung*

We consider flow simulation in highly heterogeneous media that has many practical applications in industry. To enhance mass conservation, we write the elliptic problem in a mixed formulation and introduce a robust two-grid preconditioner to seek the solution. We first need to transform the indefinite saddle problem to a positive definite problem by preprocessing steps. The preconditioner consists of a local smoother and a coarse preconditioner. For the coarse preconditioner, we design an adaptive spectral coarse space motivated by the GMsFEM (Generalized Multiscale Finite Element Method). We test our preconditioner for both Darcy flow and two phase flow and transport simulation in highly heterogeneous porous media. Numerical results show that the proposed preconditioner is highly robust and efficient.

16:00–16:30 **Two-Grid Analysis for Nonnormal Matrices**

*Yvan Notay**

Core results about the algebraic analysis of two-grid methods are extended in relations bounding the field of values (or numerical range) of the iteration matrix. On this basis, bounds are obtained on its norm and numerical radius, leading to rigorous convergence estimates. Numerical illustrations show that the theoretical results deliver qualitatively good predictions, allowing one to anticipate success or failure of the two-grid method. They also indicate that the field of values and the associated numerical radius are much more reliable convergence indicators than the eigenvalue distribution and the associated spectral radius. On this basis, some discussion is developed about the role of local Fourier or local mode analysis for nonsymmetric problems.

16:30–17:00 **A Multigrid Method for Ground State Solution of the Bose-Einstein Condensates**

*Manting Xie** ⋄ *Hehu Xie* ⋄ *Fei Xu*

In this talk, we'll propose a multigrid method to compute the ground state solution of Bose-Einstein condensations by the finite element method based on the multilevel correction for eigenvalue problems and the multigrid method for linear boundary value problems. In this scheme, obtaining the optimal approximation for the ground state solution of Bose-Einstein condensates includes a sequence of solutions of the linear boundary value problems by the multigrid method on the multilevel meshes and a series of solutions of nonlinear eigenvalue problems on the coarsest finite element space. The total computational work of this scheme can reach almost the optimal order as same as solving the corresponding linear boundary value problem. Therefore, this type of multigrid scheme can improve the overall efficiency for the simulation of Bose-Einstein condensations. Some numerical experiments are provided to validate the efficiency of the proposed method.

17:00–17:30 **Two-Grid Finite Element Methods for Parabolic Integro-Differential Equations**

*Wansheng Wang** ⋄ *Qingguo Hong*

In this talk several two-grid finite element algorithms for solving parabolic integro-differential equations (PIDEs) are proposed. In the first part of this talk, we consider two-grid finite element algorithms for the semilinear PIDEs with positive memory. With backward Euler scheme for the temporal discretization, the basic idea of the space two-grid finite element algorithms is to approximate the semilinear equations on a coarse space grid and solve the linearized equations on a finer space grid at each time step. To further decreases the amount of computational work, a space-time two-grid algorithm based on a coarse space grid with large time stepsize ΔT and a finer space grid with small time stepsize Δt for the evolutional equations is proposed in this talk. The sharp long-time stability and error estimates for the standard finite element, space two-grid finite element methods, and space-time two-grid finite element methods are derived. It is showed that the two-grid algorithms' long-time stability and error estimates are similar to those of the direct resolution of the semilinear problem on a fine grid. In the second part of this talk, we consider two-grid finite element algorithms for PIDEs with nonlinear memory. Analysis of these algorithms is given assuming a fully implicit time discretization. It is shown that these algorithms are as stable as the standard fully discrete finite element algorithm, and can achieve the same accuracy as the standard algorithm if the coarse grid size H and the fine grid size h satisfy $H = O(h^{\frac{r-1}{r}})$. Especially for PIDEs with nonlinear memory defined by a lower order nonlinear operator, our two-grid algorithm can save significant storage and computing time. Numerical experiments are given to confirm these theoretical results.

17:30–18:00 **Expandable Local and Parallel Two-Grid Finite Element Iterative Scheme for the Stokes Equations**

Yanren Hou ⋄ *Feng Shi* ⋄ *Haibiao Zheng**

In this talk, we present a novel local and parallel two-grid finite element iterative scheme for solving the Stokes equations, and rigorously establish its a priori error estimates. The scheme admits simultaneously small scales of subproblems and distances between subdomains and its expansions, and hence can be expandable. Based on the a priori error estimates, we provide a corresponding iterative scheme with suitable iteration number. The resulting iterative scheme can reach the optimal convergence orders within specific two-grid iterations ($O(|\ln H|^2)$ in 2-D and $O(|\ln H|)$ in 3-D) if the coarse mesh size H and the fine mesh size h are properly chosen. Finally, some numerical tests including 2-D and 3-D cases are carried out to verify our theoretical results.

Wednesday 16:00–18:00

16:00–16:30 **Two-Grid Methods for P_0^2-P_1 Mixed Finite Element Approximation of General Elliptic Optimal Control Problems with Low Regularity**

*Tianliang Hou** ⋄ *Haitao Leng* ⋄ *Tian Luan*

In this talk, we present a two-grid mixed finite element scheme for distributed optimal control governed by general elliptic equations. P_0^2-P_1 mixed finite elements are used for the discretization of the state and co-state variables, whereas piecewise constant function is used to approximate the control variable. We first use a new approach to obtain the superclose property between the centroid interpolation and the numerical solution of the optimal control u with order h^2 under the low regularity. Based on the superclose property, we derive the optimal a priori error estimates. Then, using a postprocessing projection operator, we get a second-order superconvergent result for the control u. Next, we construct a two-grid mixed finite element scheme and analyze a priori error estimates. In the two-grid scheme, the solution of the elliptic optimal control problem on a fine grid is reduced to the solution of the elliptic optimal control problem on a much coarser grid and the solution of a linear algebraic system on the fine grid and the resulting solution still maintains an asymptotically optimal accuracy. Finally, a numerical example is presented to verify the theoretical results.. *Tianliang Hou

16:30–17:00 **Two-Scale Finite Element Approximations for Semilinear Parabolic Equations**

*Fang Liu**

To reduce computational cost, we study some two-scale finite element approximations with the backward Euler scheme for the semilinear parabolic equations. First, a basic two-scale finite element method with the backward Euler scheme is proposed. A Boolean sum of some existing finite element approximations for the semilinear system on a coarse grid and some univariate fine grids is calculated. Second, a linearized two-scale finite element method with the backward Euler scheme is introduced. A semilinear system is solved on a coarse grid and then some linear systems are solved on some univariate fine grids. Both the two new two-scale finite element approximations with the backward Euler scheme not only have the less degrees of freedom but also achieves a good accuracy of approximation.

17:00–17:30 **An Unfitted Virtual Element Methods for the Elliptic Interface Problem**

*Feng Wang**

In this talk, we shall present an unfitted virtual element method for the elliptic interface problem. Under some mesh assumption, It is proved that the error estimates in H^1 weighted semi-norm is optimal, that is, the error is $O(h)$ and the hidden constant is independent of the contrast between the coefficients and the location of the interface. Some numerical experiments are provided to verify the theoretical findings.

17:30–18:00 **A Two-Grid Finite Difference Algorithm for Compressible Darcy-Forchheimer Model in Porous Media**

*Jintao Cui** ⋄ *Wei Liu*

In this work, a two-grid finite difference method is proposed to solve the compressible Darcy-Forchheimer model which describes the high speed non-Darcy flow in porous media. The discretized nonlinear problem on the fine grid is solved in two steps: first solving a small nonlinear system on the coarse grid; then solving a modified nonlinear problem on the fine grid. On the coarse grid, the coupled term of pressure and velocity is approximated by using the fewest number of nodes. On the fine grid, the original nonlinear term is modified with a small parameter to construct a linear block-centered finite difference scheme. Optimal order error estimates for pressure and velocity are obtained.

MS05 – Mini-Symposium on Advanced Numerical Methods for Electromagnetic Problems

Organizers: Zhiming Chen ⋄ Liwei Xu ⋄ Weiying Zheng

We propose a mini-symposium on efficient numerical methods for electromagnetic problems in DD26. The theme of the mini-symposium concentrates on the design and analysis of efficient numerical methods, and robust and fast discrete system solvers for electromagnetic problems arising in sciences and engineering applications.

Tuesday 10:00–11:30

10:00–10:30 **An Adaptive High-Order Unfitted Finite Element Method for Elliptic Interface Problems**

*Zhiming Chen**

We design an adaptive unfitted finite element method on the Cartesian mesh with hanging nodes. We derive an hp-reliable and efficient residual type a posteriori error estimate on K-meshes. A key ingredient is a novel hp- domain inverse estimate which allows us to prove the stability of the finite element method under practical interface resolving mesh conditions and also prove the lower bound of the a posteriori error estimate. Numerical examples are included.

10:30–11:00 **Parallel 3-D Adaptive Finite Element Method and Its Application on EDA Tools**

*Tao Cui**

Electronic design automation (EDA), also referred to as electronic computer-aided design (ECAD), is a category of software tools for designing electronic systems such as integrated circuits and printed circuit boards. As the VLSI technology scales down to nanoscale and the circuit's frequency reaches GHz, EDA tools play a more and more important role in today's integrated circuits (IC) industry. The finite element method (FEM) is a powerful tool for the numerical simulation of a wide range of problems. In this talk, the parallel adaptive finite element method for parasitic extraction of large scale interconnects and thermomechanical stress evaluation of 3D IC is developed to provide extremely high parallel scalability and numerical accuracy. Numerical results of some large scale adaptive finite element simulations with up to 1 billion degrees of freedom and using up to ten thousand CPU cores are presented to demonstrate that our adaptive method is robust and scalable for analysis of very complicated geometries.

11:00–11:30 **Discrete Compactness Holds for Any Order Lagrange Elements for the Classical Formulation $(\mu^{-1}\mathbf{curl}\, u, \mathbf{curl}\, v) = \omega^2(\varepsilon u, v)$ of the Maxwell Eigenproblem**

*Huoyuan Duan**

We have proven that, based on the classical variational formulation $(\mu^{-1} curlu, curlv) = \omega^2(\varepsilon u, v)$ of the Maxwell eigenproblem in the multiply connected domain with multiple connected boundary components occupying with discontinuous, anisotropic and inhomogeneous media, any order Lagrange elements in two and three dimensions, which are H^1-conforming nodal-continuous elements, holds the discrete compactness. The discrete compactness is the key to spurious-free and spectral-correct approximations of the Maxwell eigenproblem. These are the first known and the first proven results about the discrete compactness for the Lagrange elements of any order in two and three dimensions. The earliest discrete compactness result by us for the two-dimensional Lagrange elements of any order greater than or equal to two on the Clough-Tocher split of the triangle meshes has been included in our paper(by Huoyuan Duan and Zhijie Du) which has been published in Journal of Scientific Computing, 2020. Numerical results given have confirmed.

16:00–16:30 **An Efficient Iterative Method for Solving Multiple Scattering in Locally Inhomogeneous Media**

*Bo Wang**

In this talk, an efficient iterative method is proposed for solving multiple scattering problem in locally inhomogeneous media. The key idea is to enclose the inhomogeneity of the media by well separated artificial boundaries and then apply purely outgoing wave decomposition for the scattering field outside the enclosed region. As a result, the original multiple scattering problem can be decomposed into a finite number of single scattering problems, where each of them communicates with the other scattering problems only through its surrounding artificial boundary. Accordingly, they can be solved in a parallel manner at each iteration. This framework enjoys a great flexibility in using different combinations of iterative algorithms and single scattering problem solvers. The spectral element method seamlessly integrated with the non-reflecting boundary condition and the GMRES iteration is advocated and implemented in this work. The convergence of the proposed method is proved by using the compactness of involved integral operators. Ample numerical examples are presented to show its high accuracy and efficiency.

16:30–17:00 **A Pure Source Transfer Domain Decomposition Method for Helmholtz Equations in Unbounded Domain**

*Haijun Wu**

A pure source transfer domain decomposition method (PSTDDM) for solving the truncated perfectly matched layer (PML) approximation in bounded domain of Helmholtz scattering problem is proposed and analyzed.

17:00–17:30 **Double Source Transfer Domain Decomposition Method for Helmholtz Problems**

*Xueshuang Xiang**

We propose and study a double source transfer domain decomposition method (Double STDDM) for solving the truncated perfectly matched layer approximation in the bounded domain of Helmholtz problems. Based on the decomposition of the domain into non-overlapping layers and instead of transferring the source along one direction in STDDM [Z. Chen and X. Xiang, 2013], Double STDDM transfers the source in each layer along two directions, which can capture of the reflection information for heterogenous media. Double STDDM is an iterative scheme, and in each iteration, it first transfers the source from down to up and produces the Up wave (the wave propagating from down to up), and then transfers the source from up to down and produces the Down wave (the wave propagating from up to down). The output of Double STDDM is the summation of the Up and Down waves that are produced during the iteration. By using the fundamental solution of the PML equation, the convergence of Double STDDM is proved for the case of a constant wavenumber. Numerical examples are included to show the efficient performance of using Double STDDM as a preconditioner both for the problems with constant and heterogenous wavenumbers. For problems with a low velocity contrast, the number of iterations is independent of the wavenumber and mesh size, whereas for problems with a high velocity contrast, double STDDM performs much better than STDDM.

17:30–18:00 **Mathematical Analysis of Wave Radiation by a Step-Like Surface**

*Wangtao Lu**

In this talk, we propose for wave propagating in a globally perturbed half plane with a perfectly conducting step-like surface, a sharp Sommerfeld radiation condition (SRC) for the first time, an analytic formula of the far-field pattern, and a high-accuracy numerical solver. We adopt the Wiener-Hopf method to compute the Green function for a cracked half plane, a special background for the perturbed half plane. We rigorously show that the Green function asymptotically satisfies a universal-direction SRC (uSRC) and radiates purely outgoing at infinity. This helps to propose an implicit transparent boundary condition for the scattered wave, by either a cylindrical incident wave due to a line source or a plane incident wave. Then, a well-posedness theory is established via an associated variational formulation. The theory reveals that the scattered wave, post-subtracting a known wave field, satisfies the same uSRC so that its far-field pattern is accessible theoretically. For a plane-wave incidence, asymptotic analysis shows that merely subtracting reflected plane waves, due to non-uniform heights of the step-like surface at infinity, from the scattered wave in respective regions, is sufficient to produce an outgoing wave satisfying the uSRC. Numerically, we adopt a previously developed perfectly-matched-layer (PML) boundary-integral-equation method to solve the problem. Numerical results demonstrate that the PML truncation error decays exponentially fast as thickness or absorbing power of the PML increases, of which the convergence relies heavily on the Green function exponentially decaying in the PML.

Wednesday 20:00–22:00

20:00–20:30 **A Novel Boundary Integral Equation Method for Elastic Scattering Problems on a Half-Space**

*Tao Yin**

This talk will present a windowed Green function (WGF) method for the numerical solution of problems of elastic scattering by locally-rough surfaces. The proposed WGF method relies on an integral-equation formulation based on the free-space Green function, together with smooth operator windowing and efficient high-order singular-integration methods. The approach avoids the evaluation of the expensive layer Green function for elastic problems on a half-space, and it yields uniformly fast convergence for all incident angles. Numerical experiments for both two and three dimensional problems will be presented, demonstrating the accuracy and super-algebraically fast convergence of the proposed method as the window-size grows. This is a joint work with O. Bruno.

20:30–21:00 **Time-Domain Metamaterial Models and Finite Element Simulations**

*Wei Yang**

In this talk, we first introduce the development history of mematerials and give some time-domain mathematical model in metamaterials. Then, we focus on the time-domain cloaks model. The explicit expressions of the cloak parameters without the contour curve expressions of the objects and 2d arbitrary shape cloak model are established. A new time-domain finite element scheme is developed to solve the governing equations, and it's stability is also provided. Numerical results are presented to confirm the theoretical analysis and the effectiveness of our cloak model and FETD method.

21:00–21:30 **Space Splitting, Calderon Projection and Discrete Version of Integral Equation Method**

*Chunxiong Zheng**

Integral equation method is powerful to handle the PDE problem with nice local symmetry. Usually, it is coupled with some other numerical techniques to deal with more complicated PDE problems. In this talk, I will present a discrete version of integral equation method, formulating which through an operator point of view. Calderon projection is set up correspondingly. The application to PDE problems and lattice problems will be reported.

21:30–22:00 **A Charge-Conservative Finite Element Method for Inductionless Mhd Equations**

*Weiying Zheng**

A charge-conservative finite element method is proposed for solving inductionless and incompressible magnetohydrodynamic (MHD) equations. To solve the discrete problem, we propose a robust solver for the discrete problem in the framework of field-of-values-equivalence. We first study the preconditioned Krylov space method for the continuous problem in the setting of Hilbert spaces. The algebraic preconditioner for the discrete problem is then obtained by representing the preconditioner for the continuous problem in finite element spaces. By three numerical examples, the optimality of the solver to the number of unknowns is demonstrated for both stationary and time-dependent MHD problems.

MS06 – Techniques and Application of Domain Decomposition for Modelling of Environmental Flows

Organizers: Menno Genseberger ◇ Hansong Tang

In recent years, domain decomposition techniques have been incorporated in large computer codes and used for real-life applications. This minisymposium presents recent work in this regard with a twofold aim. On one hand, it illustrates the importance of domain decomposition in the application field, for instance, for modelling flexibility or parallel performance. On the other hand, the minisymposium intends to highlight the applied domain decomposition techniques, to discuss them, and, if needed, to reconsider or further improve them. The minisymposium is restricted to hydrodynamics and aerodynamics based on shallow water flow and Navier-Stokes equations, and it will yield a good basis for further discussion.

Wednesday 20:00–22:00

20:00–20:30 **Schwarz Algorithms for Ocean-Atmosphere Coupled Problems Including Turbulent Boundary Layers Parameterizations**

Sophie Thery ◇ Eric Blayo ◇ Florian Lemarié*

The interactions between atmosphere and ocean play a major role in many geophysical phenomena, covering a wide range of temporal scales (e.g. diurnal cycle,tropical cyclones, global climate...). Therefore the numerical simulation of such phenomena require coupled atmospheric and oceanic models, which properly represent the behavior of the boundary layers encompassing the air-sea interface and their two-way interactions.
We propose to couple such systems through a Schwarz iterative algorithm. In this talk we focus on a 1-D diffusion type coupled problem representing a simplified ocean-atmosphere coupling. The effects of turbulent boundary layers near the interface are parameterized using a spatially variable viscosity coefficient. We will first study the impact of the Coriolis effect and of the non-constant viscosity on the convergence of the Schwarz algorithm. We will then discuss some difficulties that arise when moving towards more realistic ocean-atmosphere coupling.

20:30–21:00 **Domain Decomposition in Shallow Water Modelling of Dutch Lakes for Multiple Applications**

*Menno Genseberger**

For the larger lakes in The Netherlands, important societal aspects are:

- safety assessments of dikes,
- operational forecasting of flooding,
- improving water quality and ecology,
- extraction of sand from pits, land reclamation, and maintenance of navigation channels.

In forecasting and impact assessments of these aspects, modeling water levels and currents is essential. For this purpose, currently there is a transition from two existing shallow water solvers Delft3D-FLOW and SIMONA/WAQUA on curvilineair, structured computational grids to the new shallow water solver D-Flow FM (Flexible Mesh) on unstructured computational grids.
In this talk some illustrative examples will be shown for Lake IJssel and Lake Marken in The Netherlands. In all three shallow water solvers domain decomposition is a key ingredient for the multiple applications:

- SIMONA/WAQUA with domain decomposition for fast computations: in operational forecasting of flooding accurate waterlevels have to be computed in a very short time.
- Delft3D-FLOW with domain decomposition for modelling different spatial and temporal scales: salt intrusion requires high resolution for mixing processes and less resolution for wind driven transport.

– D-Flow FM with domain decomposition for high detailed modelling: in an integrated approach for the different societal aspects the key idea is to have the unstructured computational grid optimized for required local resolution and computational time.

21:00–21:30 **An Improved Weakly Reflective Boundary Condition for Model Nesting in Shallow Waters**

*Qinghua Ye** ◇ *Mart Borsboom* ◇ *Erik De Goede*

Numerical modeling is becoming more and more detailed and the models are covering larger areas at a higher resolution. This results in a large computational cost of model simulations, even though the computational power is increasing. In order to efficiently run detailed model simulations for the area of interest, (offline) model nesting is commonly used, next to online nesting (i.e. domain decomposition) or unstructured model grids. One benefit of offline nesting is that if the nested, smaller detailed model is sufficiently large to contain all the modeled effects of interest, this model can run independently from the overall model it was nested in. This makes the execution of modeling studies much more efficient and flexible. Nesting of hydrodynamic models is however not trivial and the nested model's open boundary locations and types have to be chosen carefully to avoid model instabilities and inaccuracies.
In the Delft3D-FLOW model that was used in this research, water level, velocity, discharge, Riemann or Neumann boundaries condition types are available. Different types can be assigned to the different open boundaries of the model to produce complex combinations of boundary conditions, e.g., non- or weakly reflective boundary conditions. Nevertheless, undesired boundary effects can still remain if complex flow conditions exist at the model boundaries. One of the reasons is that only one velocity component could be prescribed at an open boundary, which forces the flow perpendicular to the open boundary (i.e. only the normal velocity component). Therefore, a transition zone in the nested model domain is usually necessary to eliminate the inaccuracies induced at the boundary, if these effects can be eliminated at all.
Here we imposed a new boundary condition type, water levels and two velocity components (normal and parallel to the model boundary) can be simultaneously prescribed at boundaries, i.e. a combination of Riemann and tangential velocity boundary conditions. The extra term corresponding to the tangential velocity (parallel) component was included in the momentum equations. In this way, oblique incoming flows can be represented accurately at open boundaries. Furthermore, with the prescribed water level at the boundaries, the mass conservation across the open boundaries is assured. In addition, model nesting tools were adapted to be able to produce these combined boundary conditions. With the combination of Riemann and tangential velocity type boundary conditions, very complex flow patterns can be reproduced without undesired boundary effects. The technique was tested in four different models, including a lake, a coastal and an offshore application with wind-driven currents, complex horizontal circulations, etc. Based on these tests, it could be concluded that this technique significantly increases the flexibility and robustness of boundary conditions for nesting numerical models, especially for engineering practice.

21:30–22:00 **Simulation of Environmental Flows by Coupling Navier-Stokes Equations and Their Hydrostatic Versions**

*Hansong Tang**

It is now becoming a trend to integrate different sets of partial differential equations in simulation of real-world fl ow problems. This presentation deals with coupling of the Navier-Stokes equations and their hydrostatic versions for simulation of environmental flows in oceans. It introduces the background problems and presents the methods and transmission conditions to couple these equations. Numerical algorithms are discussed, and experiments on benchmark problems are made to illustrate the performance of the conditions and the algorithms. Difficulties and future work are also discussed.

MS07 – Domain Decomposition Methods for Isogeometric and High Order Discretizations

Organizers: Luca F. Pavarino ⋄ Olof B. Widlund ⋄ Ulrich Langer

This minisymposium will focus on the latest research developments in Domain Decomposition Methods for Isogeometric Analysis (IGA) and other High Order discretizations. IGA is a recent innovative numerical framework for the discretization of Partial Differential Equations (PDEs), based on the integration of Finite Element analysis and Computer Aided Design (CAD) by employing splines and NURBs basis functions in a Galerkin or collocation scheme. The design and analysis of efficient solvers for IGA and other High Order discretizations present new difficulties, such as the high regularity of the high-order discrete spaces employed, leading to nonstandard interface problems. The talks in this minisymposium will focus on the latest developments in the design, analysis and parallel implementation of novel domain decomposition preconditioners and scalable solvers for IGA and other High Order discretizations.

Wednesday 20:00–22:00

20:00–20:30 **Block FETI--DP/BDDC Preconditioners for Mixed Isogeometric Discretizations of Three-Dimensional Almost Incompressible Elasticity**

Stefano Zampini ⋄ Olof B. Widlund ⋄ Luca F. Pavarino ⋄ Simone Scacchi*

A block preconditioner for mixed formulations of almost incompressible elasticity is presented, based on FETI–DP (dual primal finite element tearing and interconnection) and BDDC (balancing domain decomposition by constraints). The saddle point problems of the mixed problems are discretized with mixed isogeometric analysis with continuous pressure fields. The proposed preconditioner is applied to a reduced positive definite system involving only the pressure interface variable and the Lagrange multipliers of the FETI–DP algorithm, as in previous work by Tu and Li for finite element discretizations of the incompressible Stokes system. The novelty of this preconditioner consists in using BDDC with deluxe scaling for the interface pressure block as well as deluxe scaling for the FETI–DP preconditioner for the Lagrange multiplier block. A convergence rate analysis is presented with a condition number bound for the preconditioned operator which depends on the inf-sup parameter of the fully assembled problem and the condition number of a closely related BDDC algorithm for compressible elasticity. This bound is scalable in the number of subdomains, poly-logarithmic in the ratio of subdomain and element sizes, and robust with respect to material incompressibility. Parallel numerical experiments validate the theory and indicate how the rate of convergence varies with respect to the spline polynomial degree and regularity and the deformation of the domain.

20:30–21:00 **Parallel Block Preconditioners for Three-Dimensional Virtual Element Discretizations of Elliptic Equations in Mixed Form**

Simone Scacchi ⋄ Franco Dassi*

In recent years, several research groups worldwide have focused on the development of numerical methods for the approximation of partial differential equations (PDEs) on polygonal or polyhedral grids. Among the different methodologies proposed, the Virtual Element Method (VEM) represents a generalization of the Finite Element Method that can easily handle general polytopal meshes. In this talk, we first present a new VEM discretization for the solution of the mixed formulation of three-dimensional elliptic equations. Then, we focus on the parallel solution of the linear system arising from such discretization, considering both direct and iterative parallel solvers. In the latter case, we develop two block preconditioners, one based on the approximate Schur complement and one on a regularization technique. The numerical tests performed on a Linux cluster show that the proposed VEM discretization recovers the expected theoretical convergence rates and we analyze the performance of the direct and iterative parallel solvers taken into account.

21:00–21:30 **BDDC Deluxe and Isogeometric Analysis - The Impact of Fat Interfaces on Theory and Practice**

*Olof B. Widlund** ◇ *Stefano Zampini* ◇ *Luca F. Pavarino* ◇ *Simone Scacchi*

The use of BDDC deluxe interative substructuring algorithms have proven to be quite effective for several classes of elliptic problems. Thus, there has been recent studies of linear elasticity in three dimensions for both the compressible and almost incompressible cases and efforts are now focused on problems formulated in H(div) and H(curl). Except for the case of minimal continuity, the use on splines and NURBS leads to interfaces which are fat compared to those of finite elements. While estimates of the condition numbers of BDDC algorithms have much in common with what is done for finite elements, new ideas are also required and this will be a focus of this talk. The effciency of the BDDC algorithms depends to a large extent on finding small primal spaces which provide the global part of the BDDC preconditioners and we will focus on the question of how much larger the primal space needs to be for an isogeometrc analysis problem in comparison to a comparable finite element case while maintaining a satisfactory rate of convergence of the iterative algorithm.

21:30–22:00 **Spectral Equivalence of Higher-Order Tensor Product Finite Elements and Applications to Preconditioning**

*Clark Dohrmann**

In this study, we present spectral equivalence results for higher-order tensor product edge, face and interior-based finite elements. Specifically, we show for certain choices of shape functions that the mass and stiffness matrices of the higher-order elements are spectrally equivalent to those for an assembly of lowest-order elements on the associated Gauss-Lobatto-Legrendre mesh. Based on this equivalence, efficient preconditioners can be designed with favorable computational complexity. Numerical results are presented which confirm the theory and demonstrate the benefits of the equivalence results for overlapping Schwarz preconditioners.

Thursday 16:00–17:30

16:00–16:30 **Optimal Multilevel Preconditioners for Isogeometric Collocation Methods**

*Durkbin Cho**

As in the isogeometric Galerkin formulation, the isogeometric collocation stiffness matrix has a condition number that grows quickly with the inverse of mesh size h. The cost of solving the linear system of equations arising from the isogeometric collocation discretization becomes an important issue. Therefore, there is currently a growing interest in the design of efficient preconditioners/solvers for isogeometric collocation methods, in both the mathematical and the engineering communities.

In this talk, we present optimal additive and multiplicative multilevel methods, such as BPX preconditioner and multigrid V-cycle, for the solution of linear systems arising from isogeometric collocation discretizations of second order elliptic problems. These resulting preconditioners, accelerated by GMRES, lead to optimal complexity for the number of levels, and illustrate their good performance with respect to the isogeometric discretization parameters such as the spline polynomial degree p and regularity k of the isogeometric basis functions, as well as with respect to domain deformations.

16:30–17:00 **Condition Number Bounds for IETI-DP Methods That Are Explicit in h and p**

Rainer Schneckenleitner ◇ *Stefan Takacs**

Isogeometric Analysis (IgA) is a spline-based finite element approach with global geometry function. Following the usual approach, the overall computational domain is decomposed into subdomains, in IgA typically called patches, where each of them is represented by its own geometry function. A standard Galerkin discretization yields a large-scale linear system.

For the solution of that linear system, domain decomposition approaches are a canonical choice since they can be based on the subdivision of the overall domain into patches. We consider the Isogeometric Analysis Tearing and Interconnecting (IETI) method, which is a variant of the FETI-method for IgA. We will discuss convergence analysis, focusing on the dependence of the condition number of the preconditioned system on the spline degree p.
Previously, a convergence theory has been provided that is accurate concerning the dependence of the condition number on the grid size. There, an auxiliary problem with $p = 1$ was introduced. The proof uses the fact that the stiffness matrices of the original and the auxiliary problem are spectrally equivalent. The constants in this spectral equivalence are independent of the grid size but grow exponentially in p.
In this presentation, we will see a direct convergence proof, i.e., a convergence proof that is solely based on the spline spaces of interest and that does not use such an auxiliary problem. This allows us to get rid of the exponential dependence in p. We will discuss the dependence of all estimates on p. Numerical experiments, which confirm our theoretical findings, will be presented.

17:00–17:30 **Convergence Theory for IETI-DP Solvers for Discontinuous Galerkin Isogeometric Analysis That Is Explicit in h and p**

*Rainer Schneckenleitner** ⋄ *Stefan Takacs*

In this presentation, we discuss a convergence theory for a Dual-Primal Isogeometric Tearing and Interconnecting (IETI-DP) solver for isogeometric multi-patch discretizations of the Poisson problem. We consider the case of patches that are coupled with a discontinuous Galerkin (dG) formulation. dG approaches are very beneficial if the patch interfaces from two different geometry mappings are not identical or if the meshes on the interfaces do not agree. The presented theory gives condition number bounds that are explicit in the grid sizes and in the spline degrees. We give an analysis that holds for various choices for the primal degrees of freedom: vertex values, edge averages, and a combination of both. If only the vertex values or both vertex values and edge averages are taken as primal degrees of freedom, the condition number bound is the same as for the conforming case. If only the edge averages are taken, both the convergence theory and the experiments show that the condition number of the preconditioned system grows with the ratio of the grid sizes on neighboring patches.

MS08 – New Developments of Domain Decomposition Methods: Non-Standard Methods and Applications

Organizers: *Yingxiang Xu* ⋄ *Shu-Lin Wu* ⋄ *Hui Zhang*

This minisympsium is for the small streams of domain decomposition methods. We will learn a new Schwarz method based on Perfectly Matched Layers that transmit waves in all dimensions, a reflection on the general theory of Robin-type DDMs and a non-standard parallel-in-time method. Also, we will learn the DDMs for non-standard applications such as nonlinear optimization, image processing and quantum chemistry.

Thursday 16:00-18:00

16:00–16:30 **A Diagonal Sweeping Domain Decomposition Method for the Helmholtz Equation**

*Wei Leng** ⋄ *Lili Ju*

We propose a novel diagonal sweeping domain decomposition method (DDM) for solving the high-frequency Helmholtz equation in R^n. In this method, the computational domain is partitioned into checkerboard subdomains, then a set of diagonal sweeps over the subdomains are specially designed to solve the system efficiently. The computational complexity of the proposed method is $\mathcal{O}(NlogN)$, which makes it potentially superior to the time domain wave computation for very large scale problems. The sweeping nature of the method requires sequential subdomain solves, however, an efficient pipeline could be built for multiple RHSs problem so that the method is still weak scalable. The method is very suitable for solving forward problems in seismic imaging where typically hundreds of times of wave modeling are needed.

16:30–17:00 **Computational Efficiency and Robustness of Algebraic Optimized Schwarz Methods: Promises and Challenges**

*Lahcen Laayouni** ⋄ *Martin J. Gander* ⋄ *Daniel B. Szyld*

We investigate the robustness and efficiency of Algebraic Optimized Schwarz Methods (AOSM). The challenges of developing preconditioners that are not affected by the variation in the parameters of problems has been a focus in domain decomposition over the past years. The main purpose of the present work is to explore the behavior of AOSM with respect to variations in the problem parameters and their scale. The computational efficiency and robustness of AOSM depends on how effective the transmission blocks between subdomains are approximated. To obtain efficient and robust preconditioners, we approximate the transmission blocks taking into account the banded structure and the sparsity of the matrix. Techniques of sparse approximate inverses (SPAI) will be used to reduce the computational cost. We will also explore the computational behavior of AOSM on graphical processing units. With different numerical experiments we will show the applicability of AOSM to a variety of problems.

17:00–17:30 **New Coarse Corrections in Optimized Schwarz Methods for Symmetric and Non-Symmetric Problems**

Martin J. Gander ⋄ *Serge Van Criekingen**

The ORAS2 method improves the Restricted Additive Schwarz (RAS) method using an effective coarse correction and optimized transmission conditions. A PETSc implementation of ORAS2 was presented at the DD25 conference [1], and we pursue here our investigations in terms of scalability of the implementation and its extension to non-symmetric problems. The scalability is achieved through a coarse correction computed at each iteration of the solution process on a reduced-size (coarse) grid enabling a global propagation of the iterative corrections throughout the entire domain. The convergence speed of this two-level method relies heavily on the choice of the coarse grid point location. The coarse correction presented in [1] uses as coarse points the extreme grid points of the non-overlapping decomposition of RAS, that is, in 1-D, two coarse points are placed respectively left and right of each subdomain interface, and, for a rectangular decomposition in 2-D, four coarse grid points are placed around each cross point of the RAS decomposition. Then, a harmonic correction for the Laplacian is obtained by using harmonic coarse basis functions based on these coarse points (Q1 functions). This harmonic correction yields a significant reduction of the number of iterations compared to the classical option of placing the coarse points in the middle of each subdomain (again with Q1 coarse basis functions). Depending on the problem size and on the use or not of an outer Krylov acceleration, this reduction in the number of iterations can compensate the larger time per iteration due to the bigger coarse space. **We also consider a coarse correction having the same coarse space size as our new one, but with equally distributed coarse mesh points.** We compare the total solving times of **these different** approaches up to 16384 cores and compare them also with those obtained with the multigrid library Hypre as interfaced by PETSc. **Whatever coarse space is used however, with several thousands of core used, a parallel coarse solve with agglomeration of the unknowns on a small subset of the cores is required to maintain scalability.** The ORAS2 method can be extended to non-symmetric problems, and we here consider adding an advection term to the Laplacian. Similar to the harmonic correction considered in the symmetric case, we introduce, using the same coarse grid point placement, an "adapted" coarse space such that the basis functions are local (on each subdomain) solutions of the original problem. In 1-D, this approach yields the solution in one iteration as with the harmonic correction in the symmetric case. In 2-D, the local solves necessary to determine the coarse basis functions can be done with various options of local boundary conditions. One option to fix these conditions for a 2-D rectangular decomposition is to solve, on every local boundary, the 1-D advection equation obtained by lumping the coefficients in the perpendicular direction. Another, simpler, option is to take as local boundary conditions the same linear boundary conditions as in the symmetric case, so that the coarse space is adapted only in the interior of each subdomain. For the case of a rotating advection term of the type $a = (-y, x)$, we compare the results with these different methods to the classical coarse correction obtained using a single coarse point in the middle of each subdomain.[1] M.J. Gander and S. Van Criekingen. New Coarse Corrections for Optimized

Restricted Additive Schwarz Using PETSc, accepted for Domain Decomposition Methods in Science and Engineering XXV, LNCSE, Springer-Verlag, 2019

17:30–18:00 **Are Parallel Schwarz Methods for Magnetotelluric Approximations to Maxwell's Equations Scalable?**

*Alexandros Kyriakis** ⋄ *Victorita Dolean* ⋄ *Martin J. Gander*

We study the convergence properties of one-level parallel Schwarz methods applied to the one and two dimensional complex diffusion equation, which naturally arises in the magnetotelluric approximation of Maxwell equations. One level parallel Schwarz methods are not scalable in general and coarse correction techniques are needed to obtain scalability. However, it has been recently observed that parallel Schwarz algorithms are scalable under the assumption that we have chains of fixed sized subdomains. This corresponds to stripwise decompositions often encountered in waveguide problems. Using the new technique of limiting spectrum estimates, we show that also for the complex diffusion equation, the classical parallel Schwarz method and the optimized variant using Robin conditions are scalable in such configurations. We illustrate our results with numerical experiments.

Friday 10:00–12:00

10:00–10:30 **Fourier Masked Phase Retrieval: Mask Design, Blind Recovery, and Sparsity Modeling**

*Huibin Chang**

Phase retrieval plays an important role in vast industrial and scientific applications, which is essentially a non-convex and possible non-smooth optimization problem mathematically. In this talk, we mainly concern how to design structured masks for unique recovery, and convergent splitting algorithm with convergence guarantee, and further improve the quality of reconstructed images driven by the sparse prior. We first show our factional-index based mask, which can give promising recovery with only three groups of measurements. Then, we consider the bind ptychography problem, where we address a general least squares model by maximum likelihood estimation and adopt fast alternating direction method of multipliers to solve it. Under mild conditions, we establish the global convergence to stationary points. Numerically, the proposed algorithm outperforms the state-of-the-art algorithms in both speed and image quality. Then we consider a noisy phase retrieval problem with measured intensities corrupted by strong Gaussian or Poisson noises. Sparse regularization methods, e.g. Total Variation, Dictionary Learning and BM3D filters, are utilized to denoise phaseless measurements, and as a result, the quality of recovery images is greatly increased from noisy (or incomplete) data. Due to the non-convexity of established models, we also discuss how to design fast splitting algorithms with convergence guarantee for a general recovery problem. This is a joint work mainly with Stefano Marchesini in LBNL, and Yifei Lou in UT Dallas.

10:30–11:00 **A Space Decomposition Framework for Nonlinear Optimization**

*Zaikun Zhang** ⋄ *Serge Gratton* ⋄ *Luis Nunes Vicente*

Space decomposition has been a popular methodology in both the community of optimization and that of numerical PDEs. Under the name of Domain Decomposition Methods, the latter community has developed highly successful techniques like Restricted Additive Schwarz and Coarse Grid. Being crucial for the performance of Domain Decomposition Methods, these techniques are however less studied in optimization. This talk presents a framework that allows us to extend these techniques to general nonlinear optimization. We establish the global convergence and convergence rate of the framework. Numerical results, although preliminary, show the Restricted Additive Schwarz and Coarse Grid (now Coarse Space) techniques can effectively accelerate space decomposition methods as they do in Domain Decomposition Methods.

11:00–11:30 **Optimization of Two-Level Methods for DG Discretizations of Reaction-Diffusion Equations**

*Jose Pablo Lucero Lorca** ⋄ *Martin Jakob Gander*

We analyze and optimize two-level methods applied to a symmetric interior penalty discontinuous Galerkin finite element discretization of a singularly perturbed reaction-diffusion equation. Previous analyses of such methods have been performed numerically by Hemker et. al. for the Poisson problem. Our main innovation is that we obtain explicit formulas for the optimal relaxation parameter of the two-level method for the Poisson problem in 1D, and very accurate closed form approximation formulas for the optimal choice in the reaction-diffusion case in all regimes. Our Local Fourier Analysis, which we perform at the matrix level to make it more accessible to the linear algebra community, shows that for DG penalization parameter values used in practice, it is better to use cell block-Jacobi smoothers of Schwarz type, in contrast to earlier results suggesting that point block-Jacobi smoothers are preferable, based on a smoothing analysis alone. Our analysis also reveals how the performance of the iterative solver depends on the DG penalization parameter, and what value should be chosen to get the fastest iterative solver, providing a new, direct link between DG discretization and iterative solver performance. We illustrate our analysis with numerical experiments and comparisons in higher dimensions and different geometries.

11:30–12:00 **Schwarz Domain Decomposition Methods for the Fluid-Fluid System with Friction-Type Interface Conditions**

*Yingxiang Xu** ⋄ *Wuyang Li*

In this talk we consider a fluid-fluid system coupled with a friction-type interface condition which is described by a jump of the velocities along the tangential interface. Based on domain decomposition, we propose a Schwarz type iterative method to decouple the different physical process in each subdomain occupied by a single fluid, which allows solving at each subdomain only the physical process described by a Stokes problem. Using energy estimate, we prove the algorithm converges for any traction coefficient. A more detailed analysis based on Fourier analysis shows that the convergence depends on both the fluids' properties and the traction coefficient but independent of the numerical grid. In addition, towards fast convergence we also proposed a new Robin-type transmission condition and optimized the involved transmission parameters based on Fourier analysis. We finally use numerical examples to illustrate the theoretical results.

Friday 16:00–17:30

16:00–16:30 **Towards Optimized Schwarz Methods with Low-Rank Transmission Conditions**

*Michal Outrata** ◇ *Martin J. Gander*

For a coercive model problem, it has been shown in [**?**] that if a combination of Neumann and Dirichlet data is exchanged on the subdomain interfaces instead of exchanging only Dirichlet data (as in the classical alternating Schwarz method), one can obtain optimal transmission conditions in the sense that the resulting Schwarz methods - the optimal Schwarz methods - converge in finite number of steps, and can thus not be further improved within the given class of the methods. This makes the optimal Schwarz methods essentially direct solvers. The transmission conditions obtained however are non-local and thus such optimal methods are expensive to use in practice. In [**?**], the optimal transmission conditions are replaced by several types of local approximations of the optimal transmission conditions all of which impose a very appealing zero structure on the discretized transmission operator, ensuring its structural sparsity . These local approximations of the transmission conditions are determined to optimize the convergence behavior of the Schwarz methods rather than to approximate the optimal transmission condition, which leads to the class of optimized Schwarz methods. We propose here a new type of approximations: instead of structural sparsity, we impose data sparsity of the discretized transmission operator. We start by studying a straight forward low-rank approximation of the transmission operator, and then also consider more sophisticated hierarchical formats such as HODLR and $\mathcal{H}$-matrices. Following the idea of optimized Schwarz methods, the focus is on optimizing the convergence behavior rather than minimizing the approximation error. We will present numerical results for the Dirichlet problem together with theoretical background, and we will compare the different data-sparsity formats, both to each other and to the other Schwarz methods.

16:30–17:00 **A Superlinear Convergence of Parareal Schwarz Waveform Relaxation Algorithm**

Martin Gander ◇ *Yao-Lin Jiang* ◇ *Bo Song**

The Parareal Schwarz Waveform Relaxation algorithm is a new space-time parallel algorithm for the solution of evolution partial differential equations. It is based on a decomposition of the entire domain both in space and in time into smaller space-time subdomains, and then computes by an iteration in parallel on all these small subdomains a better and better approximation of the overall solution. The initial conditions in the subdomains are updated using a parareal mechanism, while the boundary conditions are updated using Schwarz waveform relaxation techniques. A first precursor of this algorithm was presented fifteen years ago, and while the method works well in practice, the convergence of the algorithm is not yet understood, and to analyze it is technically difficult. We present in this talk for an accurate superlinear convergence estimate when the algorithm is applied to the heat equation. We illustrate our analysis with numerical experiments including cases not covered by the analysis, which opens up many further research directions.

17:00–17:30 **A Domain Decomposition Method for the Poisson--Boltzmann Solvation Model in Quantum Chemistry**

*Chaoyu Quan** ◇ *Benjamin Stamm* ◇ *Yvon Maday*

A special domain decomposition method, called ddLPB, is developed for solving the Poisson–Boltzmann solvation model in quantum chemistry. In this method, the domain (solute molecule) is decomposed into balls and the linearized Poisson–Boltzmann equation is transformed into a system of coupled subequations restricted in these balls. Each local subequation can be solved explicitly, using the Galerkin spectral method with the spherical harmonics as basis functions. The coupling conditions between the subequations are discretized, which yields a global linear system that could be solved by the GMRES algorithm. A series of numerical experiments will be presented to show the robustness and efficiency of this method.

MS09 – Space--Time FEM/BEM: Theory and Applications

Organizers: Ulrich Langer ◇ Olaf Steinbach

In recent years, space-time discretization methods became very popular for the solution of time-dependent partial differentil equations. This is mainly due to the applicability of adaptive resolutions in space and time simultaneousy, and the parallel solution in space and time. Within this minisymposium we will present some of the recent developments of space-time finite and boundary element methods in theory and applications.

Friday 16:00–18:00

16:00–16:30 **A Modified Hilbert Transform and Space-Time Continuous Galerkin Finite Element Methods for the Second-Order Wave Equation**

Marco Zank ◇ Olaf Steinbach*

For the discretisation of time-dependent partial differential equations, the standard approaches are explicit or implicit time stepping schemes together with finite element methods in space. An alternative approach is the usage of space-time methods, where the space-time domain is discretised, and the resulting global linear system is solved at once. In any case, CFL conditions play a decisive role for stability. In this talk, the model problem is the scalar wave equation. The starting point is a space-time variational formulation, where a modified Hilbert transform is used such that ansatz and test spaces are equal. A conforming discretisation of this space-time variational formulation is introduced, leading to a Galerkin finite element method, which is unconditionally stable, i.e. no CFL condition is required. Additionally, numerical examples for a one-dimensional spatial domain and a two-dimensional spatial domain are presented.

16:30–17:00 **The Advection Curse of Multigrid-In-Time Methods**

Thibaut Lunet ◇ Martin J. Gander ◇ Scott MacLachlan*

Numerical methods that discretize space and time simultaneously, in order to induce parallel computations, have been the focus of renewed attention over last two decades. Among all these parallel-in-time (PinT) methods, the parareal algorithm and more generally multigrid-in-time methods have been of great interest, since they can benefit from parallel computation techniques that have been developed in general for domain decomposition and multigrid methods.
We consider here Parareal and similar algorithms (e.g MGRIT) that work quite well for parabolic problems, but have been shown by numerous analyses and experiments to be less efficient for hyperbolic problems, where there is no physical diffusion. Introducing numerical diffusion through the space-time discretization permits, to some extent, to reestablish convergence properties of the iterates computed in parallel to the time-sequential solution. This raises the important question if one should not just use a suffciently diffusive discretization scheme that accurately approximates the problem as soon as the space-time mesh sizes are small enough, in order to get multigrid-in-time convergence for a hyperbolic problem.
We provide some elements for an answer by first focusing on the linear advection problem, and studying how accurate general space-time discretization schemes are, compared to each other. We present a criterion that determines, for a given accuracy on one wavelength of the solution, how many grid points are required. This allows us to rank the different discretization schemes, and distinguish those which are more efficient to solve the pure advection problem from the less efficient ones. We then extend our study to the Parareal algorithm, used together with numerical schemes that are well suited for advection. These are in our opinion important building blocks to answer the question if Parareal (and MGRIT) can be effectively used to solve hyperbolic problems.

17:00–17:30 **Locally Stabilized Space-Time Finite Element Methods on Anisotropic Hexahedral Decompositions**

Andreas Schafelner ◇ Ulrich Langer*

We present locally stabilized, conforming space-time finite element methods for parabolic evolution equations on hexahedral decompositions of the space-time cylinder. Tensor-product decompositions allow for anisotropic a priori error estimates, that are explicit in spatial and temporal meshsizes. Moreover, tensor-product finite elements are suitable for anisotropic adaptive mesh refinement strategies provided that an appropriate a posteriori discretization error estimator is available. The large-scale system of space-time finite element equations is then solved by means of the Flexible Generalized Minimal Residual (FGMRES) method preconditioned by space-time algebraic multigrid. We present and discuss numerical results for several examples possessing different features.

This work was supported by the Austrian Science Fund (FWF) under grant W1214, project DK4.

17:30–18:00 **A Space-Time FETI Method for Incompressible Flow Problems**

*Douglas R. Q. Pacheco** ⋄ *Olaf Steinbach*

Space-time finite element formulations differ from conventional methods by treating time and space in a similar manner. This means that time is considered as an additional dimension, so that the whole space-time domain is discretised with finite elements. Once the structure of time steps/slabs is lost, it becomes of utmost importance to employ appropriate solution techniques to efficiently tackle the large algebraic system resulting from the space-time discretisation. In this context, we present a new finite element tearing and interconnecting method for a stable space-time formulation of the Stokes system. Two settings are considered: First with pressure continuity enforced on the interfaces via Lagrange multipliers, then the simpler case where the pressure is continuous per subdomain, but discontinuous across interfaces. Since our problem is parabolic and non-symmetric, an appropriate iterative procedure must be derived for solving the global problem, whereas the local subproblems are solved directly. Numerical examples showcase the potential of our approach.

Friday 20:00–22:00

20:00–20:30 **Exponential Fitting for Space-Time Convection Diffusion Problems**

*Ludmil Zikatanov**

We introduce a class of numerical methods for convection diffusion equations in arbitrary spatial dimensions. Targeted applications include the Nernst-Plank equations for transport of species in a charged media and the space-time discretizations of such equations. The numerical schemes that we consider are descendants of the popular, one-dimensional, first order, exponentially fitted Scharfetter-Gummel method in semiconductor devices modeling (1969). We illustrate how such exponentially fitted methods are derived in several simple, typical, and instructive cases. We also reveal intricate connections with other families of discretization spaces usually used as building blocks in the Finite Element Exterior Calculus. These findings lead, in a natural way, to higher order exponentially fitted discretizations in any spatial dimensions. We state several theoretical results regarding stability and errors for the resulting numerical schemes. Distinctive features of the proposed methods are: (1) monotonicity (in the lowest order case); (2) errors depending on the flux (a function often smoother than the solution). Our numerical tests verify the theory on examples from space-time formulation of parabolic problems. This is a joint work with R. E. Bank (UCSD) and P. S. Vassilevski (LLNL).

20:30–21:00 **PFASST and Finite Elements**

*Robert Speck** ⋄ *Ruth Schoebel*

The efficient use of modern supercomputers has become one of the key challenges in computational science. For the numerical solution of time-dependent processes, time-parallel methods have opened new ways to overcome scaling limits. Inspired by the classical Parareal method and nonlinear multigrid ideas, the "parallel full approximation scheme in space and time" (PFASST) allows to integrate multiple time-steps simultaneously using "multi-level spectral deferred corrections" (MLSDC) with different coarsening strategies in space and time. In numerous studies, this approach has been successfully coupled to space-parallel solvers which use finite differences, spectral methods or even particles for discretization in space. In this talk, we report on using MLSDC and PFASST in time together with the finite element method in space. In particular, we discuss modifications necessary to treat the mass matrix appropriately on all levels of the space-time hierarchy. While more natural, avoiding the inversion of the mass matrix introduces convergence problems of the iterative schemes in time. We examine this issue and show its impact along the lines of the diffusion equation and a nonlinear reaction-diffusion model. The algorithms are implemented using FEniCS together with the prototyping framework pySDC, which allows users to test new ideas or to analyze issues like this one without great effort.

21:00–21:30 **Mesh Refinement in 4D**

*David Lenz**

In this talk, we present recent progress in mesh generation and refinement algorithms for four-dimensional conforming simplicial meshes. Most existing meshing software does not contain support for meshes of dimension greater than three, which often forces researchers of space-time methods to create their own meshing utilities, focus on tensor-product elements, or conduct numerical experiments in two spatial dimensions. This problem can be mitigated, however, by looking to existing models of high-dimensional geometry. We propose the concept of a combinatorial map as a data model for high-dimensional meshes. A combinatorial map is a mathematical object defined by permutations on a set of basic objects which can represent subdivided manifolds of any dimension. The benefits and drawbacks of this model will be illustrated with examples, using the implementation supported by the Computational Geometry Algorithms Library (CGAL). Finally, we show how this model can be used to create space-time meshes from spatial meshes and refine existing space-time meshes.

21:30–22:00 **Space-Time Boundary and Finite Element Domain Decomposition Methods for Parabolic Problems**

*Olaf Steinbach**

For the numerical solution of parabolic problems we consider space-time finite and boundary element methods where the underlying variational problem is formulated in anisotropic Sobolev spaces. Unique solvability of the continuous variational formulations is then based on suitable inf–sup stability conditions, and using a modified Hilbert transformation we finally end up with a Galerkin–Bubnov discretization. Since the numerical analysis of space-time finite and boundary element methods is based on the same variational principles, the coupling of both follows the same lines. In addition, we also consider domain decomposition methods, which, in the most general case, are based on a decomposition of the space-time domain into rather general non-overlapping subdomains.

MS10 – Multilevel Domain Decomposition Methods and Parallel Implementations

Organizers: Jakub Šístek ⋄ Stefano Zampini

The almost exponential growth in computing power of the largest supercomputers has been maintained for the last decade mainly by increasing concurrency at different hardware levels, especially increasing core counts of CPUs and incorporating accelerators, such as GPUs. These hardware developments pose new challenges to domain decomposition (DD) algorithms and their massively parallel implementations. One of the main issues we are facing is solving coarse problems in DD in a scalable way to exploit the mathematical scalability of the algorithms.

The minisymposium brings together researchers working on techniques addressing the issue of the coarse problem solution mainly by employing several levels in the DD methods. The topics include developments in Multilevel BDDC for symmetric and nonsymmetric problems. An emphasis is put on parallel implementations of the methods and applications to large-scale problems.

Tuesday 16:00–17:30

16:00–16:30 **Substructured Two-Level and Multi-Level Domain Decomposition Methods**

Gabriele Ciaramella ⋄ Tommaso Vanzan*

Two-level domain decomposition methods are very powerful techniques for the efficient numerical solution of partial differential equations. A two-level domain decomposition method requires two main components: a one-level preconditioner (or its corresponding smoothing iterative method), which is based on domain decomposition techniques, and a coarse correction step, which relies on a coarse space. The coarse space must properly represent the error components that the chosen one-level method is not capable to deal with. In the literature most of the works introduced efficient coarse spaces obtained as the span of functions defined on the entire space domain of the considered partial differential equation. Therefore, the corresponding two-level preconditioners and iterative methods are defined in volume.

In this talk, a new class of substructured two-level methods is introduced, for which both domain decomposition smoothers and coarse correction steps are defined on the interfaces. This approach has several advantages. On the one hand, it allows one to use some of the well-known efficient coarse spaces proposed in the literature. On the other hand, the required computational effort is cheaper than the one required by classical volumetric two-level methods. Moreover, our new substructured framework can be efficiently extended to a multi-level framework, which is always desirable when the high dimension or the scarce quality of the coarse space prevent the efficient numerical solution. Numerical experiments demonstrate the effectiveness of the proposed new numerical framework.

16:30–17:00 **Multilevel Iterative Solver and a Posteriori Algebraic Error Estimator with p-Robust Behavior**

Jan Papež ⋄ Ani Miraçi ⋄ Martin Vohralík*

We consider conforming finite element discretizations of arbitrary polynomial degree $p \geq 1$ of the Poisson problem on a hierarchy of nested, unstructured simplicial meshes. We design a multilevel iterative algebraic solver and we show that this solver contracts the algebraic error on each iteration by a factor bounded independently of p. We show that this result is equivalent to deriving a multilevel a posteriori estimator of the algebraic error that is reliable and efficient (represents a two-sided bound on the error), with a constant independent of the degree p.

The p-robustness results rely on the work of Schöberl et al. [IMA J. Numer. Anal., 28 (2008)] for one given mesh. We combine this with the design of an algebraic residual lifting constructed over the hierarchy of meshes, in the spirit of Papež et al. [HAL Preprint 01662944, (2017)]. This includes a global coarse-level solve of the lowest-order, with local higher-order contributions from the subsequent mesh levels. These higher-order contributions are given as solutions of mutually independent Dirichlet problems posed over patches of elements around vertices. This residual lifting is the core of the a posteriori estimator and determines the descent direction for the next iteration of the proposed multilevel solver. Its construction can be seen as one geometric V-cycle multigrid step with zero pre- and one post-smoothing by damped additive Schwarz. Numerical tests are presented to illustrate the theoretical findings.

17:00–17:30 **BDDC Methods and GPUs with PETSc. Current Status and Future Perspectives.**

*Stefano Zampini**

Balancing Domain Decomposition by Constraints (BDDC) methods have proven to be powerful algorithms for preconditioning linear systems arising from the finite element discretization of elliptic PDEs, with excellent weak scalability properties, robustness with respect coefficient heterogeneities, and favorable coarsening features. In this talk we will review the current status of the implementation provided in the open source Portable and Extensible Toolkit for Scientific computing (PETSc), and describe recent developments exploiting Graphical Processing Units (GPUs). Future work toward a matrix-free implementation will be also discussed.

Tuesday 20:00–22:00

20:00–20:30 **A Parallel Multilevel BDDC Solver and Its Application to Adaptive FEM**

*Jakub Šístek** ⋄ *Pavel Kůs*

We briefly recall the algorithm of the Multilevel Balancing Domain Decomposition based on Constraints (Multilevel BDDC) [2]. Despite the fact that the extension of the method to multiple levels worsens the convergence, it can significantly improve the scalability of the method. Our open-source implementation of the Multilevel BDDC method, the BDDCML library, will be presented. Next, we will discuss the combination of the method with the finite element method using an adaptive mesh refinement (AMR). AMR is challenging in the context of distributed memory parallel FEM in general. The particular focus of the talk will be on the incorporation of hanging nodes into the domain decomposition solver and on the treatment of disconnected subdomains, a typical output of the employed mesh partitioning based on space-filling curves used by the *p4est* library [1]. We have enhanced the BDDC method to support disconnected subdomains by analysing subdomain mesh graphs, while the hanging nodes are incorporated naturally by the non-overlapping domain decomposition. We provide experiments for the Poisson problem using up to 1.3 billion unknowns and 2048 CPU cores. Details can be found in [3].

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20:30–21:00 **Multilevel BDDC for Nonsymmetric Systems from Incompressible Flows**

*Martin Hanek** ⋄ *Jakub Šístek* ⋄ *Pavel Burda*

We deal with numerical simulations of incompressible flows. Multilevel Balancing Domain Decomposition based on Constraints (BDDC) is applied to non-symmetric systems arising from the linearization of incompressible Navier-Stokes equations. For linearization, we use Picard iteration, and the linear system is solved by the BiCGstab method using one step of BDDC as the preconditioner. Numerical results for a benchmark problem of 3-D lid-driven cavity and for an industrial problem of flow in a hydrostatic bearing are presented.

21:00–21:30 **A Multigrid Method for $H(\mathbf{curl})$ with Nonoverlapping Domain Decomposition Smoothers**

*Duk-Soon Oh**

We introduce a $\mathcal{V}-$cycle multigrid method for $H(\mathbf{curl})$ problems arising from $3D$ edge elements. Due to the fact that conventional smoothers do not work well for problems posed in $H(\mathbf{curl})$ vector fields, a special approach for smoothers in the multigrid methods is essential. We suggest smoothers using nonoverlapping domain decomposition smoothers, similar to those for $H(\mathrm{div})$ problems introduced in DD 24.

21:30–22:00 **Construction of Grid Operators for Multilevel Solvers by Means of a Neural Networks Approach**

*Claudio Tomasi** ⋄ *Rolf Krause*

Multigrid methods are among the most successful strategies for solving linear systems arising from the discretization of linear elliptic partial differential equations. These methods require to transfer information between different levels through transfer operators. These operators are crucial for fast convergence of Multigrid.
We propose a Neural Network (NN) approach to find these operators, where the mass and stiffness matrices of the discretized problem are given as input. In the construction of the training set we consider only 2-grid scenarios which have a good frequency decomposition.
Therefore we provide the NN with examples coming from optimal multigrid settings, allowing to predict transfer operators which will give a faster convergence with an error reduction that is mesh size-independent.
Starting with a Finite Element discretization of a problem on a fine grid, we ask the network to recursively get transfer operators. The procedure continues until the problem dimension allows a direct solving on the coarsest grid. We investigate the accuracy of the transfer operator predicted by the NN, testing it with different network architectures. This NN approach for the construction of grid operators can then be extended for an automatic building of multilevel solvers, thus allowing for a portable solution in scientific computing.

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MS11 – Domain Decomposition Methods in HPC: Extremely Scalable Implementations and Their Application

Organizers: *Axel Klawonn* ◇ *Oliver Rheinbach*

Nonoverlapping as well as overlapping domain decomposition (DD) methods proved to be robust and efficient parallel linear or nonlinear solvers. Their favorable divide and conquer structure makes them extremely suitable for modern parallel computer architectures and carefully designed coarse spaces deliver robustness and numerical scalability up to large numbers of subdomains and compute cores. In recent years, approximating the global coarse solves by applying efficient parallel solvers as, e.g., algebraic multigrid or DD recursively, facilitated the efficient use of hundreds of thousands of compute cores. In this minisymposium, several state-of-the-art DD approaches, their efficient implementation, and their application to hard real-world problems are presented and discussed.

Wednesday 16:00–18:00

16:00–16:30 **Preconditioning the Coarse Problem of BDDC - Comparison of Three-Level, Algebraic Multigrid, and Vertex-Based Preconditioners**

Martin Lanser ◇ *Axel Klawonn* ◇ *Oliver Rheinbach** ◇ *Janine Weber*

A comparison of three Balancing Domain Decomposition by Constraints (BDDC) methods with approximate coarse space solvers is presented. The comparison is made for a BDDC method with an algebraic multigrid preconditioner for the coarse problem, a three-level BDDC method, and a BDDC method with a vertex-based coarse preconditioner which was recently introduced by Clark Dohrmann, Kendall Pierson, and Olof Widlund. All approaches are presented in a common framework and compared using a highly scalable PETSc-based BDDC implementation avoiding the construction of Schur-Complements. Condition number bounds are provided for all approaches. Numerical results showing the parallel scalability of all variants are presented for the equations of linear elasticity.

16:30–17:00 **A Three-Level Extension for the Fast and Robust Overlapping Schwarz (FROSch) Preconditioner**

*Friederike Röver** ◇ *Alexander Heinlein* ◇ *Axel Klawonn* ◇ *Oliver Rheinbach*

The Fast and Robust Overlapping Schwarz (FROSch) framework is parallel implementation of the GDSW preconditioner in the Trilinos software library. To improve the parallel scalability of the two-level method a three-level extension has recently been introduced. The introduction of the third level reduces the size of problem on the coarsest level. A further improvement of the scalability can be obtained by recently introduced reduced coarse spaces. Results for the two- and the three-level methods are presented, showing the extended parallel scalability of the new method.

17:00–17:30 **Large Eddy Simulation of the Wind Flow in a Realistic Full-Scale Urban Community with a Scalable Parallel Algorithm**

Zhengzheng Yan ◇ *Rongliang Chen** ◇ *Xiao-Chuan Cai*

Accurate knowledge of the wind flow field in large urban areas is the basis of numerous applications such as building wind vulnerability analysis and air pollution forecasting. In actual urban settings, the tall buildings and complex structures make the air flow turbulent, usually requiring a high-resolution temporal and spatial computation grid, which results in computationally challengings. Current supercomputers, in which massive processors work simultaneously to produce exceptional computational power and reduce the total computational time, are able to solve such large problems. But in order to efficiently use the supercomputer, scalable parallel algorithms are requested. In this work, we present a scalable domain decomposition method based 3D incompressible Navier-Stokes solver for solving unsteady, complex wind flows around urban communities. A large eddy simulation (LES) with Smagorinsky modeling is employed for the simulation of the highly turbulent wind flows. The algorithm is first validated by a benchmark case where the numerical results are compared with the wind tunnel measured data. Then, we simulate the wind flow field of a realized actual-scale urban community in the downtown area of Shenzhen China with a group of buildings. The algorithm is tested on the top two supercomputers of China: the Tianhe-2A and the Sunway TaihuLight. The numerical results show that the simulation results match well with the measured results for the benchmark case, some reasonable, detailed, and complex flow structures are obtained for the urban community simulation case, and the algorithm scales up to 16,384 processor cores for a grid with over 20 million unstructured cells.

17:30–18:00 **A Frugal FETI-DP and BDDC Coarse Space for Heterogeneous Problems - Comparison with Competing Coarse Spaces and Parallel Results**

*Martin Lanser** ◇ *Alexander Heinlein* ◇ *Axel Klawonn* ◇ *Janine Weber*

The convergence rate of domain decomposition methods is generally determined by the eigenvalues of the preconditioned system. For second-order elliptic partial differential equations, coefficient discontinuities with a large contrast can lead to a deterioration of the convergence rate. Only by implementing an appropriate coarse space or second level, a robust domain decomposition method can be obtained. In this talk, a new frugal coarse space for FETI-DP (Finite Element Tearing and Interconnecting - Dual Primal) and BDDC (Balancing Domain Decomposition by Constraints) methods is presented, which has a lower set-up cost than competing adaptive coarse spaces and is robust for a more general class of problems than classical FETI-DP and BDDC coarse spaces. Furthermore, a reduction of the size of the frugal coarse space is discussed. Also inexact BDDC variants based on an approximation of the frugal coarse problem are presented, using either a vertex-based approach or an AMG (algebraic multigrid) preconditioner. The new coarse space is compared to adaptive coarse spaces as well as classical coarse spaces, and parallel scalability up to 262 144 parallel cores for a parallel BDDC implementation is shown.

Wednesday 20:00–22:00

20:00–20:30 **FE2TI -- A Software for Large Scale Simulations of Sheet Metal Forming with Contact Using Computational Homogenization and Domain Decomposition Methods**

*Matthias Uran** ◇ *Axel Klawonn* ◇ *Axel Lanser* ◇ *Oliver Rheinbach*

The finite element simulation of sheet metal forming of dual-phase (DP) steel including its microstructure requires a homogenization approach since a brute force discretization would lead to unfeasibly large systems of equations and would also require knowledge about the complete microstructure, which is usually not given. In this talk, we present our FE2TI software, which combines a highly scalable implementation of the well-known FE^2 homogenization approach with different domain decomposition as well as multgrid methods. In the FE^2 method, the microstructure is represented by a representative volume element (RVE), which is discretized separately from the macroscopic problem. On the RVE level, a Newton-Krylov-FETI-DP (NK-FETI-DP) as well as a more recent nonlinear FETI-DP method (NL-FETI-DP-1) have been used as an efficient solver. Recently, we have included a macroscopic contact algorithm in our FE2TI software in order to solve realistic deformation processes such as the Nakajima test. This is a material test, where the sample sheet is first clamped between a blank holder and a die and then a hemispherical punch is driven into the sample sheet until it cracks. On the one hand, such a realistic problem can be simulated by exploiting the symmetric experimental setup, as this keeps the size of the macro problem small. On the other hand, we have incorporated a Newton-Krylov-BDDC (NK-BDDC) approach in order to solve the homogenized contact problem on the macroscale efficiently without exploiting symmetry. We show the different results obtained from simulations with and without exploiting symmetry on up to 60 000 MPI ranks using an elasto-plastic material model on the microscopic level.

20:30–21:00 **Globalization of Nonlinear FETI-DP Methods**

*Stephan Köhler** ⋄ *Oliver Rheinbach*

Nonlinear FETI-DP (Finite Element Tearing and Interconnecting − Dual-Primal) methods [**?**] are domain decomposition methods for the solution of nonlinear problems from the discretization of PDEs. In these methods, before linearization, the global problem is decomposed into concurrent nonlinear problems. Then, different local elimination strategies can be used, in the sense of nonlinear preconditioning [**?**]. This talk discusses the globalization of Nonlinear FETI-DP methods with SQP methods [**?**] and (exact) penalty methods [**?**], which make explicit use of the nonlinear elimination.

21:00–21:30 **FROSch Preconditioners for Land Ice Simulations of Greenland and Antarctica**

*Alexander Heinlein** ⋄ *Mauro Perego* ⋄ *Sivasankaran Rajamanickam*

Greenland and Antarctic ice sheets store most of the fresh water on earth and mass loss from these ice sheets significantly contributes to sea-level rise. The simulation of temperature and velocity of the ice sheets gives rise to large highly nonlinear systems of equations. The solution of the associated tangent problems, arising in Newton's method, is challenging also because of the strong anisotropy of the meshes. We first consider simulations of the ice velocity of Antarctica and the ice temperature of Greenland. We use one-level Schwarz preconditioners as well as GDSW (Generalized Dryja–Smith–Widlund) type preconditioners from the Trilinos package FROSch (Fast and Robust Schwarz), scaling up to 32 k processor cores (8 k MPI ranks and 4 OpenMP threads) for the finest Antarctica mesh; the corresponding velocity problem contains 566 M degrees of freedom. We then study the coupled velocity and temperature problem for the Greenland ice sheet. To the best of our knowledge, it is the first time that a scalable monolithic two-level preconditioner has been used for this multiphysics problem. We present strong scaling results, up to 4 k MPI ranks, using a monolithic GDSW type preconditioner with decoupled extensions from the FROSch package.

21:30–22:00 **Comparison of Monolithic and Block Preconditioners with GDSW-Type Coarse Spaces for Incompressible Fluid Flow Problems**

*Christian Hochmuth** ⋄ *Alexander Heinlein* ⋄ *Axel Klawonn*

The construction of a monolithic two-level overlapping Schwarz preconditioner with Generalized–Dryja–Smith–Widlund (GDSW) type coarse spaces is outlined. A comparison of results of this monolithic approach and widely used block preconditioners for the Navier–Stokes equations is presented. As block preconditioners the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) and the Least-Squares Commutator (LSC) block preconditioners are used. These block preconditioners are built from approximate LU block factorizations and a suitable approximation of the Schur complement. The corresponding inverses are approximated with two-level overlapping Schwarz preconditioners. For the second level, GDSW-type coarse spaces are used and different coarse space variants are compared. Results of a P2–P1 and a stabilized P1–P1 finite element discretization as well as different recycling strategies for the preconditioners will be discussed. The Trilinos packages FROSch and Teko are used to construct the two-level Schwarz precconditioners and the block-preconditioners, respectively.

MS12 – Asynchronous Iterative Methods

Organizers: Christian Glusa ⋄ Daniel B. Szyld

Classical synchronous iterative methods alternate between local computation and boundary data exchange. In asynchronous iterative methods this dependency is relaxed and processing units are allowed to use whatever data is available at the beginning of a computation phase. Originally called 'Chaotic Relaxation' for fixed-point iterations, asynchronous iterative methods are used in various areas of high-performance computing and numerical optimization. In this minisymposium, recent research is presented both on the theory and implementation of asynchronous iterative methods.

Thursday 20:00–22:00

20:00–20:30 **Acceleration of the Convergence of the Asynchronous Restricted Additive Schwarz Method**

*Damien Tromeur-Dervout**

This talk focuses on the acceleration of the convergence of the asynchronous Restricted Additive Schwarz (RAS) iterates. Schwarz methods with asynchronous communications are becoming particular interesting with the development of high performance computers with several thousand of cores and with more and more complex hierarchical communication networks. In these context the use of global reduction operation such as the dot products involving in the GMRES acceleration can be the bottleneck for the performance. For asynchronous RAS , some boundary conditions of some artificial interfaces generated by the domain decomposition cannot been updated for some iterates. In these context the Aitken acceleration should not be applicable as it is based on the pure linear convergence of the RAS., i.e. there exists a linear operator P independant of the iteration that links the error with the searched solution at the artificial interfaces of two consecutives RAS iterates. In the context of asynchronous iteration we do not have a priori the pure linear convergence property as some boundary conditions have not been updated (randomly). We develop a mathematical model of the Asynchronous RAS allowing to set the percentage of the number of the randomly chosen local artificial interfaces where boundary conditions are not updated. Then we show how this ratio deteriorates the convergence of the Asynchronous RAS and how some regularization techniques on the traces of the iterative solutions at artificial interfaces allow to accelerate the convergence to the searched solution.

20:30–21:00 **Asynchronous One- and Two-Level Domain Decomposition Solvers**

*Christian Glusa**

Parallel implementations of linear iterative solvers generally alternate between phases of data exchange and phases of local computation. Increasingly large problem sizes on more heterogeneous systems make load balancing and network layout very challenging tasks. In particular, global communication patterns such as inner products become increasingly limiting at scale. In this talk, we explore the use of asynchronous domain decomposition solvers based on one-sided MPI primitives. We will discuss practical issues encountered in the development of a scalable solver and show experimental results obtained on a variety of state-of-the-art supercomputer systems.

21:00–21:30 **Algebraic View of Optimized Schwarz Methods**

*Daniel B. Szyld**

Optimized Schwarz methods (OSM) are based on domain decomposition where the transmission conditions on the artificial interfaces are of Robin type, i.e., with a parameter which can be optimized. We discuss new proofs of convergence of an asynchronous version of OSM which are completely algebraic, that is, they apply to matrices which may or may not come from discretizations of differential equations. We assume optimal transmission conditions on the artificial interfaces.

21:30–22:00 **Asynchronous Two-Level Optimized Schwarz Method**

*Faycal Chaouqui** ⋄ *Daniel B. Szyld*

Exascale computing with hundreds of thousands of cores is around the corner. Asynchronous iterations bring promise since they avoid waiting for idle time, i.e., time for which some processors are not being used but could be. We explore here such iterations for the Optimized Schwarz Method for the solution of large sparse linear systems arising from the discretization of PDEs. In particular, we treat one and two-level cases. The latter ensures weak scaling. Moreover, a particular coarse grid is considered in order to ensure the convergence of the two-level method iteratively, and hence it is more suitable in the asynchronous framework. We provide some numerical experiments that illustrate our proposed approach.

MS13 – Domain Decomposition Methods for Optimal Control Problems

Organizers: Gabriele Ciaramella ⋄ Felix Kwok

Optimal control and PDE-constrained optimization problems are of fundamental importance in several application areas. For these reasons, an intense research is boosting the development of efficient numerical strategies capable to deal with large-scale optimization and control problems. Among these strategies, domain decomposition methods (DDMs) represent one of the most powerful and probably the most versatile techniques for the efficient solution of large-scale problems emerging in different disciplines. On the one hand, DDMs are effective linear solvers and efficient preconditioners. On the other hand, the incredible versatility of DDMs has boosted their use for the efficient numerical treatment of heterogeneous multi-physics problems, problems in computational chemistry, optimization and optimal control problems.
In this minisymposium, recent advances of domain decomposition methods for the efficient solution of PDE-constrained optimization and control problems are presented. A particular focus will be put on time and space parallel methods, linear and non-linear preconditioning, and optimized transmission conditions. All these aspects and new research perspectives will be posed and discussed in light of the most recent developments.

Friday 20:00–22:00

20:00–20:30 **A Time Parallelisation Method for Identification.**

*Julien Salomon**

Several methods have been developed over the last twenty years to parallelise the solving of the evolution equation with respect to time. The great majority of these methods are based on the decomposition of the resolution interval into sub-intervals, processed in parallel. These methods are therefore not directly applicable to situations in which no relevant solving interval is known a priori. This is the case of identification problems, where one intends to reconstruct the state of a system from partial data, assuming the known model. A usual algorithm for this kind of problem consists in using an Luenberger observer, i.e. adding a data attachment term to the model. Suitable assumptions can then be made to prove the convergence of the observer to the observed system. We will present a time parallelization method adapted to this type of problem. Our approach is based on a two-stage partitioning of the information flow which allows to obtain convergence without additional assumptions. In addition and despite the addition of a parallelization loop, this method preserves the convergence rate of the observer under consideration.

20:30–21:00 **Optimized Schwarz Method for Diffusion-Reaction PDE-Constrained Optimal Control Problems**

Luca Mechelli ⋄ Gabriele Ciaramella ⋄ Laurence Halpern ⋄ Reinhard Racke*

We study the convergence behavior of the optimized Schwarz method for computing the solution of the optimality system of a 1D diffusion reaction PDE-constrained optimal control problem with a quadratic cost functional. We first show the impossibility of using a continuous Fourier analysis in time, then we perform a discrete Fourier analysis, proving the convergence of the method. Furthermore, we compute an approximation of the optimal Robin parameter to achieve best performances in terms of convergence. Finally, numerical tests confirm the predicted behavior of the method.

21:00–21:30 **Optimized Schwarz Methods for Complex Elliptic Problems**

*Bérangère Delourme**

The solution of optimal control problems with distributed control leads to, according to Benamou-Després, the solution of an elliptic problem with pure imaginary coefficients. Domain decomposition methods with complex Robin coefficients have been proposed and analyzed from various points of view.
We extend here the method to a more general complex elliptic problem. For both overlapping

and not overlapping subdomains,we give a complete analysis of the best approximation problem, which optimizes the convergence rate of the algorithm. In particular, we prove the existence of an optimal complex parameter and we obtain practical analytic and asymptotic formulas, useful as the mesh size tends to 0.

21:30–22:00 **Nonlinear Precondtioners for a PDE-Constrained Optimization Problems**

*Gabriele Ciaramella** ⋄ *Felix Kwok*

In the last decades, impressive progress has been made in the numerical solution of PDE-constrained optimization problems. However, solving such problems is not an easy task if highly non-linear PDEs are considered or if non-differentiable penalization terms are included in the cost functional. In these cases, classical Newton-type methods need strong initialization and globalization procedures to converge. However, these are generally computationally expensive. In this talk, we propose a new class of nonlinear domain decomposition preconditioners that greatly improve the convergence of classical Newton-type methods. Extensive numerical experiments demonstrate the effectiveness of the proposed new numerical framework.

Saturday 20:00–21:00

20:00–20:30 **Time-Parallelization of Data Assimilation Problems**

*Sebastian Reyes-Riffo** ⋄ *Felix Kwok* ⋄ *Julien Salomon*

We propose a procedure to couple unbounded in time data assimilation problems with time-parallel algorithms. In order to do so, we design a procedure called *Diamond strategy*, that consists in splitting the infinite time interval into bounded *windows* and then apply, following a sequential order, the time-parallel solver on each. The problem we consider deals with the identification of a system state by using a Luenberger observer. This strategy gives rise to an exponential rate of convergence, that we aim at preserving when coupling it with the Parareal algorithm. We provide a way to estimate, on each window, the number of parareal iterations required to preserve the observer rate of convergence. In addition, we estimate the theoretical efficiency of the whole procedure.

20:30–21:00 **Space-Time Finite Element Solvers for Parabolic Optimal Control Problems**

*Huidong Yang** ⋄ *Ulrich Langer* ⋄ *Olaf Steinbach* ⋄ *Fredi Tröltzsch*

In this talk, we will present some numerical methods for optimal control of parabolic PDEs. In particular, we aim to minimize certain objective functionals subject to linear and nonlinear parabolic PDEs, and with proper constraints on the control variables. The objective functional may involve a Lipschitz continuous and convex but not Fréchet differentiable term, and lead to spatio-temporally sparse optimal control. The space-time finite element discretization of the optimality system, including both the state and adjoint state equations, relies on a Galerkin-Petrov variational formulation employing piecewise linear finite elements on unstructured simplicial space-time meshes. The nonlinear optimality system is solved by means of the semismooth Newton method, whereas the linearized coupled state and adjoint state systems are solved by an algebraic multigrid preconditioned GMRES method.

MS14 – Adaptive Coarse Spaces, Multipreconditioning and Reduced Basis Methods

Organizers: Martin J. Gander ⋄ Axel Klawonn ⋄ Oliver Rheinbach

In classical domain decomposition methods, coarse spaces are used to provide scalability. It was discovered however over the past decade that coarse spaces can do much more: they can make domain decomposition methods robust for problems with large contrasts in the coefficients, they can treat error components for which the underlying domain decomposition method is not effective, and they can even transform divergent domain decomposition methods into convergent ones, without Krylov acceleration. These new coarse spaces use techniques from multiscale finite elements, and are also related to reduced basis methods and multi preconditioning. This minisymposium brings researchers working on these various aspects together to exchange the most recent ideas in this field.

Saturday 20:00–22:00

20:00–20:30 **Adaptive Coarse Spaces for Schwarz Methods Based on Decompositions of the Domain Decomposition Interface**

Alexander Heinlein ⋄ Axel Klawonn ⋄ Jascha Knepper ⋄ Oliver Rheinbach ⋄ Olof B. Widlund*

We propose robust coarse spaces for two-level overlapping Schwarz preconditioners which are based on decompositions of the domain decomposition interface. These coarse spaces can be seen as extensions of energy-minimizing coarse spaces of GDSW (Generalized–Dryja–Smith–Widlund) type. The resulting two-level methods are robust for second-order elliptic problems, even in presence of highly heterogeneous coefficient functions, and reduce to standard GDSW type algorithms if no additional coarse basis functions are used. Numerical results for heterogeneous diffusion and linear elasticity model problems in two and three dimensions are presented for structured as well as unstructured domain decompositions.

20:30–21:00 **Convergence Properties of New Coarse Spaces for the Additive Schwarz Method**

*Martin J. Gander ⋄ Bo Song**

The Additive Schwarz method (AS) does not converge in general when used as a stationary iterative method, it was designed to be used as a preconditioner for a Krylov method. In the two level variant of AS, a coarse grid correction is added to make the method scalable. We introduced in DD24 the notation of complete, optimal and optimized coarse spaces for AS, and derived a convergence factor of our new two level method in the two subdomain case in two spatial dimensions. This showed that using a coarse correction, AS can be made into a convergent stationary iterative solver. In this talk, we will present a general convergence result of our new two level AS method in the many subdomain case without cross points in two spatial dimensions. We will also give a theoretical comparison of our new coarse spaces with GenEO recently proposed by Spillane, Dolean, Hauret, Nataf, Pechstein, and Scheichl. We will illustrate our theoretical results with numerical experiments.

21:00–21:30 **Adaptive Reduced Basis Domain Decomposition Methods**

Andreas Buhr ⋄ Martin Gander ⋄ Mario Ohlberger ⋄ Stephan Rave ⋄ Felix Schindler*

Snapshot-based model order reduction techniques such as Reduced Basis (RB) methods have been successfully applied in a wide range of application areas to obtain parametrized reduced order models which act as efficient high-quality surrogates for the full order model in many-query or realtime simulation applications. Recently, several localized RB techniques have been introduced, e.g. [OhlbergerSchinder2015], [BuhrEngwerOhlbergerRave2017], where local RB approximation spaces are constructed by a spatial partitioning of the computational domain and solving adequate local problems on the individual subdomains. While originally motivated by the theory of numerical multiscale methods, these schemes show striking similarities with two-level Schwarz methods: The adaptation procedures in localized RB schemes yield approximation spaces strongly related to the spaces appearing in multi-preconditioned Krylov methods. The offline initialization of the local RB spaces is closely related to the construction of multiscale coarse spaces such as DtN, GenEO or SHEM. In this talk I will give an overview on (localized) RB techniques and discuss their relation with DD methods. In particular, I will discuss the convergence of localized RB schemes from the perspective of DD methods.

21:30–22:00 **On Optimal Coarse Grid Correction for the Optimized Schwarz Method**

*Faycal Chaouqui** ⋄ *Daniel B. Szyld*

We present a new optimal coarse space correction for the optimized Restricted Additive Schwarz method. We use coarse spaces defined by harmonic extensions of interface and surface functions to the subdomains' interior. In particular, we show that these coarse spaces yield convergence in a single iteration when fully used. We then explain how to choose and implement approximations of these coarse spaces utilizing the operator's spectral information. Numerical examples are provided to illustrate the performance of the ideas presented.

MS15 – Domain Decomposition Methods for Uncertainty Quantification and Applications

Organizers: Xuemin Tu ⋄ Qifeng Liao

Most numerical simulations of physical systems are with different sources of uncertainties, which include geometrical uncertainty, initial and boundary data uncertainty, model structural uncertainty, parametric uncertainty and so on. The computational methods to study the uncertainty propagation, interplay, and predictions are usually much more expensive than the original forward model simulations. Domain decomposition methods have made significant progress on the forward model simulations. In this mini-symposium, we present recent research in computational methods for uncertainty quantification based on the domain decomposition methods.

Thursday 20:00–21:30

20:00–20:30 **Domain Decomposition for Implicit Sampling Methods**

Xuemin Tu ⋄ Yiying Cheng*

In this talk, we focus on the uncertainty quantification of inverse problem, under the Bayesian framework. The estimation is given by a posterior probability density. Implicit sampling method can be used to sample the posterior density, which combines the prior information about the parameter with the noisy data. For large scale problems, sampling the posterior can be an extremely challenging task. Using a simple example, we show that the domain decomposition methods can be used to improve the efficiency of the implicit sampling methods in several ways.

20:30–21:00 **A Class of Conforming Multiscale Finite Element Method for Elliptic Problems with Multiscale Coefficients**

Zhiwen Zhang ⋄ Houde Han*

We develop a conforming Multiscale finite element method for second order elliptic equations with rough or highly oscillatory coefficients. Our method is based on an iterative approach that enables us to obtain more accurate boundary conditions for the local cell problems. Several numerical examples are provided to show the accuracy and convergence of the proposed method.

21:00–21:30 **D3M: A Deep Domain Decomposition Method for Partial Differential Equations**

Ke Li ⋄ Kejun Tang ⋄ Tianfan Wu ⋄ Qifeng Liao*

A state-of-the-art deep domain decomposition method (D3M) based on the variational principle is proposed for partial differential equations (PDEs). The solution of PDEs can be formulated as the solution of a constrained optimization problem, and we design a hierarchical neural network framework to solve this optimization problem. Our contribution is to develop a systematical computational procedure for the underlying problem in parallel with domain decomposition. Our analysis shows that the D3M approximation solution converges to the exact solution of the underlying PDEs. The accuracy and the efficiency of D3M are validated and demonstrated with numerical experiments.

Friday 10:00–12:00

10:00–10:30 **The Direct Method of Lines for Elliptic Problems in Star-Shaped Domains**

Zhizhang Wu ⋄ Zhongyi Huang ⋄ Wei-Cheng Wang ⋄ Yi Yang*

We generalize the direct method of lines for solving elliptic problems with singularities. We first deduce the variational-differential form of the anisotropic Laplace equation in the curvilinear coordinate and obtain the direct method of lines for the Laplace equation in the anisotropic case; and then we discuss how the direct method of lines should cope with Neumann boundary conditions as exterior boundary conditions for problems in a general star-shaped domain; and subsequently the application of the direct method of lines on the Poisson equation is deduced; and by combining the idea of domain decomposition, the direct method of lines for elliptic problems with multiple singular points is proposed in the end. Numerical results show that the generalized direct method of lines can effectively solve these kinds of singular elliptic problems and maintain the advantages of the original method.

10:30–11:00 **A Model Reduction for Nonlinear Multiscale Parabolic Problems**

Lijian Jiang ⋄ *Mengnan Li**

In this talk, I present a model reduction technique for solving nonlinear multiscale parabolic problems. The proposed method combines Constraint Energy Minimizing Generalized Multiscale Finite Element Method (CEM-GMsFEM) and Dynamic Mode Decomposition(DMD) to reduce the computational complexity. The CEM-GMsFEM is used to represent the solution on a coarse grid with multiscale basis functions computed offline. Using the CEM-GMsFEM to solve nonlinear multiscale model involves calculating the residual and the Jacobian on a fine grid. This is computationally expensive because the evaluation of the nonlinear term requires computing the full-dimensional model. To overcome the challenge, the DMD method is used to estimate nonlinear term and decomposes the nonlinear system into spatiotemporal coherent structures for short-term future state prediction. The proposed approach avoids the evaluation of the nonlinear term in the online stage. In order to achieve a full coarse model for the nonlinear problem, we utilize a coarse-scale observation in DMD. CEM-DMD uses models and data to simulate multi-scale nonlinear parabolic equations. When the model is unknown, we use Koopman operator and deep learning to predict the evolution of multi-scale nonlinear dynamic system only by using data.

11:00–11:30 **Clustered Active-Subspace Based Gaussian Process Emulator for High-Dimensional and Complex Models**

*Junda Xiong** ⋄ *Jinglai Li*

Quantifying uncertainties in physical or engineering systems often requires a large number of simulations of the underlying computer models that are computationally intensive. Emulators or surrogate models are often used to accelerate the computation in such problems, and in this work we focus on the Gaussian Process (GP) emulator for its ability to quantify the approximation error in the emulator itself. However, a major limitation of the GP emulator is that it can not handle problems of very high dimensions, and to address the issue we employ the active subspace method to reduce the dimensionality of the problem and then construct the GP emulator in the reduced space. In particular, to address the issue that the simulation model may admit different low dimensional structures in different parameter regimes, we propose a clustered active subspace method which identifies the local low-dimensional structures as well as the parameter regimes they are in (represented as clusters), and then construct low dimensional and local GP emulators within the clusters. Specifically we design a clustering method based on the gradient information to identify these clusters, and a local GP construction procedure to build the GP emulator within a local cluster. With numerical examples, we demonstrate that the proposed method is effective when the underlying models are of complex low dimensional structures.

11:30–12:00 **Domain Decomposed Uncertainty Analysis Based on RealNVP**

Ke Li ⋄ *Kejun Tang* ⋄ *Qifeng Liao**

The domain decomposition uncertainty quantification method (DDUQ) (SIAM J. SCI. COMPUT (37) pp. A103-A133) provides a decomposed strategy to conduct uncertainty analysis for complex engineering systems governed by PDEs. In DDUQ, uncertainty analysis on each local component is independently conducted in an "offline" phase, and global uncertainty analysis results are assembled using precomputed local information in an "online" phase through importance sampling. The performance of DDUQ relies on the coupling surrogates and probability density estimation during the importance sampling procedure. Since coupling surrogates can give high-dimensional interface parameters, we in this work develop a RealNVP based interface coupling strategy, which dramatically improve the efficiency of DDUQ.

MS16 – Polygonal Finite Elements, DG, and Related Methods

Organizers: Eun-Jae Park ◇ Dong-wook Shin ◇ Lina Zhao

Recently, polygonal finite elements have received great attention. Polygonal and polyhedral meshes offer a very convenient framework for mesh generation, mesh refinements and coarsening, mesh deformations, fracture problems, composite materials, and topology optimizations. This mini-symposium is about Polygonal finite elements, DG, and related methods. This includes fast numerical methods such as domain decomposition and multigrid. We hope that this MS is to bring together experts as wells as junior researchers to discuss new types of questions at the foundation and applications.

Thursday 16:00–18:00

16:00–16:30 **Pointwise A-Posteriori Error Analysis for a Discontinuous Galerkin Approximation of an Elliptic Obstacle Problem**

Blanca Ayuso de Dios ◇ Kamana Porwal ◇ Thirupathi Gudi*

We consider a piecewise linear discontinuous Galerkin finite element method for an elliptic obstacle problem over polyhedral domains in $\mathbb{R}^d$, $d = 2, 3$ which enforces the unilateral constraint solely at the nodes. We derive upper and lower a posteriori estimates for the maximum norm error which generalize the standard residual-type estimators for unconstrained problems by additional terms addressing the non-linearity. To avoid geometric mesh constraints (typical of the a priori error analyses) the analysis hinges on the construction of suitable barrier functions by correcting an appropriate averaged discrete solution and an application of the continuous maximum principle. We present several numerical experiments in $d = 2$ to verify the theory and illustrate the reliability and efficiency properties of the proposed estimators.

16:30–17:00 **Agglomeration-Based Solvers for High-Order Discontinuous Galerkin Methods on Polygonal and Polyhedral Grids**

Paola F. Antonietti ◇ Paul Houston ◇ Giorgio Pennesi ◇ Endre Süli*

In this talk, we present a survey of agglomeration-based fast solution algorithms for high-order discontinuous Galerkin finite element methods which employ general polygonal/polyhedral elements. In particular, we present and analyse a family of multigrid schemes on nested and non-nested agglomerated meshes [1] as well as Schwarz-type domain decomposition preconditioners [3]. The performance of the proposed solvers will be tested on both two- and three-dimensional test cases. The design of quadrature rules over general polygonal/polyhedral elements, that is a key ingredient in the design and efficient implementation of fast solvers, will also be discussed. More precisely, we present efficient quadrature rules for the numerical approximation of integrals of polynomial functions over general polygonal/polyhedral elements that do not require an explicit construction of a sub-tessellation into triangular/tetrahedral elements [2].

References

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[3] Antonietti, P.F. and Houston, P. and Pennesi, G. and Süli, E. An agglomeration-based massively parallel non-overlapping additive Schwarz preconditioner for high-order discontinuous Galerkin methods on polytopic grids, *Math. Comp.* (2020), 89:2047–2083.

17:00–17:30 **An Analysis of Staggered DG Method for Coupling of the Stokes and Darcy-Forchheimer Problems**

*Lina Zhao** ⋄ *Eric Chung* ⋄ *Eun-Jae Park* ⋄ *Guanyu Zhou*

In this talk we present a staggered discontinuous Galerkin method for the Stokes and Darcy-Forchheimer problems coupled with the Beavers–Joseph–Saffman conditions. Staggered continuity properties are imposed for all the variables involved and the interface conditions are enforced by switching the roles of the variables met on the interface, which eliminate the hassle of introducing additional variables. Our method earns the salient features that it can be flexibly applied to rough grids such as the highly distorted grids and the polygonal grids, in addition, hanging nodes can be automatically incorporated in our method. In addition, the method allows nonmatching grids on the interface thanks to the special inclusion of the interface conditions, which is highly appreciated from a practical point of view. A new discrete trace inequality and a generalized Poincaré-Friedrichs inequality are proved, which enables us to prove the optimal convergence estimates under reasonable regularity assumptions. Finally, several numerical experiments are provided to illustrate the good performances of the proposed method and the numerical results indicate that the proposed method is accurate and efficient.

17:30–18:00 **Staggered Discontinuous Galerkin Methods for the Stokes Equations on General Polygonal Meshes**

*Dohyun Kim** ⋄ *Eun-Jae Park* ⋄ *Lina Zhao*

In this talk, we introduce arbitrary high order staggered discontinuous Galerkin (SDG) method on general polygonal meshes for the pseudostress-velocity formulation of the Stokes equations. Pseudostress-velocity formulation of the Stokes equations has two variables and therefore, computationally less expensive compared to stress-velocity-pressure formulation. SDG methods use a subspace of piecewise polynomial space on sub-triangles derived from the polygonal mesh. We impose a staggered continuity for each discrete function space which makes SDG methods stable without special treatment. Hybridization techniques are introduced so that the standard nodal basis can be used and the size of the resulting linear system is reduced. Stability and optimal convergence for all variables will be provided. Several numerical experiments will be presented to verify theoretical results and some observations on robustness to mesh distortion will be given.

Friday 10:00–12:00

10:00–10:30 **Primal-Dual Weak Galerkin Finite Element Methods**

*Junping Wang**

The weak Galerkin (WG) finite element method is a generic numerical method for partial differential equations. The essence of WG is to reconstruct differential operators in the usual variational forms for the PDE by using a framework that mimics the theory of distributions for piecewise polynomials. The regularity requirements (such as H^1, H^2, $H(div)$, or $H(curl)$) for the underlying approximating functions are compensated by some carefully-designed stabilizers (or smoothers). The framework yields discrete weak differential operators (e.g., weak gradients, weak curl, weak Laplacian etc) which are employed for PDE discretization. The computation of the discrete differential operators involves the solution of inexpensive problems defined locally on each element. Due to its structural flexibility, the WG finite element method is well suited to most PDEs by providing the needed stability and accuracy. In this talk, the speaker will discuss a primal-dual weak Galerkin (PD-WG) approach for some model PDE problems for which the usual numerical methods are difficult to apply. The speaker will first demonstrate the basic ideas of PD-WG by using a linear transport problem, and will then apply the method div-curl systems with either tangential or normal boundary conditions.

10:30–11:00 **A High Order Discontinuous Galerkin Method with Skeletal Multipliers for Convection-Diffusion-Reaction Problems**

*Mi-Young Kim** ⋄ *Dong-wook Shin*

A high order discontinuous Galerkin method with skeletal multipliers (DGSM) is developed for diffusion problem. Skeletal multiplier is introduced on the edge/face of each element through the definition of a weak divergence and weak derivative in the method. The local weak formulation is derived by weakly imposing the Dirichlet boundary condition and continuity of fluxes and solutions on the edges/faces. The global weak formulation is then obtained by adding all the local problems. Equivalence of the weak formulation and the original problem is proved. Stability of DGSM is shown and an error estimate is derived in a broken norm. A DGSM for linear convection-diffusion-reaction problems is also derived. An explanation on algorithmic aspects is given. Some numerical results are presented. Singularities due to discontinuities in the diffusion coefficients are accurately approximated. Internal/boundary layers are well captured without showing spurious oscillations. Robustness of the method in increasingly small diffusivity is demonstrated on the whole domain.

11:00–11:30 **A FETI-DP Formulation for the Stokes Problem with a Discontinuous Viscosity**

*Eun-Hee Park** ⋄ *Hyea Hyun Kim*

In this talk we will discuss a non-overlapping domain decomposition method for the Stokes problem with a discontinuous viscosity. There are two key ingredients in the proposed method: one is an inf-sup stable finite element for the Stokes problem and the other is a preconditioning procedure based on the FETI-DP approach. Theoretical results for the condition number estimate of the preconditioned problem will be presented along with numerical results.

11:30–12:00 **A Hybrid Difference Method for the Second-Order Elliptic Problems**

*Dong-wook Shin** ⋄ *Eun-Jae Park*

In this talk, we consider a hybrid difference method (HDM) for the second-order elliptic problems. This method can be viewed as a finite difference version of hybrid discontinuous Galerkin methods (Jeon-Park, SIAM J. Numer. Anal., 2010). The HDM named the hybrid spectral difference (HSD) method was introduced and analyzed by Jeon-Park-Shin (Comput. Methods Appl. Math., 2017). The HSD method is locally conservative and allows arbitrarily high-order approximations. Also, the method has great reduction in global degrees of freedom and its convergence is proved in the discrete energy norm. However, this is not conforming method and needs a postprocessing to evaluate the approximation value at arbitrary point in the domain in $\mathbb{R}^d$ ($d = 2, 3$). In the present work, the HSD method is developed and generalized to be a conforming method and to prove the optimal rate of convergence in the semi H^1 norm. Numerical results are presented to show the performance of the proposed method, which support our theoretical findings.

MS17 – Nonlinear Preconditioning and Applications

Organizers: *Xiao-Chuan Cai* ◇ *Martin J. Gander*

Preconditioning is an established field of research in the case of linear problems, and there is even a conference dedicated to this topic: Preconditioning 2019 was the 11th in the series last year in Minnessotta. Preconditioning of nonlinear problems is however much less explored. Since the introduction of ASPIN (Additive Schwarz Preconditioned Inexact Newton), more and more people started to investigate nonlinear preconditioning techniques. The goal of this minisymposium is to bring together researchers in this field and to obtain an overview of the most recent developments of the algorithms and applications.

Thursday 16:00–18:00

16:00–16:30 **Nonlinear Preconditioning Strategies for CFD**

Li Luo ◇ *Xiao-Chuan Cai** ◇ *David Keyes*

We consider numerical computation of several fluid flows whose solutions have certain level of singularities induced by either the geometry and/or the material coefficients. This includes driven cavity flows at high Reynolds number, blood flows in arteries with stenosis, and two-phase flows in porous media. The classical inexact Newton method doesn't converge well for all these problems because of the non-smoothness in the sequence of approximate solutions created by the inexact Newton iteration. We discuss several versions of the nonlinear elimination preconditioning strategy tailored to the particular types of singularities in the fluid flows.

16:30–17:00 **Cycles in Newton-Raphson-Accelerated Alternating Schwarz**

*Conor McCoid** ◇ *Martin J. Gander*

Newton-Raphson may be used to accelerate Schwarz methods. While this can speed up convergence, it can also cause the methods to exhibit problems particular to Newton-Raphson. This talk explores several examples where acceleration of alternating Schwarz by Newton-Raphson leads to period doubling cycles. We also examine the sufficient conditions for convergence and how these might be used to create more robust algorithms.

17:00–17:30 **Additive and Hybrid Nonlinear Two-Level Schwarz Methods and Energy Minimizing Coarse Spaces for Unstructured Grids**

*Martin Lanser** ◇ *Alexander Heinlein*

Nonlinear domain decomposition (DD) methods, such as, e.g., ASPIN (Additive Schwarz Preconditioned Inexact Newton), RASPEN (Restricted Additive Schwarz Preconditioned Inexact Newton), Nonlinear-FETI-DP, or Nonlinear-BDDC methods, can be reasonable alternatives to classical Newton-Krylov-DD methods for the solution of sparse nonlinear systems of equations, e.g., arising from a discretization of a nonlinear partial differential equation. These nonlinear DD approaches are often able to effectively tackle unevenly distributed nonlinearities and outperform Newton's method with respect to convergence speed as well as global convergence behavior. Furthermore, they often improve parallel scalability due to a superior ratio of local to global work. Nonetheless, as for linear DD methods, it is often necessary to incorporate an appropriate coarse space in a second level to obtain numerical scalability for increasing numbers of subdomains. In addition to that, an appropriate coarse space can also improve the nonlinear convergence of nonlinear DD methods. In this talk, four variants how to integrate coarse spaces in nonlinear Schwarz methods in an additive or multiplicative way using Galerkin projections are introduced. These new variants can be interpreted as natural nonlinear equivalents to well-known linear additive and hybrid two-level Schwarz preconditioners. Furthermore, they facilitate the use of various coarse spaces, e.g., coarse spaces based on energy-minimizing extensions, which can easily be used for irregular domain decompositions, as, e.g., obtained by graph partitioners. In particular, Multiscale Finite Element Method (MsFEM) type coarse spaces are considered, and it is shown that they outperform classical approaches for certain heterogeneous nonlinear problems.

17:30–18:00 **Error Bounds on Solutions of PDEs**

*Lulu Liu** ⋄ *David Keyes*

In many multiphysics applications, the ultimate quantity of interest can be written as a linear functional of the solution to the discretized governing nonlinear partial differential equations and finding a sufficiently accurate pointwise solution may be regarded simply as a step toward that end. We derive *a posteriori* error bounds for linear functionals corresponding to quantities of interest using two kinds of nonlinear preconditioning techniques. Nonlinear preconditioning, such as INB-NE and MSPIN, may be effective in improving global convergence for Newton's method. It may allow to avoid stagnation of the nonlinear residual norm and reduce the number of solutions of large ill-conditioned linear systems involving a global Jacobian otherwise required at each nonlinear iteration. We illustrate the effectiveness of the new bounds using canonical nonlinear PDE models: a flame sheet model and a nonlinear coupled lid-driven cavity problem.

Friday 16:00–18:00

16:00–16:30 **Nonlinear FETI-DP - Tailoring the Nonlinear Elimination Set**

*Axel Klawonn** ⋄ *Martin Lanser* ⋄ *Oliver Rheinbach* ⋄ *Matthias Uran*

Highly scalable and robust Newton-Krylov domain decomposition approaches are widely used for the solution of nonlinear implicit problems, e.g., in structural mechanics. In general, in these methods, the nonlinear problem is first linearized and afterwards decomposed into subdomains. By changing this order, i.e., by first decomposing the nonlinear problem, many nonlinear domain decomposition methods have been designed in the last two decades. These methods often show a higher robustness compared with classical Newton-Krylov variants due to a better resolution of local nonlinear effects by nonlinear subdomain problems. Additionally, the balance between local work, communication, and synchronization is usually more favorable for modern computer architectures. In this talk, nonlinear FETI-DP (Finite Element Tearing and Interconnecting - Dual Primal) and BDDC (Balancing Domain Decomposition by Constraints) methods - which are formulated in a common framework and can both be interpreted as nonlinear right-preconditioners - will be discussed. Especially in Nonlinear-FETI-DP, the choice of the so-called nonlinear elimination set, e.g., the set of degrees of freedom which are eliminated in a nonlinear fashion, and the choice of the coarse space have a huge impact on the nonlinear convergence behavior of the method. These effects will be discussed and an automatic selection of the nonlinear elimination set will be presented.

16:30–17:00 **Minisymposium MS17: On the Choice of Robin Parameters for the Optimized RASPEN Method**

*Yaguang Gu** ⋄ *Felix Kwok*

In this talk, we consider the nonlinear elliptic equations which are of the form $-\nabla \cdot (a(u)\nabla u) = f$. We present an optimized RASPEN (ORASPEN) method, which is similar to ASPIN and RASPEN, but uses optimized transmission conditions. Note that for nonlinear problems, the diffusivity depends on the unknown itself. A constant Robin parameter, i.e., $p = const.$, is therefore not optimal for problems with large jumps in diffusivity. Naturally, we consider using a spatially varying parameter p that is proportional to the diffusivity. Furthermore, we also consider making the parameter p adapted as a function of the solution and linearize it within Newton steps. Finally, we present numerical examples showing an improvement in convergence behaviour when adaptive Robin parameters are used.

17:00–17:30 **Nonlinear Preconditioned Semismooth Newton Algorithms for Nonsmooth Systems**

Haijian Yang ⋄ *Feng-Nan Hwang** ⋄ *Xiao-Chuan Cai*

This talk aims to study a nonlinearly preconditioned semismooth Newton algorithm for solving nonsmooth large, sparse, nonlinear systems of equations. Such problems represent a broad range of applications in computational sciences and engineerings, such as complementarity problems, variational inequality problems, and inequality-constrained optimization problems that can be also reformulated as systems of non-smooth equations. Addition to the high dimensionality, such characteristics make the optimization problems more challenging to solve. Due to its strong theoretical foundation and also reliability and efficiency from the numerical aspects, the semi-smooth Newton method is one of the most popular methods for the non-smooth system. However, it stills suffers from the convergence issue when the nonlinearity of the system is not well balanced. Nonlinear preconditioning technique provides alternative other than globalization techniques, e.g., linesearch or trust region not only to enhance the robustness of Newton type method but also to improve the efficiency of the method. In this talk, we first use a simple example to illustrate the idea of right nonlinear preconditioning that can be interpreted as the nonlinear change of coordinates so that the problem can be easier solved on the new coordinate than on the original one. Next we introduce a nonlinear elimination preconditioner in conjunction with semismooth Newton algorithms followed by showing some numerical results. The key point of new proposed algorithm is that before performing the global Newton update, we first identify the to-be-eliminated components that causes Newton method's slow convergence, and then remove the component corresponding to the strong local nonlinearity by using a subspace correction. Using the obstacle problem and the flow control problem with inequality constraints as numerical examples, we show that the new approach is more robust and efficient than the standard semismooth Newton method.

17:30–18:00 **Substructured Nonlinear Preconditioning**

*Pratik M. Kumbhar** ◇ *Faycal Chaouqui* ◇ *Martin J. Gander* ◇ *Tommaso Vanzan*

When solving nonlinear partial differential equations, domain decomposition (DD) methods can be used in two ways: one can use them directly as nonlinear iterative solvers [1], or as preconditioners for Newton's method [2]. Dolean et al proposed in [3] to use Restricted Additive Schwarz (RAS) as a nonlinear preconditioner for Newton's method, in a spirit very similar to what is done to precondition linear systems for Krylov methods. They called their method RASPEN (Restricted Additive Schwarz Preconditioned Exact Newton). In this method, all the components are defined directly by the underlying iterative method in volume. In this talk, we first introduce a substructured version of RASPEN, and study carefully the impact of the substructuring on implementation details and performance. We then introduce and analyze new substructured nonlinear Dirichlet-Neumann and Neumann-Neumann preconditioners for Newton's method. We present a computationally efficient way of calculating the Jacobian matrices needed in all these substructured methods. Numerical experiments confirm the validity of our approach, showing a significantly better performance in terms of computational time and convergence properties of the substructured approach compared to RASPEN in volume.
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20:00–20:30 **A Nonlinearly Preconditioned Inexact Newton Method Based on Nonlinear Elimination**

*Lulu Liu ◇ Feng-Nan Hwang ◇ Li Luo ◇ David Keyes**

Nonlinear preconditioning techniques such as ASPIN and MSPIN deal with strong nonlinearities by replacing the nonlinear functions. However, the Jacobian of the preconditioned system is expressible only in the form of matvecs and it is therefore a challenge to precondition further. In contrast, we propose a new preconditioned inexact Newton method based on nonlinear elimination. It keeps the form of Jacobian corresponding to the original nonlinear function, and provides a better globally corrected Newton direction by removing the local high nonlinearity in the Jacobian system. In this paper, we illustrate the robustness and effectiveness of the proposed algorithm using the transonic full potential equation.

20:30–21:00 **A Multi-Layer Nonlinear Elimination Preconditioned Inexact Newton Method for a Steady-State Incompressible Flow in 3D**

Li Luo ◇ Xiao-Chuan Cai ◇ Zhengzheng Yan ◇ Lei Xu ◇ David Keyes*

We develop a multi-layer nonlinear elimination preconditioned inexact Newton method for nonlinear algebraic system of equations, and illustrate on a three-dimensional steady-state incompressible Navier-Stokes flow at high Reynolds numbers. Nonlinear steady state problems are often more difficult to solve than time-dependent problems because the Jacobian matrix is less diagonally dominant, and a good initial guess from the previous time step is not available. For such problems, Newton-like methods may suffer from slow convergence or stagnation even with globalization techniques such as line search. We introduce a cascadic multi-layer nonlinear elimination approach based on feedback from intermediate solutions to improve the convergence of Newton iteration. Numerical experiments show that the proposed algorithm is superior to the classical inexact Newton method and other single layer nonlinear elimination approaches in terms of the robustness and efficiency. Using the proposed nonlinear preconditioner with a highly parallel domain decomposition framework, we demonstrate that steady solutions of the Navier-Stokes equations with Reynolds number as large as 7,500 can be obtained for the lid-driven cavity flow problem in 3D without the use of any continuation methods.

21:00–21:30 **A True Matrix-Free Multigrid Preconditioner for Globalized Jacobian-Free Newton-Krylov Methods**

Hardik Kothari ◇ Alena Kopaničáková ◇ Rolf Krause*

Jacobian-free Newton-Krylov (JFNK) methods are popular in practice for solving large-scale nonlinear problems. They provide convergence properties of Newton's method without explicit knowledge of the Jacobian. The method is especially useful when the Jacobian is not trivial to compute or computationally expensive to evaluate and/or to store. At each nonlinear iteration, JFNK employs a matrix-free Krylov subspace method in order to solve the arising linear systems of equations. The convergence rate of Krylov methods usually deteriorates as problem-size increases, unless a robust preconditioning strategy is employed. There have been many attempts to create a preconditioner for the Krylov subspace methods that do not require an explicit representation of the Jacobian. However, it often becomes necessary to evaluate the full Jacobian matrix at least once or partial Jacobian multiple times in order to construct suitable preconditioners. In this work, we discuss a multigrid preconditioner for the matrix-free Krylov subspace methods. Our multigrid method does not require the representation of the Jacobian at any level of the multigrid hierarchy. As in the standard multigrid method, this method also relies on four building blocks: a hierarchy of meshes, transfer operators, smoothers, and a coarsest level solver. Here, we create a hierarchy of nested meshes and utilize the L^2-projection based transfer operators to pass the information between the meshes on subsequent levels. In this method, it is essential to assemble the gradients on coarser levels, to this aim we use also the projection operator to transfer the iterates on coarse levels. High-frequency components of the error are tackled by Chebyshev smoother as it has good smoothing properties and it does not require the representation of the Jacobian matrix. In the standard multigrid method, the remaining low-frequency components of the error are usually annihilated by a direct solver on the coarsest level. We propose to replace the direct solver with a matrix-free Krylov subspace method, thus giving rise to a truly Jacobian-free multigrid preconditioner. In this talk, we will discuss all building blocks of our multigrid preconditioner in detail and demonstrate how it can be utilized in combination with globalized variants of JFNK methods, such as, line-search and the trust-region methods. Finally, the robustness and the efficiency of the proposed multigrid method will be demonstrated through several numerical examples, e.g., Bratu equation, minimal surface equation, and hyperelastic problems.

21:30–22:00 **A Parallel and Fully Implicit Constraint-Preserving Simulator for Multiphase Flow in Porous Media**

*Haijian Yang**

Fully implicit methods are drawing more attention in scientific and engineering applications due to the allowance of large time steps in extreme-scale simulations. When using a fully implicit method to solve two-phase flow problems in porous media, one major challenge is the solution of the resultant nonlinear system at each time step. To solve such nonlinear systems, traditional nonlinear iterative methods, such as the class of the Newton methods, often fail to achieve the desired convergent rate due to the high nonlinearity of the system and/or the violation of the boundedness requirement of the saturation. In the talk, we reformulate the two-phase model as a variational inequality that naturally ensures the physical feasibility of the saturation variable. The variational inequality is then solved by an active-set reduced-space method with a nonlinear elimination preconditioner to remove the high nonlinear components that often causes the failure of the nonlinear iteration for convergence. To validate the effectiveness of the proposed method, we compare it with the classical IMplicit Pressure-Explicit Saturation (IMPES) method for two-phase flow problems with strong heterogeneity.

MS18 – Average Schwarz Method and Numerical Homogenization: Theory and Implementation

Organizers: *Guanglian Li* ⋄ *Leszek Marcinkowski* ⋄ *Talal Rahman*

Additive Average Schwarz method was proposed by Bjørstadt, Dryja, Vainniko in early 90ties. The method is one of the simplest of all additive Schwarz preconditioners because it is easy to construct and quite straightforward to analyze. Unlike most additive Schwarz preconditioners, its local subspaces are defined on non-overlapping subdomains, and it requires no explicit coarse grid as its coarse space is simply defined as the range of an averaging operator. A closely related type of methods is the numerical homogenization methods, which have been studies for over half century in different research fields aiming at solving challenging multiscale problems efficiently. They have been applied to many practical applications successfully, including reservoir simulation, porous media and material science. The construction of coarse space or multiscale space plays a key role in several numerical homogenization methods, e.g., (Generalized) Multiscale Finite Element Methods (GMsFEMs) and Localized Orthogonal Decomposition (LOD) Methods.
The goal of this minisymposium is to present recent theoretical results and implementation issues on Average Schwarz method and Numerical Homogenization, and to arise attention of both communities on the theoretical development of each other.

Tuesday 20:00–22:00

20:00–20:30 **Additive Average Schwarz with Enriched Coarse Space for Nonconforming Finite Elements**

*Salah Alrabeei** ⋄ *Mahmood Jokar* ⋄ *Leszek Marcinkowski*

An additive average Schwarz method is improved by enriching the coarse spaces by two adaptive eigenspaces. This algorithm is used to solve second and fourth order elliptic problems with highly varying and discontinuous coefficients. Our analysis and implementation show that the condition number of the preconditioned problem has third order of convergence which is independent of varying and discontinuous coefficients.

20:30–21:00 **Localized Eigenstates by Domain Decomposition**

*Daniel Peterseim** ⋄ *Robert Altmann* ⋄ *Patrick Henning*

This talk discusses the numerical approximation of low-energy eigenstates of the linear random Schrödinger operator. Under oscillatory high-amplitude potentials with a sufficient degree of disorder it is known that these eigenstates localize in the sense of an exponential decay of their moduli. We propose a reliable numerical scheme which provides localized approximations of such localized states. The method is based on an inverse iteration and a domain decomposition preconditioner inspired by numerical homogenization. The practical performance of the approach is illustrated in various numerical experiments in two and three space dimensions and also for a non- linear random Schrödinger operator.

21:00–21:30 **Wavelet-Based Edge Multiscale Finite Element Method for Helmholtz Problems in Perforated Domains**

*Guanglian Li** ⋄ *Shubin Fu* ⋄ *Richard Craster* ⋄ *Sebastien Guenneau*

We introduce a new efficient algorithm for Helmholtz problems in perforated domains with the design of the scheme allowing for possibly large wavenumbers. Our method is based upon the Wavelet-based Edge Multiscale Finite Element Method (WEMsFEM) as proposed recently. For a regular coarse mesh with mesh size H, we establish $\mathcal{O}(H)$ convergence of this algorithm under the resolution assumption, and with the level parameter being sufficiently large. The performance of the algorithm is demonstrated by extensive 2-dimensional numerical tests including those motivated by photonic crystals.

21:30–22:00 **Computational Multiscale Methods and Numerical Homogenization**

*Sai-Mang Pun**

In this talk, we will introduce a computational multiscale method based on the recently developed Constraint Energy Minimizing Generalized Multiscale Finite Element Method and employ it to solve the problems with multiscale features arising in engineering applications. The proposed method can be viewed as a generalization of numerical homogenization and it makes use of the idea of energy minimization with suitable constraints to generate multiscale basis functions for approximating desired physical quantities. These basis functions are constructed by solving a class of local auxiliary optimization problems based on dominant modes containing local information on the heterogeneity. Techniques of oversampling are adapted to enhance the computational performance in terms of accuracy. Numerical results will be provided to demonstrate the performance of the proposed method.

Wednesday 20:00–22:00

20:00–20:30 **A MsFEM Approach with High-Order Legendre Polynomials**

*Frederic Legoll**

The Multiscale Finite Element Method (MsFEM) is a Finite Element type approximation method for multiscale PDEs, where the basis functions used to generate the approximation space are precomputed as solutions to problems posed on local elements and ressembling the global problem of interest. These basis functions are thus specifically adapted to the problem at hand. Once these local basis functions have been computed, a standard Galerkin approximation of the global problem is performed. Many ways to define these basis functions have been proposed in the literature over the past years.
In this work, we introduce and analyze a specific MsFEM variant, the construction of which is inspired by component mode synthesis techniques [Hetmaniuk and Lehoucq, M2AN 2010; Hetmaniuk and Klawonn, ETNA 2014]. However, in contrast to these approaches, we do not solve local eigenvalue problems but rather consider enrichment by highly oscillatory basis functions that are harmonic (with respect to the operator at hand) and satisfy Dirichlet boundary conditions (on the boundary of the local elements) given by (possibly high order) Legendre polynomials.
Motivation for this new approach, a priori and a posteriori error estimates and numerical results will be discussed.
Joint work with U. Hetmaniuk (University of Washington), C. Le Bris and P.-L. Rothe (ENPC and Inria).

20:30–21:00 **Adaptive Average Schwarz Method for Crouzeix-Raviart Discretization of Multiscale Elliptic Problem**

*Leszek Marcinkowski** ⋄ *Talal Rahman*

In our talk we present an extension of the average Additive Schwarz Method for nonconformin Crouzeix-Raviart discretization of a second order elliptic problem in two dimensions with highly varying coefficients. We proposed larger enriched coarse space constructed by a simple procedure which enables us enrich the coarse space of the classical average ASM for CR discretization by the local spaces spanned by eigenfunctions of specially defined local problems. The new coarse space allows us to obtain the same bounds on condition number of the parallel average Schwarz preconditioner as in the classical average additive Schwarz method for conforming P_1 element which was proposed by Bjørstad, Dryja, Vainikko in 1990s.

21:00–21:30 **Spectral Additive Schwarz Methods for Hybrid Discontinuous Garlerkin Discretizations**

*Yi Yu** ⋄ *Maksymilian Dryja* ⋄ *Marcus Sarkis*

Additive Average Schwarz methods were introduced by Bjorstad, Dryja and Vainikko 97' and designed for solving 2D and 3D elliptic problems with discontinuous coefficients across subdomains using classical discretizations. Spectral Additive Schwarz Methods is an upgrade version which can solve 2D and 3D elliptic problems with discontinuous coefficients across elements. These methods use non-overlapping subdomains and the subdomain iteration is via the coarse space. The methods do not require a coarse triangulation and the coarse problem can be seen as inverting a diagonal matrix plus a sparse low-rank matrix associated to few generalized eigenvectors obtained purely inside the subdomains. The condition number estimate of the preconditioned systems is of the same order as the classical Additive Schwarz methods with minimum overlap and with a coarse space, that is, O(H/h), independent of coefficients. In this talk we design and analyze this type of preconditioning for some Hybrid Discontinuous Garlekin discretizations with heterogeneous coefficients, possibly with high-contrast coefficients and with the use of local generalized eigenvalues problems in order to recover the robust O(H/h) condition number estimate independently of the coefficients.

21:30–22:00 **CR, P1 and Iterative Methods for Heterogeneous Problems**

*Juan Galvis**

In this talk we explore the performance of iterative methods for the solution of heterogeneous diffusion problems when they are numerically solved using CR and P1 finite element spaces. We present novel iterative methods that explore some relations between CR and P1 finite element methods in some particular meshes. Analysis is presented together with numerical verifications of the results.

Thursday 20:00–22:00

20:00–20:30 **Domain Decomposition Preconditioning for the Integral Equation Formulations of the Forward and Inverse Scattering Problems**

*Carlos Borges** ⋄ *George Biros*

In this work, we propose domain decomposition preconditioners for the solution of the integral equation formulation of forward and inverse acoustic scattering problems with point scatterers in two dimensions. For the forward problem, we extend to integral equations the domain decomposition based preconditioning techniques presented for partial differential equations in "A restricted additive Schwarz preconditioner for general sparse linear systems", SIAM Journal on Scientific Computing, 21 (1999), pp. 792–797. To further improve the preconditioning strategy, we combine this domain decomposition preconditioner with a low-rank correction, which is easy to construct, forming a new preconditioner. For the inverse problem, we propose a preconditioner for the Gauss-Newton Hessian that is contracted using the forward problem preconditioner. Numerical results shows the performance of both preconditioning strategies.

20:30–21:00 **Implementation Issues of Average Schwarz Method**

*Talal Rahman**

In the talk we will present implementation aspects of Average Schwarz method. We will focus on details of implementation of this method. Namely, we will show that the method is very simple to implement in general.

21:00–21:30 **FEM Solver for Cahn-Hilliard Equation**

*Siamak Faal** ⋄ *Marcus Sarkis*

Since their introduction in the late 1950's, the Cahn-Hilliard equation has played an important role in understanding phase transition phenomena that is observed in materials. In particular, the Cahn-Hilliard equation describes the process of phase separation in which a mixture of two materials separate or fuse to form pure material domains. In this talk, we present possible schemes to obtain stable and robust Newton iterations when solving the Cahn-Hilliard Equation with nonlinear potential function. Our focus is mainly on formulating suitable preconditioners to solve the resultant linear system based on multigrid approach.

21:30–22:00 **25 Years with Additive Average - a Brief Review**

*Petter E. Bjørstad**

The Additive Average Schwarz coarse space was introduced as a robust and simple approach at the DD-8 Conference in Beijing 1995. Since then, the method has been extended and analyzed by several authors over the last 25 years. In this talk, we review the original paper and some of the later developments.

MS19 – Solution Techniques for Nonstandard Discretization Methods: Theory and Applications

Organizers: Blanca Ayuso de Dios ⋄ Susanne C. Brenner

The aim of the session is to bring together experts who are active in the construction and analysis of solution techniques for non-standard discretizations. This includes the development of domain decomposition and multilevel type preconditioners, and also the design of adaptivity techniques.
Over the last decade, there has been un upsurge on the development of novel, “non standard” discretizations for complex partial differential equations (PDEs) featuring for instance multiple fields and/or multiple scales, due to many applications in material science, cell biology and continuum mechanics. In some instances it is essential to build structure preserving discretizations, while in others the complexity of the problem itself precludes the use of conventional discretizations. Among the advanced and innovative methods, one finds various discontinuous Galerkin methods, nonconforming approximations, isogeometric analysis, unfitted approaches or virtual element methods, to mention a few.
This mini-symposia will bring together experts in the different topics in the field to facilitate the discussion in identifying common points in the design of solution techniques for non-standard methods. Sample topics include the design, the theoretical analysis and issues related to the implementation and applications of the various solution techniques.

Thursday 19:30–22:00

19:30–20:00 **On the Computation of the Eigenvalues of the curl Operator**

Ana Alonso Rodríguez ⋄ Jessika Camaño*

We present a new algorithm for the finite element approximation of the eigenvalue problem for the **curl** operator in a multiply-connected domain Ω. If the domain is simply connected the **curl** operator is self-adjoint when restricted to the space of vector fields $\mathbf{v}$ that satisfy the boundary condition $\mathbf{curl}\,\mathbf{v}\cdot\mathbf{n} = 0$. When Ω is not simply connected additional constraints must be imposed: a viable choice is the vanishing of the line integrals of $\mathbf{v}$ on suitable homological cycles lying on the boundary (see [2]). The new algorithm that we propose is based on the weak formulation and finite element approximation of this problem analyzed in [1]. The algorithm exploits the Hodge decomposition of the finite element space. To reduce the dimension of the generalized matrix eigenvalue problem it uses a tree-cotree decomposition of the graph relating the degrees of freedom of the Lagrangian finite elements and those of the first family of Nédélec finite elements. Some numerical experiments are presented to assess the performance of the method.

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20:00–20:30 **Auxiliary Space Preconditioning for Elasticity**

Lukas Kogler ⋄ Joachim Schoeberl*

It is of advantage to be able to use solvers developed for standard Finite Elements also for hybrid discretizations such as the Hybrid Discontinuous Galerkin (HDG) or the Mass Conserving Mixed Stress (MCS) method.
In this talk we present auxiliary space preconditioning techniques for such methods: The space is split into an embedded global low order space, plus local sub-spaces.

For the auxiliary space we have adapted our parallel algebraic multigrid method. It uses displacement and rotational degrees of freedom. We explicitely construct grid transfer operators preserving rigid body modes. Strong connections are defined by bending and stretching energies. The definition of the embedding is given by the dual basis, which is also used in a generic implementation in NGSolve.
We present numerical examples demonstrating the robustness and scalability of our method. They were run within our finite element software NGSolve on the Vienna Scientific Cluster VSC4 supercomputer.

20:30–21:00 **Additive Schwarz Preconditioners for C^0 Interior Penalty Methods for the Obstacle Problem of Clamped Kirchhoff Plates**

*Kening Wang** ◇ *Susanne C. Brenner* ◇ *Li-Yeng Sung*

The discrete variational inequalities resulting from C^0 interior penalty methods for the obstacle problem of clamped kirchhoff plates can be solved by the primal-dual active set algorithm. We develop and analyze additive Schwarz preconditioners for the auxiliary systems that appear in each iteration of the primal-dual active set algorithm. Numerical results corroborate our theoretical estimates.

21:00–21:30 **A Unified Framework of A Posteriori Error Estimates by Preconditioning**

*Yuwen Li** ◇ *Ludmil Zikatanov*

In this talk, we discuss a posteriori error estimates of finite element methods in the framework of preconditioning. This framework derives from several popular tools for developing preconditioners and yields classical as well as new a posteriori error estimators. Examples include the $H(\mathrm{curl})$, $H(\mathrm{div})$, and Hodge Laplacian problems.

21:30–22:00 **Additive Schwarz Preconditioners for a Localized Orthogonal Decomposition Method.**

*Jose Garay** ◇ *Susanne Brenner* ◇ *Li-Yeng Sung*

The solution of multiscale elliptic PDEs with non-separable scales and high contrast in the coefficients by standard Finite Element Methods (FEM) is typically prohibitively expensive since it requires the resolution of all characteristic lengths to produce an accurate solution. Numerical homogenization methods such as Localized Orthogonal Decomposition (LOD) methods provide access to feasible and reliable simulations of such multiscale problems. These methods are based on the idea of a generalized finite element space where the generalized basis functions are obtained by modifying standard coarse FEM basis functions to incorporate relevant microscopic information in a computationally feasible procedure. Using this enhanced basis one can solve a much smaller problem to produce an approximate solution whose accuracy is comparable to the solution obtained by the expensive standard FEM. We present a variant of LOD that uses domain decomposition techniques to compute the basis corrections and we also provide a two-level preconditioner for the resulting linear system.

MS20 – Cross Points in Domain Decomposition Methods

Organizers: Kévin Santugini-Repiquet ⋄ Laurence Halpern

In contrast to multigrid methods, domain decomposition methods with their aggressive coarsening strategy lead to cross points where more than two subdomains meet. The treatment of cross points in iterative domain decomposition methods is non-trivial, and often domain decomposition preconditioners diverge at cross points when used as stationary iterations, which leads for example to the coloring constant in the condition number estimate of Additive Schwarz, or the logarithmic term in the condition number estimate of FETI and Neumann-Neumann methods. We are interested in bringing together people who have worked on improving the cross point treatment in domain decomposition methods, and to get an up to date view of what the state of the art currently is.

Thursday 20:00–22:00

20:00–20:30 **Non-Overlapping Domain Decomposition Methods for the Cross-Points Problem Using Nodal Finite Elements in the Approximation of the Helmholtz Equation**

*Yassine Boubendir**

The design of non-overlapping domain decomposition methods leads to challenging mathematical problems related to the assignment of degrees of freedom at cross points. In order to avoid this difficulty, we have introduced a new approach to the treatment of cross points which consists of preserving the finite element equation at the level of these points, i.e., of taking a common value for the degrees of freedom located on the nodes at the junction of several subdomains. In this talk, we review this technique in a context of local transmission conditions. In addition, we show how to extend it to some particular non local transmission conditions. Numerical results validate these methods.

20:30–21:00 **Generalized Optimised Schwarz Method for Arbitrary Non-Overlapping Sub-Domain Partitions**

Xavier Claeys ⋄ Emile Parolin*

The Optimized Schwarz Method (OSM) is a well established domain decomposition (DDM) strategy for solving frequency domain wave propagation problems such as Helmholtz equation [2, 4].
In this method, the wave equation is solved independently in each subdomain imposing impedance conditions at the boundary. Coupling between subdomains is obtained via an exchange operator that swaps traces on each side of each interface. Whenever the subdomain partition does not involve any junction i.e. point where at least three subdomains abut, this strategy can be very efficient provided that the impedance of local subproblems is chosen wisely [3].
The situation is different when there are junctions and the presence of such points can spoil the consistency of the method, even for common geometric configurations [1]. The treatment of junctions in OSM has been the subject of many contributions and, although convincing numerical remedies are now available in the case of right-angled junctions, no generic satisfactory approach has been proposed so far.
In this talk we will present a new variant of OSM where the exchange operator is defined through layer potentials and appears as a good candidate for dealing with junctions. We shall discuss in detail the properties of the operator associated to this new method.

This work was supported by the project NonlocalDD from the French National Research Agency (ANR) through grant ANR-15-CE23-0017-01.

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21:00–21:30 **A Simple Finite Difference Discretization for Ventcell Transmission Conditions at Cross Points**

*Martin J. Gander** ◇ *Laurence Halpern*

The discretization of optimized transmission conditions at cross points in domain decomposition is not a straighforward task. This is even more so for higher order transmission conditions which include also tangential derivatives. We show in this presentation a simple finite difference discretization for second order transmission conditions of Ventcell type. We illustrate our approach with numerical experiments.

21:30–22:00 **Optimized Schwarz Algorithms for DDFV Discretization**

*Stella Krell**

We introduce a non-overlapping optimized Schwarz method for anisotropic diffusion problems. We present a discretization of the algorithm using discrete duality finite volumes (DDFV for short), which are ideally suited for anisotropic problem on general meshes. We prove the convergence using energy estimates for general decompositions including cross points and fully anisotropic diffusion. We illustrate our analysis with numerical experiments for the model problem covered by our analysis, and also in situations that go beyond our analysis, with an application to anisotropic image reconstruction.

Friday 20:00–22:00

20:00–20:30 **A Non-Overlapping Domain Decomposition Method with High-Order Transmission Conditions and Crosspoint Treatment for Helmholtz Problems**

*Axel Modave** ◇ *Xavier Antoine* ◇ *Anthony Royer* ◇ *Christophe Geuzaine*

The parallel finite-element solution of large-scale time-harmonic problems is addressed with a non-overlapping Schwarz domain decomposition method (DDM). It is well known that the efficiency of this method strongly depends on the transmission condition enforced on the interfaces between the subdomains. Local conditions based on high-order absorbing boundary conditions (HABCs) have proved to be well suited, as a good compromise between basic impedance conditions and non-local conditions. However, a direct application of the approach for configurations with interior cross points *(where more than two subdomains meet)* and boundary cross points *(points that belong to both the exterior boundary and at least two subdomains)* is suboptimal and, in some cases, can lead to wrong results. In this work, we extend this approach to efficiently deal with cross points for lattice-type partitioning. The proposed cross-point treatment relies on corner conditions developed for HABCs. Two-dimensional numerical results with a nodal finite-element discretization are proposed to validate the approach, including scalability studies with respect to frequency, mesh size and the number of subdomains.

20:30–21:00 **Corners and DDM for the Helmholtz Problem**

*Anouk Nicolopoulos** ◇ *Bruno Després* ◇ *Bertrand Thierry*

We derive new transmission conditions (TC) for solving the Helmholtz equation

$$(-\Delta - \omega^2)u = f \text{in } \mathbf{R}^2, \lim_{\|\mathbf{x}\|\to\infty} \nabla u(\mathbf{x}) - \mathbf{i}\omega u(\mathbf{x}) = 0,$$

with domain decomposition methods (DDM) which admit mesh constraints such as corners. With respect to previous work by Joly-Lohrengel-Vacus [1] and current work by Modave-Geuzaine- Antoine [2] where the study concerns right-angle corners or right-angle cross-points, our frame- work adresses more general mesh features. The central idea for the development of the new corner relations is that the algebraic pro- perties of convergent DDMs for the Helmhotz equation must be preserved by the new conditions : each DDM is endowed with a decreasing energy. The starting point is the coercive 2nd order ABC

$$\left(1 - \frac{1}{2\omega^2}\right)\partial_{\mathbf{n}} u - \mathbf{i}\omega u = 0 \quad \text{on } \Gamma := \partial\Omega,$$

where Ω is a polygonal computational domain that encloses the support of the source f. Because of the derivation of $\partial_{\mathbf{n}} u$ along Γ in the absorbing condition, additional relations have to be prescribed at the corners of Γ, where the outgoing normal $\mathbf{n}$ is not defined. We obtain a glocal ABC that writes

$$\partial_{\mathbf{n}} u - \mathbf{i}\omega T u = 0 \quad \text{on } \Gamma,$$

where the result of the operator T applied to a given $v \in L^2(\Gamma)$ is given by the solution to a coercive variational formulation. Different DDMs are derived based on this ABC. Then, we generalize the construction for a TC with corners on the interfaces between subdomains. Since the corresponding operator T depends on the interior angle of the considered corner, it leads to two different operators T_i and T_j when considering Ω_i or Ω_j as the interior, and the proof of convergence requires particular attention. The theoretical properties are illustrated by numerical simulations.

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21:00–21:30 **Novel Non-Local Impedance Operators for Non-Overlapping DDM Applied to Wave Propagation Problems**

Xavier Claeys ⋄ *Francis Collino* ⋄ *Patrick Joly* ⋄ *Emile Parolin**

The pioneering work of B. Després [1] have shown that it is mandatory, at least in the context of wave equations, to use impedance type transmission conditions in the coupling of sub-domains in order to obtain convergence of non-overlapping domain decomposition methods (DDM). In later works [2, 3], it was observed that using non-local impedance operators could lead to geometric convergence, a property which is unattainable with local operators. This result was recently extended to arbitrary geometric partitions, including configurations with cross-points, with provably uniform stability with respect to the discretization parameter [4].
In this talk we present a novel strategy to construct suitable non-local impedance operators that satisfy the theoretical requirements of [4]. It is based on the resolution of elliptic auxiliary problems posed in the vicinity of the transmission interfaces. The definition of the operators is generic, with simple adaptations to the acoustic or electromagnetic settings, even in the case of heterogeneous media. Besides, no complicated tuning of parameters is required to get efficiency. The implementation in practice is straightforward and applicable to sub-domains of arbitrary geometry, including ones with rough boundaries generated by automatic graph partitioners. Numerical results will be provided to illustrate the robustness of the proposed approach.

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21:30–22:00 **A Unified Convergence Theory for Robin-Schwarz Methods - Continuous and Discrete, Including Crosspoints**

*Clemens Pechstein**

This talk deals with non-overlapping Schwarz methods with generalized Robin transmission conditions for wave propagation problems in variational form. A convergence result is shown that applies (unlike many previous results) to the continuous case with crosspoints, while not requiring any regularity of the solution. The key ingredient, inspired by a recent paper by X. Claeys on a Schwarz method in the context of integral equations, is a global interface exchange operator. The same theory can be applied to the discrete case, where the exchange operator is allowed to be even local. The resulting scheme can be viewed as a generalization of the 2-Lagrange-multiplier method introduced by S. Loisel.

Saturday 21:00–22:00

21:00–21:30 **A Non-Overlapping Domain Decomposition Method with Perfectly Matched Layer Transmission Conditions**

*Anthony Royer** ⋄ *Axel Modave* ⋄ *Eric Béchet* ⋄ *Christophe Geuzaine*

It is well-known that the convergence rate of non-overlapping domain decomposition methods (DDMs) for the Helmholtz equation strongly depends on the transmission condition enforced at the interfaces between the subdomains. Transmission operators based on high-order absorbing boundary conditions (HABCs) or perfectly matched layers (PMLs) have proved to be well-suited for configurations with layered domain partitions. Unfortunately, the extension of these approaches for more general partitions with cross-points (where more than two subdomains meet) is rather tricky and requires some care. In this work, we present a non-overlapping domain decomposition method with PML transmission conditions for chequerboard (cartesian) decompositions. In such decompositions, each subdomain is surrounded by PMLs associated to edges and corners, respectively called *edge* PMLs and *corner* PMLs. The continuity of interface Dirichlet traces between a subdomain and *edge* PMLs, and between each *edge* PML and their two neighbor *corner* PMLs, is enforced with Lagrange multipliers. This coupling strategy offers the benefit of naturally computing Neumann traces, which allows to use the PMLs as discrete operators approximating the exact Dirichlet-to-Neumann maps. Solvability and stability of the resulting saddle-point problem is numerically studied and two possible Lagrange multiplier finite element spaces are presented and compared. The approach developed here shares many similarities with the cross-point treatment applied to DDMs with HABC-based transmission conditions detailed in [Modave et al., Comput. Methods in Appl. Mech. Engrg. 368 (2020) 113162].

21:30–22:00 **Asymptotic Preserving Cell-Centered Finite Difference Discretizations of Robin Transmission Conditions for Singularly Perturbed Elliptic Equations**

*Stephan B. Lunowa** ⋄ *Martin J. Gander* ⋄ *Christian Rohde*

We consider the linear stationary advection diffusion equation in one dimension as a prototypical application for non-overlapping domain decomposition methods with Robin transmission conditions. We discretize the elliptic equation by a cell-centered finite volume method with upwind fluxes and study the effect of different choices for Robin transmission conditions. We analyze the necessary and sufficient conditions to obtain a consistent and stable algorithm, even in the singularly perturbed case. Furthermore, we characterize the requirements to obtain convergence of the iterations. In particular, when diffusion is absent, convergence in two iterations can be achieved. Finally, we provide some numerical examples, which confirm the theoretical results.

MS21 – Domain-Decomposition Methods for Coupled Problems in Fluid Dynamics

Organizers: *Anyastassia Seboldt* ⋄ *Martina Bukač*

Many applications from geomechanics, aerodynamics and biomedical engineering require the accurate and stable simulation of complex multiphysics processes. Examples include fluid-structure interaction models (e.g., valvular modeling), fluid-porous or poroelastic medium interaction models (e.g, groundwater flow, fracture propagation), as well as the transport problems (e.g., transport of drugs or chemicals). The numerical simulation of coupled problems has received considerable attention in recent years, but still remains a significant challenge in the mathematical and computational sciences. Substantial effort is allocated to the design of adaptable and robust numerical methods for coupled problems due to their intricate multiphysics nature and often strong nonlinearity. This minisymposium focuses on the domain decomposition methods and computational techniques used for solving coupled problems in various applications. Possible topics include but are not limited to: Fluid-structure interaction; Porous and poroelastic medium flow; Numerical analysis of domain decomposition methods; Validation and verification of numerical solvers; Higher-order partitioned methods for coupled problems.

Friday 10:00–12:00

10:00–10:30 **Nonconforming Time Discretization Based on Optimized Schwarz Waveform Relaxation for the Stokes-Darcy System**

Thi-Thao-Phuong Hoang ⋄ *Hemanta Kunwar* ⋄ *Hyesuk Lee**

We consider an optimized Schwartz waveform relaxation (OSWR) method based on Robin transmission conditions for the time-dependent Stokes-Darcy system. We show that the OSWR algorithm converges with appropriate choices of transmission parameters for the semi-discrete and fully discrete systems and that the parameters can be optimized to improve the convergence rates of the algorithm. In this approach, the local problems are still time-dependent and thus nonconforming discretizations in both space and time can be considered to efficiently simulate relevant applications with discontinuities in model parameters. Numerical results with nonconforming time grids are presented to demonstrate the accuracy and efficiency of the algorithm.

10:30–11:00 **Stable and Accurate Partitioned Methods for Fluid Structure Interaction**

*Jeffrey Banks**

In this talk I will discuss computational challenges associated with partitioned solvers for fluid-structure interaction (FSI). The discrete formulation of the fluid/solid interface conditions has a strong influence on the overall stability of the approach, and partitioned FSI solvers are historically found to suffer when the so-called added-mass effects are large. These difficulties have their origin in the fact that the reaction of an immersed body to an applied force depends not only on the mass of the body but also on the mass of the fluid displaced by the body through its motion. Traditional approaches do not properly account for the fluid added mass, and can therefore experience a situation where the over-reaction of a light solid to an applied fluid force leads in turn to an even larger reaction from the fluid and so on. I will present recent work concerning the development and analysis of a class of provably stable partitioned FSI solvers that avoid added-mass instabilities. The approach is based on embedding the evolutionary character of the fully coupled differential operator near interfaces. Results for incompressible flow regimes will be presented, and stability of the FSI coupling will be discussed using normal-mode stability theory.

11:00–11:30 **A Global-In-Time Domain Decomposition Method for the Coupled Nonlinear Stokes and Darcy Flows**

*Thi-Thao-Phuong Hoang** ⋄ *Hyesuk Lee*

We study a decoupling iterative algorithm based on domain decomposition for the time-dependent nonlinear Stokes-Darcy model, in which different time steps can be used in the flow region and in the porous medium. The coupled system is formulated as a space-time interface problem based on the interface condition for mass conservation. The nonlinear interface problem is then solved by a nested iteration approach which involves, at each Newton iteration, the solution of a linearized interface problem and, at each Krylov iteration, parallel solution of time-dependent linearized Stokes and Darcy problems. Consequently, local discretizations in time (and in space) can be used to efficiently handle multiphysics systems of coupled equations evolving at different temporal scales and with discontinuous parameters. Numerical results with nonconforming time grids are presented to illustrate the performance of the proposed method.

11:30–12:00 **Interface Flux Recovery Coupling Method for the Ocean-AtmosphereSystem**

*K. Chad Sockwell** ⋄ *Kara Peterson* ⋄ *Paul Kuberry* ⋄ *Pavel Bochev*

Typical atmosphere-ocean coupling schemes that transfer fluxes between models can be related to a single iteration of an iterative method. In general, this type of coupling leads to restrictions to attain stability and conservation. In this talk, we present an alternative coupling method which implements the so-called bulk condition common in the atmosphere-ocean coupling. We refer to this method as the Bulk-Interface-Flux-Recovery method (Bulk-IFR). In the Bulk-IFR method, the two model equations and the interface conditions are combined into a monolithic system. Then, in a similar manner to FETI methods, a Schur complement is used to define a linear system for the interface flux. The specific form of the bulk condition results in the interface flux being treated as a new variable. This is in contrast to FETI methods where the interface flux is a Lagrange multiplier. Finally, the linear system defined on the interface is solved to recover the exact flux of the discrete monolithic model, while remaining non-intrusive to the two model components that it couples. The method is applied to a simplified atmosphere-ocean model of heat transfer with explicit time stepping. This results in an explicit synchronous partitioned coupling algorithm. Accuracy, stability, and conservation results are presented for the Bulk-IFR method applied to the simplified atmosphere-ocean heat transfer test case.

Friday 20:00–22:00

20:00–20:30 **Parallel Block-Preconditioned Monolithic Solvers for Fluid-Structure-Interaction Problems**

*Thomas Wick** ⋄ *Daniel Jodlbauer* ⋄ *Ulrich Langer*

In this presentation, we consider the solution of fluid-structure interaction problems using a monolithic approach for the coupling between fluid and solid subproblems. The coupling of both equations is realized by means of the arbitrary Lagrangian-Eulerian framework and a nonlinear harmonic mesh motion model. Monolithic approaches require the solution of large, ill-conditioned linear systems of algebraic equations at every Newton step. Direct solvers tend to use too much memory even for a relatively small number of degrees of freedom, and, in addition, exhibit superlinear grow in arithmetic complexity. Thus, iterative solvers are the only viable option. To ensure convergence of iterative methods within a reasonable amount of iterations, good and, at the same time, cheap preconditioners have to be developed. We study physics-based block preconditioners, which are derived from the block LDU-factorization of the FSI Jacobian, and their performance on distributed memory parallel computers in terms of two- and three-dimensional test cases permitting large deformations.

20:30–21:00 **Fully Discrete Loosely Coupled Robin-Robin Scheme for Incompressible Fluid-Structure Interaction: Stability and Error Analysis**

*Rebecca Durst** ⋄ *Erik Burman* ⋄ *Miguel Fernandez* ⋄ *Johnny Guzman*

We consider a fully discrete loosely coupled scheme for incompressible fluid-structure interaction based on the time semi-discrete splitting method introduced in [Burman, Durst & Guzm ́an, arXiv:1911.06760]. The splittling method uses a Robin-Robin type coupling that allows for a segregated solution of the solid and the fluid systems, without inner iterations. For the discretisation in space we consider piecewise affine continuous finite elements for all the fields and ensure the infsup condition by using a Brezzi-Pitkaranta type pressure stabilization. The interfacial fluid-stresses are evaluated in a variationally consistent fashion, that is shown to admit an equivalent Lagrange multiplier formulation. We prove that the method is unconditionally stable and robust with respect to the amount of added-mass in the system. Furthermore, we provide an error estimate that shows the error in the natural energy norm for the system is order $\sqrt{T}(\sqrt{k}+h)$ where k is the time step size and h is the spatial mesh size and T is the final time.

21:00–21:30 **Refactorization of the Midpoint Rule**

*Catalin Trenchea**

An alternative formulation of the midpoint method is employed to analyze its advantages as an implicit second-order absolutely stable timestepping method. Legacy codes originally using the backward Euler method can be upgraded to this method by inserting a single line of new code. We show that the midpoint method, and a theta-like generalization, are B-stable. We outline two estimates of local truncation error that allow adaptive time-stepping.

21:30–22:00 **Monolithic Divergence-Conforing HDG Scheme for FSI: Linear Model and Efficient Block Preconditioning**

*Guosheng Fu** ◇ *Wenzheng Kuang*

We present a novel monolithic divergence-conforming HDG scheme for a linear fluid- structure interaction (FSI) problem with a thick structure. Our fully discrete scheme produces an exactly divergence-free fluid velocity approximation, is energy-stable and optimal convergent which is robust with respect to the material parameters. The resulting linear system, which is symmetric and indefinite, is solved using a preconditioned MinRes method with a robust block algebraic multigrid (AMG) preconditioner.

MS22 – Robust Solvers for Multiphysics Problems

Organizers: Xiaozhe Hu ⋄ Carmen Rodrigo

The simulation of multi-physics problems, where different models interact to describe a complex process, has recently received a lot of attention. These problems are often modeled by coupled systems of partial differential equations. Advances in the improved understanding of the modeling of such physical processes are crucial. However, mathematical modeling at appropriate scales is impossible without further developments in numerical approximations, and large-scale computational algorithms. The design of efficient solvers is an essential aspect in the numerical simulation of multi-physics problems, since an efficient and accurate solution method is crucial to carry out large-scale simulations, necessary to obtain realistic results and to deal with real-life applications. The focus of the minisymposium is on the design and practical implementation of robust solvers for multiphysics problems.

Saturday 20:00–22:00

20:00–20:30 **Robust Preconditioners for Perturbed Saddle-Point Problems: Application to the Four-Field Biot Equations**

Wietse M. Boon ⋄ Miroslav Kuchta ⋄ Kent-Andre Mardal ⋄ Ricardo Ruiz-Baier*

We present robust solvers for a class of perturbed saddle-point problems arising in the study of the four-field formulation of Biot's consolidation problem for linear poroelasticity (using displacement, filtration flux, total pressure and fluid pressure). The stability of the continuous variational problems, which hinges upon using adequately weighted spaces, is addressed in detail using the Brezzi conditions and an additional, necessary inf-sup condition. Using this analysis, we propose a preconditioner that is robust with respect to relevant material parameters, including nearly incompressible solids and low permeabilities. The performance of our preconditioner is demonstrated through several numerical experiments.

20:30–21:00 **A Stabilized Hybrid Mixed Finite Element Method and Robust Preconditioners for Poroelasticity**

Chunyan Niu ⋄ Hongxing Rui ⋄ Xiaozhe Hu*

In this talk, we present a stabilized hybrid mixed finite element method for Biot's model. The hybrid P1-RT0-P0 discretization of the displacement-pressure-Darcy's velocity system of Biot's model is not uniformly stable with respect to the physical parameters, resulting in some issues in numerical simulations. To alleviate such problems, following [Rodrigo, et al. 2018], we stabilize the hybrid scheme with face bubble functions and show the well-posedness with respect to physical and discretization parameters, which provide optimal error estimates of the stabilized method. Then a perturbation of the bilinear form allows for the elimination of the bubble functions. The usage of hybridization removes the normal continuity across the interior edges of RT0 element, so the mass matrix corresponding to Darcy's velocity is block diagonal. Therefore, the unknowns of Darcy's velocity and the bubble functions can be eliminated by static condensation. The resulting system has the same number of degrees of freedom as the P1-RT0-P0 discretization, which is widely used in practice. The eliminated system is proved to be well-posed with respect to the discretization as well as the physical parameters. Thus, based on the framework of preconditioner, we develop robust block preconditioners to solve the resulting linear systems of equations efficiently.

21:00–21:30 **A Finite-Element Framework for a Mimetic Finite-Difference Discretization of Maxwell's Equations**

Casey Cavanaugh ⋄ James H. Adler ⋄ Xiaozhe Hu ⋄ Ludmil T. Zikatanov*

Maxwell's equations are a system of partial differential equations that govern the laws of electromagnetic induction. In order to preserve important underlying physical properties while solving the system numerically, a mimetic finite-difference (MFD) discretization of the equations is considered. In this talk, it is shown that the MFD formulation is equivalent to a structure-preserving finite-element (FE) scheme using mass-lumping and a scaling involving properties of the computational mesh. This connection permits a transparent theoretical analysis of the MFD method using the FE framework, and allows for further exploration of robust linear solvers for the discretized system. In particular, block preconditioners designed for FE formulations are modified and applied to the MFD system. Numerical tests are presented to verify the accuracy of the MFD scheme and to confirm the robustness of the preconditioners.

21:30–22:00 **Monolithic Multigrid for a Reduced-Quadrature Discretization of Poroelasticity**

*Peter Ohm** ◇ *James Adler* ◇ *Yunhui He* ◇ *Xiaozhe Hu* ◇ *Scott MacLachalan*

In this talk we discuss a monolithic multigrid method for a recently developed stabilized discretization of the poroelastic equations. The discretization is well-posed with respect to the physical and discretization parameters. We take advantage of a reduced quadrature approximation to create a more "solver-friendly" discretization while preserving the well-posedness properties of the underlying discretization. We use geometric multigrid with divergence-preserving interpolation operators to create a robust monolithic multigrid solver. Both Braess-Sarazin and Vanka relaxation schemes are considered, utilizing local Fourier analysis of the system to optimize relaxation parameters. Numerical results agree with the theoretical predictions provided by the local Fourier analysis of the system.

Contributed Sessions

CT01 – Tuesday 10:00–12:00

10:00–10:30 **Convergence Study of The Closest Point Method Coupled with The Classical Schwarz Method**

*Alireza Yazdani** ⋄ *Ronald Haynes* ⋄ *Steven Ruuth*

The discretization of surface intrinsic PDEs has challenges that one might not face in the flat space. The closest point method (CPM) is an embedding method that represents surfaces using a function that maps points in the flat space to their closest points on the surface. This mapping brings intrinsic data onto the embedding space, allowing us to numerically approximate PDEs by the standard methods in the tubular neighbourhood of the surface. Here, we solve the surface intrinsic positive Helmholtz equation by the CPM paired with finite differences which usually yields a large, sparse, and non-symmetric system. Domain decomposition methods, especially Schwarz methods, are robust algorithms to solve these linear systems. While there have been substantial works on Schwarz methods, Schwarz methods for solving surface differential equations have not been widely analyzed. In this work, we first investigate the classical Schwarz method on curves in 2D, and then the CPM coupled with the classical Schwarz method will be studied in terms of convergence and effective parameters.

10:30–11:00 **Optimal Model Order Reduction on Riemannian Manifolds of Port-Hamiltonian Systems**

*Kang-Li Xu** ⋄ *Yao-Lin Jiang*

Model order reduction (MOR) is effective to approximate large-scale systems by lower order systems. An effective MOR method should not only possess good approximation precision, but also be able to preserve the essential properties of the original system, such as stability, passivity and so on. In this report, we explore the structure-preserving and passive-preserving MOR methods of port-Hamiltonian systems on different Riemannian manifolds. First, we turn the constrained $\mathcal{H}_2$ optimal MOR problem into an unconstrained Riemannian optimization problem on Stiefel manifolds, and then establish a global convergence Riemannian optimization algorithm by designing a new Riemannian modified Fletcher-Reeves conjugate gradient scheme. Additionally, we further discuss the Riemannian optimization MOR methods of port-Hamiltonian systems by function approximation techniques. In this way, the $\mathcal{H}_2$ optimal MOR problem is posed on a product manifold composed of four Riemannian manifolds. The proposed algorithms can preserve the structure and the passivity of the original system. Finally, the numerical results are provided to demonstrate the effectiveness of our algorithms.

11:00–11:30 **Error Indicators and Adaptive Refinement of Finite Element Thin-Plate Splines**

*Lishan Fang** ⋄ *Linda Stals*

The thin-plate spline is a technique for interpolating and smoothing surface over scattered data in many dimensions. It is a type of polyharmonic splines that appears in various applications, including image processing and correspondence recovery. The thin-plate spline has some favourable properties like being insensitive to noise in data. However, its system of equations is dense and the size depends on the number of data points, which is impractical for large data sets. A finite element thin-plate spline was developed to approximate the thin-plate spline with piecewise linear basis functions. The resulting system of equations is sparse and the size depends only on the number of nodes in the finite element grid.
However, a solution with high accuracy will still require a fine grid. Adaptive refinement adapts the precision of the solution within sensitive regions and refines only in sensitive regions. This reduces the computational costs and memory requirements while still retaining the required

accuracy. Error indicators identify which regions to refine during the adaptive refinement process. Traditional error indicators might not work for the finite element thin-plate spline as it utilises data that might be perturbed by noise. In this talk, I will give a brief introduction of the problem and challenges and present some error indicators for the finite element thin-plate spline.

11:30–12:00 **Numerical Algorithms for Biot Model and Applications in Brain Edema Simulation**

*Mingchao Cai** ◇ *Guoliang Ju* ◇ *Jingzhi Li* ◇ *Jing Tian*

In this paper, we develop parameter-robust numerical algorithms for Biot model and apply the algorithms in brain edema simulations. By introducing an intermediate variable, we derive a multiphysics reformulation of the Biot model. Based on the reformulation, the Biot model is viewed as a generalized Stokes subproblem combining with a reaction–diffusion subproblem. Solving the two subproblems together or separately leads to a coupled or a decoupled algorithm. We conduct extensive numerical experiments to show that the two algorithms are robust with respect to the key physical parameters. The algorithms are applied to study the brain swelling caused by abnormal accumulation of cerebrospinal fluid in injured areas. The effects of the key physical parameters on brain swelling are carefully investigated. It is observed that the permeability has the biggest influence on intracranial pressure (ICP) and tissue deformation; the Young's modulus and the Poisson ratio do not affect the maximum value of ICP too much but have big influence on the tissue deformation and the developing speed of brain swelling.

CT02 – Wednesday 16:00–18:00

16:00–16:30 **Domain Decomposition of Time-Dependent Stochastic PDEs**

*Abhijit Sarkar** ◇ *Sudhi Sharma* ◇ *Ajit Desai* ◇ *Mohammad Khalil* ◇ *Chris Pettit* ◇ *Dominique Poirel*

Scalable non-overlapping domain decomposition algorithms based on intrusive spectral stochastic finite element method are reported in order to handle high resolution spatial discretization and large number of random parameters [1,2]. For two-dimensional scalar and coupled systems of SPDEs, two-level domain decomposition preconditioners with vertex-based coarse grid corrections show excellent scalable performance. The performance of these algorithms degrades for three-dimensional SPDEs due to complex geometrical subdomain interfaces and coupling in the stochastic space. A two-level domain decomposition preconditioner based on a wirebasket-based coarse grid demonstrates better scalabilities for three-dimensional SPDE systems, especially for coupled SPDE systems in linear elasticity. The usefullness of these algorithms is demonstrated for time-dependent SPDEs.

References:

(1) Desai, A., Khalil, M., Pettit, C., Poirel, D. and Sarkar, A., 2018, Scalable Domain Decomposition Solvers for Stochastic PDEs in High Performance Computing, Journal of Computer Methods in Applied Mechanics and Engineering, 335, 194-222.
(2) Desai, A., 2019, Scalable Domain Decomposition Algorithms for Uncertainty Quantification in High Performance Computing. Ph.D. thesis, Carleton University.

16:30–17:00 **Domain Decomposition Algorithms for Physics-Informed Neural Networks**

*Hyea Hyun Kim** ◇ *Hee Jun Yang*

Domain decomposition algorithms are proposed when training neural networks as a solution of partial differential equations. The neural network consists of smaller independent networks, that are trained as solutions of partial differential equations in smaller subdomains. The solution of the whole domain can then be obtained by solving the smaller problems iteratively. The use of smaller independent networks makes the parameter training faster but the convergence of the iteration very slow. To accelerate the convergence, a coarse neural network with a smaller parameter set is included. The coarse neural network is then trained as a solution of partial differential equations in the whole domain using a coarse data set. The coarse solution then can be used to accelerate the convergence in the iterative scheme. Numerical results are included to show the efficiency of the proposed method.

17:00–17:30 **A Two-Level Substructuring Method Tailored to Schur Based Domain Decomposition Methods**

*Yannis El Gharbi** ⋄ *Pierre Gosselet* ⋄ *Augustin Parret-Fréaud* ⋄ *Christophe Bovet*

The method presented here is a parallel mesh generation method leading to subdomains well-suited to Schur based domain decomposition solvers such as the FETI and BDD methods. Starting from a coarse mesh, subdomains meshes are created in parallel through hierarchical mesh refinements and mesh deformation techniques. The proposed methodology aims at limiting the occurrence of known pathological situations (jagged interfaces, misplaced heterogeneity with respect to the interfaces, ...) that penalize the convergence of the solver because of a strong degradation of the condition number of the problem. Some cures (based for instance on multipreconditioning, GenEO spectral coarse spaces, ...) were developed earlier for these situations but they require important additional computational resources. The goal, here, is to reach the solution without the help of these expensive methodologies, while making parallel the usually expensive steps dealing with mesh generation and splitting. The method has a good parallel efficiency in terms of strong and weak scaling and improves the convergence rates of the FETI method when dealing with very strong hetrogeneity ratios. We are currently investigating other solver improvements that could be obtained from this substructuring framework.

17:30–18:00 **Numerical Homogenization of Fractal Interface Problems**

Martin Heida ⋄ *Ralf Kornhuber** ⋄ *Joscha Podlesny* ⋄ *Harry Yserentant*

We consider a scalar elliptic model problem with jump conditions on a fractal network of interfaces inspired by frictional fault systems in the geosciences, and derive an associated asymptotic limit problem. The resulting solution space is characterized in terms of generalized jumps and gradients, and we prove continuous embeddings into L^2 and H^2, $s < 1/2$. Based on novel projection operators with suitable stability and approximation properties, we also derive finite element methods allowing for scale-independent error estimates together with successive subspace correction methods with scale-independent convergence rates. Our theoretical findings are illustrated by numerical computations.

CT03 – Thursday 16:00–17:30

16:00–16:30 **Developing an Adaptive Finite Element Solver for the High Frequency Time-Harmonic Maxwell Equation**

*Sebastian Kinnewig** ⋄ *Thomas Wick*

The time harmonic Maxwell equation (THM) is an ongoing research topic in computational science and applied mathematics since it has many applications in modern physics. In this work we present an adaptive and parallel finite element solver for the THM and compare different preconditioner, as a programming platform deal.II was used. We show numerically that standard preconditioners like ILU and the additive Schwarz method lead to very slow convergence for iterative solvers like GMRES, especially for high wave numbers. Even more we show that also more specialized methods like the Schur complement method and a geometric multigrid method also lead to very slow convergence from the iterative solver for high wave numbers. As an example for a highly adapted solver for the THM we present a block preconditioner which also preforms good for high wavenumbers. Additionally we discuss briefly further approaches to solve high frequency problems more efficiently.

16:30–17:00 **Optimal Complex Damping Parameters Minimizing Red-Black SOR Multigrid Smoothing Factors for Complex-Shifted Linear Systems**

*L. Robert Hocking** ⋄ *Chen Greif*

We derive optimal complex damping parameters minimizing smoothing factors associated with multigrid using red-black successive over-relaxation or damped Jacobi smoothing applied to a class of linear systems arising from discretized linear partial differential equations with a complex shift. Our analysis yields analytical formulas for smoothing factors as a function of the complex damping parameter, which may then be efficiently numerically minimized. Our results are applicable to second-order discretizations in arbitrary dimensions, and generalize earlier work of Irad Yavneh on optimal damping parameters in the real case. Our analysis is based on deriving a novel connection between the performance of SOR as a smoother and as a solver, and is validated by numerical experiments on problems in two and three spatial dimensions, using both vertex- and cell-centered multigrid, with both constant and variable coefficients. In the variable coefficient case we assign different damping parameters to different grids points, which our framework allows us to do efficiently. While we focus on anisotropic damped Helmholtz equations in our numerical experiments, our approach can be applied more generally to other complex-shifted linear systems.

17:00–17:30 **Domain Decomposition Preconditioner for a Thin Membrane Diffusion Problem**

*Piotr Krzyzanowski**

Diffusion through a thin membrane, which leads to jump penalty weak formulation, is considered. The problem is discretized with a variant of the composite h-p discontinuous Galerkin method. An additive Schwarz method preconditioner is introduced, and its convergence properties are investigated. Numerical experiments indicate the convergence rate of the method is independent of the penalty parameter and jumps in the diffusion coefficient.

CT04 – Friday 16:00–18:00

16:00–16:30 **Exploiting Morton Curves for Domain Decomposition of Staggered Grid Discretizations in CFD**

*Jonas Thies** ⋄ *Sven Baars* ⋄ *Fred Wubs*

Hierarchical Interface Decomposition (HID) is a technique that starts out with a non-overlapping distribution of grid cells (elements) among subdomains. The nodes living on the interfaces are then classified into vertices, edges and faces. Based on such a decomposition, one may e.g. implement a direct solution technique (solving the Schur complement for the interface nodes with a direct solver), or an approximate one, like a two-level Schwarz preconditioner or an incomplete factorization that somehow approximates the Schur complement by a sparser matrix. It turns out that a naive HID may lead to singular submatrices for a class of discretizations of the incompressible Navier-Stokes equations known as “staggered grid” approaches. For instance, the Arakawa C-grid discretization on a structured rectangular grid has this problem for a Cartesian partitioning of the domain. In this talk we show how skew-Cartesian partitioning in two and three space dimensions can overcome this issue, and propose a highly efficient partitioning algorithm based on space-filling curves and an incomplete index space to construct a feasible partitioning that can also be used in a multi-level context. As an application, we show a fully implicit solver for steady flow problems on structured grids which scales to thousands of cores.

16:30–17:00 **Scalable Newton-Krylov Solvers for Cardiac Reaction-Diffusion Models**

*Ngoc Mai Monica Huynh**

The development of effective solvers for the solution of mathematical models of the cardiac electro-mechanical activity has increasingly grown in the last decade. In particular, the need to handle the multiscale systems arising from the discretization of such models has required the development of specific techniques to both accurately represent physiological data and reduce the computational costs of the resulting large-scale simulations.

In this work, we focus on the numerical simulation of the cardiac electrical activity, by solving a system constituted by a nonlinear parabolic reaction-diffusion equation describing the propagation of the electric impulse in the cardiac tissue, coupled with the Roger-McCulloch ionic membrane model.
Applying a finite element discretization in space and the Backward Euler method in time, we need to solve at each time step a nonlinear system. Therefore, for the solution of this monolithic discretized nonlinear system, we propose here Newton-Krylov type methods, where the decomposition of the problem is performed after the Newton linearization. Fast convergence is ensured by preconditioning with non-overlapping domain decomposition techniques.

We investigate numerically the quasi-optimality and scalability of these preconditioners by implementing a three-dimensional parallel code based on the PETSc library from the Argonne National Laboratory. The results obtained provide a basis for an extension of this study to the inclusion of more complex realistic membrane models and to monolithic discretizations of cardiac electro-mechanical models.

This is a joint work with Simone Scacchi (Univ. of Milan) and Luca F. Pavarino (Univ. of Pavia).

17:00–17:30 **Domain Decomposition Method for Nuclear Core Reactor Simulations with Low-Regularity Solution**

*François Madiot** ◇ *P. Ciarlet* ◇ *M.H. Do* ◇ *L. Giret* ◇ *E. Jamelot* ◇ *F.D. Kpadonou*

The behaviour of a nuclear reactor core depends on the nuclear chain reaction that can be described by the neutron transport equation. This equation is a balance statement that conserves neutrons. It governs the neutron flux density, which depends on 7 variables: 3 for the space, 2 for the motion direction, 1 for the energy (or the speed), and 1 for the time. In the steady-state case, one must solve an eigenvalue problem. The energy variable is discretized using the multigroup theory (G groups). Concerning the motion direction, an inexpensive approach to approximate the transport equation is to solve the simplified PN equations ($\frac{1}{2}(N+1)$ coupled diffusion equations).
It can be shown that the basic building block which allows to solve the general multigroup simplified PN equations, is the so-called neutron diffusion equation ($G = 1$, $N = 1$): it is an eigenvalue problem set in a bounded domain of $\mathbb{R}^3$ and involving an elliptic operator [3]. More precisely, one looks for the smallest eigenvalue of this problem, whose inverse is called the criticality factor. For the core simulations, the model commonly involves three or more intersecting, highly heterogeneous, material components. As a result, the solution to the diffusion equation is of low regularity, and our focus will be on the numerical solution of the problem in this setting. In particular, we propose a domain decomposition method to solve the problem numerically [1, 2] and some associated a posteriori estimates [4].

References

[1] P. Ciarlet, Jr., E. Jamelot, F.D. Kpadonou, *Domain Decomposition Methods for the diffusion equation with low-regularity solution*, Computers Math. Applic., **74**, pp. 2369-2384 (2017).

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17:30–18:00 **Aitken-Schwarz DDM for Hybrid EMT-TS Electrical Network Simulation**

*Héléna Shourick** ⋄ *Damien Tromeur-Dervout* ⋄ *Laurent Chedot*

Simulating the power grid is essential to ensure grid security. More particularly, now that the network includes more components based on power electronics, it is important to be able to size them well so that they are not damaged by electrical disturbances. These components imply faster dynamics than before. Network simulation methods must adapt to these changes. Certain areas of the network require a high level of detail in the simulation as well as the ability to model faster dynamics (Electromagnetic Transient (EMT)) than on the rest of the network (dynamic phasor model (TS)). Hybrid simulation is a promising lead for maintaining high performance simulation speed while including more detail. The goal of this work is therefore to be able to simulate an electrical network using two different types of modeling. To do this, an overlapping Schwarz domain decomposition method is used. We examine the influence of the cutting location on the method, as well as the influence of two different models (EMT-TS) on the information to be exchanged. We show on a linear electric circuit the convergence property of the DDM with and without overlap and use the pure linear divergence of the method to accelerate it towards the solution with the Aitken acceleration technique.

CT05 – Friday 20:00–22:00

20:00–20:30 **Parallel Space--Time Methods for Evolutionary Reaction--Diffusion Problems**

*Andrés Arrarás** ⋄ *Francisco J. Gaspar* ⋄ *Laura Portero* ⋄ *Carmen Rodrigo*

In this talk, we discuss efficient space–time domain decomposition methods to approximate the solution of time-dependent reaction–diffusion problems. The proposed strategy is based on a suitable combination of domain decomposition splitting time integrators with the so-called parallel-in-time schemes. More precisely, the method of lines approach permits to reduce the initial-boundary value problem under consideration to a system of stiff ordinary differential equations. In this context, we exploit the idea of time parallelization by using the parareal algorithm to numerically solve such a system. It is well known that this algorithm combines time stepping on fine and coarse temporal grids by introducing the corresponding fine and coarse propagation operators, respectively. The novelty of this work lies in the fact that domain decomposition splitting methods are used as such operators. Since these methods are related to an overlapping decomposition of the spatial domain, spatial parallelization can also be exploited. In consequence, the resulting algorithms allow for parallelization both in time and space. However, unlike related existing methods (e.g., parareal Schwarz waveform relaxation methods), they do not require any iteration to adjust the boundary conditions of the subdomains. During the talk, we discuss the convergence properties of the proposed schemes and include a collection of numerical experiments that illustrate their numerical behaviour. Finally, we reformulate the parareal method as a two-level multigrid reduction in time (MGRIT) technique, and describe how to extend the new algorithms to a multilevel framework by using a hierarchy of coarser grids.

20:30–21:00 **Robust Preconditioning: Optimal Control of the Convection-Diffusion Equation with Limited Observation**

*Jarle Sogn** ⋄ *Kent-Andre Mardal* ⋄ *Stefan Takacs*

We consider an optimization problem with a Convection–Diffusion–Reaction equation as constraint. A Schur complement preconditioner is proposed and we show that the problem is well-posed. The preconditioner is robust with respect to all the problem parameters and the condition number of the preconditioned problem bounded by 4.05. We provide conditions for inf-sup stable discretizations and present one such discretization for box domains with constant convection. The preconditioner requires a fourth order problem to be solved. For this reason, we use Isogeometric Analysis as method of discretization. To efficiently realize the preconditioner we consider geometric multigrid with a standard Gauss-Seidel smoother as well as a new macro Gauss-Seidel smoother. The latter smoother provides good results with respect to both the geometry mapping and the polynomial degree.

21:00–21:30 **Fully Discrete Schwarz Waveform Relaxation Analysis for the Heat Equation on a Bounded Domain**

*Khaled Mohammad** ⋄ *Ronald D. Haynes*

Schwarz waveform relaxation algorithms are widely used to solve time dependent partial differential equations in parallel. The physical domain is divided into a finite number of overlapping or non overlapping subdomains. Artificial boundary conditions are imposed to transmit information from one subdomain to the others using an iterative process, until convergence is achieved. A lot of work has been done to prove the convergence of this algorithm at a continuous and semi-discrete level, but few at the fully discrete level. Recently, others have obtain a fully discrete convergence analysis on two overlapping, but infinite, subdomains. In the DD25 proceedings, we provided an analysis of a fully discrete classical Schwarz Waveform algorithm for the heat equation on two bounded subdomains. Here we present the extension of these results to allow for an arbitrary number of bounded subdomains.

CT06 – Saturday 20:00–22:00

20:00–20:30 **Accelerated Additive Schwarz Methods for Convex Optimization**

*Jongho Park**

Based on an observation that additive Schwarz methods for general convex optimization can be interpreted as gradient methods, we propose an acceleration scheme for additive Schwarz methods. Adopting acceleration techniques developed for gradient methods such as momentum and adaptive restarting, the convergence rate of additive Schwarz methods is greatly improved. The proposed acceleration scheme does not require any a priori information on the levels of smoothness and sharpness of a target energy functional, so that it can be applied to various convex optimization problems. Numerical results for linear elliptic problems, nonlinear elliptic problems, nonsmooth problems, and nonsharp problems are provided to highlight the superiority and the broad applicability of the proposed scheme.

20:30–21:00 **On Asynchronous Multi-Subdomain Methods for Parabolic Problems**

Mohamed Laaraj ⋄ *Karim Rhofir**

It is worthwhile noting that asynchronous fixed point iterations are not only a family of algorithms suitable for asynchronous computations on multiprocessors, but also a general framework in order to formulate general iteration methods associated with a fixed point mapping on a product space, including the most standard ones such as the successive approximation method (linear or nonlinear Jacobi / Gauss-Seidel) as well as its variants among many others. In this work, we present and analyze asynchronous iterations for boundary-value problems of parabolic type based on overlapping domain decomposition. We start by giving some new technical results for maximum principle and introduce some notations used in the sequel to give the problem formulation. We define the linear mapping T which defines the substructured solution process. Then we define the linear fixed point mapping $\tilde{T}$ which is the composition of T with a suitable restriction operator R. We prove that $\tilde{T}$ is a linear mapping in a suitable function space context and study the contraction convergence property with respect to a weighted norm well adapted to this situation. We finally introduce an affine mapping whose linear part is $\tilde{T}$ and whose fixed point is the solution of the problem. We state in the closing proposition the convergence of asynchronous iterations applied to the approximation of this affine fixed point mapping. We give also the block versions can be related to the additive and multiplicative Schwarz method

and there algebraic analogous.
Our results apply to a wide range of problems and are particularly well suited to pseudo stationary problems resulting from implicit or semi-implicit schemes of evolution equations. For the repetitive solution of large scale problems that are generally well conditioned, the asynchronous fixed point method can be competitive with other methods.

21:00–21:30 **Domain Decomposition for Unstructured Nonlinear Programming on Parallel Vector Architectures**

*Michel Schanen** ⋄ *Daniel Adrian Maldonado* ⋄ *François Pacaud* ⋄ *Mihai Anitescu*

Typically, sparse direct linear solvers are at the core of nonlinear programming for unstructured problems such as power systems or other network problems. Interior-point methods are the current state-of-the-art for solving optimal power flow and a host of other NLP problems. However, the reliance on indefinite inertia revealing linear solvers makes these solutions hard to port to parallel vector based architectures (e.g. GPUs, AVX, SIMD). In order to efficiently solve this problem we designed and implemented a highly parallel block-Jacobi preconditioned BiCGSTAB iterative solver. Together with a reduced space method, this solution shows good performance on highly parallel vector architectures and serves as the foundation of our optimal power flow solver ExaPF.jl, entirely developed in Julia.

21:30–22:00 **Automating Domain Decomposition Preconditioners with Code Generation**

*Igor Baratta** ⋄ *Adeeb Arif Kor* ⋄ *Elson José da Silva*

In this work, we present a framework for the development and experimentation of domain decomposition methods for finite element discretizations using automatic code generation and distributed computing, on top of the FEniCS computing platform. We use a domain-specific language, the Unified Form Language, to represent the variational formulation of the local, global and coarse problems in a near-mathematical notation and the FFCx, the new version of the FEniCS Form Compiler, to generate high-performance concrete low-level kernel implementations. These low-level kernels are then used to assemble the finite element systems and to build the domain decomposition preconditioners. We will also present a simple and efficient Python library that leverages the dispatch mechanism for NumPy's high-level array functions to represent distributed vectors and matrices simply and intuitively. Finally, we will provide some examples that highlight the expressivity of our framework for describing optimized transmission conditions and building two-level methods. All the examples will be accompanied by parallel scalability data.

List of Speakers

Alonso Rodríguez, Ana MS19
Alrabeei, Salah MS18
Antonietti, Paola F. MS16
Arrarás, Andrés .... CT
Ayuso de Dios, Blanca MS16
organizer of MS19

Banks, Jeffrey MS21
Baratta, Igor .... CT
Birken, Philipp MS02
Bjørstad, Petter E. MS18
Boon, Wietse M. MS22
Bootland, Niall MS03
Borges, Carlos MS18
Boubendir, Yassine MS20
Bouziani, Nacime MS03
Brenner, Susanne C.
organizer of MS19
Bukač, Martina
organizer of MS21

Cai, Mingchao .... MS04, CT
Cai, Xiao-Chuan MS17
organizer of MS01, MS17
Cavanaugh, Casey MS22
Chang, Huibin MS08
Chaouqui, Faycal MS12, MS14
Chen, Rongliang MS11
Chen, Zhiming MS05
organizer of MS05
Cho, Durkbin MS07
Chung, Eric .... PT, MS01
Ciaramella, Gabriele MS10, MS13
organizer of MS13
Claeys, Xavier MS20
Clement, Simon MS02
Cui, Jintao MS04
Cui, Tao MS05

Dai, Ruiyang MS03
Delourme, Bérangère MS13
Dohrmann, Clark MS07
Dolean, Victorita .... PT
organizer of MS03
Dostál, Zdeněk .... PT
Duan, Huoyuan MS05
Durst, Rebecca MS21
Dwarka, Vandana MS03

El Gharbi, Yannis .... CT

Faal, Siamak MS18
Fang, Lishan .... CT
Fu, Guosheng MS21
Fu, Shubin MS04

Galvis, Juan MS18
Gander, Martin J. MS20
organizer of MS02, MS14, MS17
Garay, Jose MS19
Genseberger, Menno MS06
organizer of MS06
Gillman, Adrianna .... PT
Glusa, Christian MS12
organizer of MS12
Gong, Shihua MS03
Graham, Ivan G.
organizer of MS03
Gu, Linyan MS01
Gu, Yaguang MS17

Halpern, Laurence .... PT
organizer of MS20
Hanek, Martin MS10
Heinlein, Alexander MS01, MS11
Hennicker, Julian MS02
Hoang, Thi-Thao-Phuong MS21
Hochmuth, Christian MS11
Hocking, L. Robert .... CT
Hou, Tianliang MS04
Hu, Xiaozhe
organizer of MS22
Huynh, Ngoc Mai Monica .... CT
Hwang, Feng-Nan MS17

Joly, Patrick .... PT

Keyes, David MS17
Kim, Dohyun MS16
Kim, Hyea Hyun .... CT
Kim, Mi-Young MS16
Kinnewig, Sebastian .... CT
Klawonn, Axel MS17
organizer of MS01, MS11, MS14
Knepper, Jascha MS14
Kogler, Lukas MS19
Köhler, Stephan MS11
Kopaničáková, Alena MS01
Kornhuber, Ralf .... CT
Kothari, Hardik MS17
Krause, Rolf .... PT
Krell, Stella MS20
Krzyzanowski, Piotr .... CT

Kumbhar, Pratik M. MS17
Kwok, Felix MS02
Kyriakis, Alexandros MS08

Laadhari, Aymen MS02
Laayouni, Lahcen MS08
Langer, Ulrich .... PT
organizer of MS07, MS09
Lanser, Martin MS11, MS17
Lee, Chang-Ock MS01
Lee, Hyesuk MS21
Lee, Pilhwa MS04
Legoll, Frederic MS18
Leng, Wei MS08
Lenz, David MS09
Li, Guanglian MS18
organizer of MS18
Li, Ke MS15
Li, Mengnan MS15
Li, Yuwen MS19
Liao, Qifeng MS15
organizer of MS15
Liu, Fang MS04
Liu, Lulu MS17
Lu, Wangtao MS05
Lucero Lorca, Jose Pablo MS08
Lunet, Thibaut MS09
Lunowa, Stephan B. MS20
Luo, Li MS17

Ma, Chupeng MS03
Madiot, François .... CT
Marcinkowski, Leszek MS18
organizer of MS18
McCoid, Conor MS17
Mechelli, Luca MS13
Modave, Axel MS20
Mohammad, Khaled .... CT

Nicolopoulos, Anouk MS20
Niu, Chunyan MS22
Notay, Yvan MS04

Oh, Duk-Soon MS10
Ohm, Peter MS22
Outrata, Michal MS08

Pacheco, Douglas R. Q. MS09
Papež, Jan MS10
Park, Eun-Hee MS16
Park, Eun-Jae
organizer of MS16
Park, Jongho .... CT
Parolin, Emile MS20
Pavarino, Luca F.
organizer of MS07
Pechstein, Clemens MS20
Peterseim, Daniel MS18
Pun, Sai-Mang MS18

Quan, Chaoyu MS08

Rahman, Talal MS18
organizer of MS18
Rave, Stephan MS14
Reyes-Riffo, Sebastian MS13
Rheinbach, Oliver MS11
organizer of MS11, MS14
Rhofir, Karim .... CT
Rodrigo, Carmen
organizer of MS22
Röver, Friederike MS11
Royer, Anthony MS20

Salomon, Julien MS13
Santugini-Repiquet, Kévin
organizer of MS20
Sarkar, Abhijit .... CT
Scacchi, Simone MS07
Schafelner, Andreas MS09
Schanen, Michel .... CT
Schneckenleitner, Rainer MS07
Schönlieb, Carola-Bibiane .... PT
Seboldt, Anyastassia
organizer of MS21
Sheng, Hailong MS01
Shin, Dong-wook MS16
organizer of MS16
Shourick, Héléna .... CT
Šístek, Jakub MS10
organizer of MS10
Sockwell, K. Chad MS21
Sogn, Jarle .... CT
Song, Bo MS08, MS14
Speck, Robert MS09
Steinbach, Olaf MS09
organizer of MS09
Szyld, Daniel B. MS12
organizer of MS12

Takacs, Stefan MS07
Tang, Hansong MS02, MS06
organizer of MS06
Thery, Sophie MS06
Thies, Jonas .... CT
Tomasi, Claudio MS10
Trenchea, Catalin MS21
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Xie, Manting MS04
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Yokota, Rio MS01
Yu, Yi MS18

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Zank, Marco MS09
Zhang, Hui MS03
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Zhang, Zaikun MS08
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Zhao, Lina MS16
  organizer of MS16
Zheng, Chunxiong MS05
Zheng, Haibiao MS04
Zheng, Weiying MS05
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Zhong, Liuqiang
  organizer of MS04
Zikatanov, Ludmil MS09