Free actions of locally compact groups on $C^*$-algebras

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Abstract: We shall consider separable $C^*$-dynamical systems $(A, G, \alpha)$ for which the induced action of the group $G$ on the primitive ideal space Prim$(A)$ of the $C^*$-algebra $A$ is free. We shall discuss how the representation theory of the associated crossed product $C^*$-algebra $A \rtimes_\alpha G$ depends on the representation theory of $A$ and the properties of the action of $G$ on Prim$(A)$ and the spectrum $\hat{A}$.

After surveying some earlier results, we shall describe some recent joint work with Astrid an Huef. The main tools are the notion of strength of convergence in orbit spaces and the notions of upper and lower multiplicities for irreducible representations. We apply these ideas to give necessary and sufficient conditions, in terms of $A$ and the action of $G$, for $A \rtimes_\alpha G$ to be (i) a continuous trace $C^*$-algebra, (ii) a Fell algebra and (iii) a bounded trace $C^*$-algebra. For the case of amenable $G$, we can apply a result of Leung and Ng to determine when $A \rtimes_\alpha G$ is (iv) a liminal $C^*$-algebra and (v) a Type I $C^*$-algebra. The results in (i) and (iii)-(v) extend some earlier special cases in which the $C^*$-algebra $A$ was assumed to have the corresponding property.
On the deformation space of Clifford-Klein forms of Heisenberg groups

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Abstract: Let $H$ be an arbitrary closed connected subgroup of the connected, simply connected Heisenberg $G = H_{2n+1}$. We exhibit in this paper a complete description of the deformation space $T(\Gamma, G, H)$ and the Moduli space $M(\Gamma, G, H)$ of a discontinuous abelian subgroup $\Gamma$ of $G$ for the homogeneous space $G/H$. The topological features of such a space, namely the topological stability, the rigidity and the local rigidity are also studied.

Functions on a topological group

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Abstract: I shall survey some of the recent results obtained jointly with colleagues on semigroup compactifications of a topological group $G$ and on Banach algebras related to $G$. The common method with all these results is the construction at each time of an appropriate function (or operator when we are dealing with an algebra of operators) to deal with the problem.

Co-authors: Alaste, Bouziad, Monfared, Neufang, Protasov, Salmi, Vedenjuoksu.
Characterizing admissible matrix groups

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Abstract: Admissible matrix groups are subgroups of $GL(n, \mathbb{R})$ whose canonical action on $\mathbb{R}^n$ give rise to a wavelet-type inversion formula. Such groups were studied, for instance, by Keith Taylor, Guido Weiss and their respective collaborators. A group is admissible whenever almost all stabilizers are compact, the semidirect product with the vector group is nonunimodular, and the dual orbit space is well-behaved. The first two conditions are known to be necessary. There exist several formulations for the last condition in the literature, which turn out to be sufficient (assuming the first two conditions to be fulfilled). In the first half of the talk, we give a precise necessary and sufficient formulation for the third condition, namely that after removal of an invariant set of measure zero, the Lebesgue measure is countably separated.

In the second half of the talk, we consider abelian admissible matrix groups in more detail. We first show how to reduce the discussion to closed simply connected and connected abelian matrix groups, and draw some conclusions for the special case that all group elements have real spectrum.

(Joint work with Margarida Miro, UAB, Barcelona)

Duals of introverted subspaces and module homomorphisms

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Abstract: Let $A$ be a Banach algebra. We will discuss various topological center problems for duals of introverted subspaces of $A^*$. We will also consider how these problems are related to algebras of module homomorphisms on $A^*$. The talk is based on joint work with M. Neufang and Z-J. Ruan.
Weak spectral synthesis in commutative Banach algebras

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Abstract: Let $A$ be a semisimple and regular commutative Banach algebra with structure space $\Delta(A)$. Given any closed subset $E$ of $\Delta(A)$, there exist a largest and a smallest ideal of $A$, $k(E)$ and $j(E)$, with hull equal to $E$. Then $E$ is called a spectral set if $k(E) = j(E)$. Generalizing this notion, $E$ is called a weak spectral set if $k(E) \setminus j(E)$ is nilpotent (equivalently, there exists $m \in \mathbb{N}$ such that $a^m \in j(E)$ for every $a \in k(E)$). There are plenty of examples of weak spectral sets which fail to be spectral sets, for instance spheres in $\mathbb{R}^n = \Delta(L^1(\mathbb{R}^n))$ for $n \geq 3$. Also, the concept of Ditkin set allows an appropriate generalization. The talk will report on recent results in this area, such as injection theorems for weak spectral sets and weak Ditkin sets and a Ditkin-Shilov type theorem. We also discuss the Fourier algebra of a locally compact group.

On Hardy’s uncertainty principle for nilpotent Lie groups

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Amenability of L1-algebras on commutative hypergroups

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Abstract: The L1-algebras of hypergroups have properties which are very different from groups. We present new results on the amenability and weak amenability of the L1-algebra of commutative hypergroups. Moreover we investigate the alpha-amenability and the existence of nonzero bounded point derivations. The examples, which are presented, will be from the class of polynomial hypergroups.

Means on translation-invariant Banach spaces defined on hypergroups

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(Joint work with Mahatheva Skantharajah)
Fixed point property and the Fourier algebra of a locally compact group

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Abstract: In recent years, there have been considerable interests in the study of when a Banach space has the weak fixed point property, i.e. whenever $T$ is a non-expansive mapping on a non-empty weakly compact convex subset $K$ of a Banach space into $K$, $K$ contains a fixed point for $E$. In this talk, I shall discuss some recent joint work with Michael Leinert related to fixed point property of the Fourier algebra of a locally compact group.

Property $T$-like for a unital $C^*$-algebra

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Abstract: In this talk, we first look at a weaker form of property $T$ for a $C^*$-algebra. And then strong property $T$ for a unital $C^*$-algebra is introduced. Some permanence properties for both property ($T$) and strong property ($T$) are discussed. We will also relate them to the property ($T$) of the unitary group of a $C^*$-algebra.

(Joint with C.K. Ng and N.C. Wong)
Disjointness preserving operators on group algebras

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Abstract: Let $G$ and $H$ be locally compact amenable groups. A linear operator $T : L^1(G) \to L^1(H)$ is called a disjointness preserving operator if $TuTv = 0$ in $L^1(H)$ whenever $uv = 0$ in $L^1(G)$. We show that a bounded linear disjointness preserving operator from $L^1(G)$ onto $L^1(H)$ is a weighted bounded homomorphism.

Intertwining Operators for Exponential Solvable Lie groups

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Flat orbits, minimal ideals and spectral synthesis

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Joint work with D. Alexander and J. Ludwig

Abstract: Let $G = \exp \mathfrak{g}$ be a connected, simply connected, nilpotent Lie group and $\omega$ be a continuous, symmetric weight with polynomial growth on $G$. Let $l \in \mathfrak{g}^*$ be such that its co-adjoint orbit $\Omega(l)$ is flat and let $\pi_i$ denote the corresponding element of $\hat{G}$ obtained by the Kirillov theory. The minimal ideal $j(\{\pi_i\})$ with hull $\{\pi_i\}$ of the weighted group algebra $L^1_\omega(G)$ is then characterized. If $N = \exp \mathfrak{g}(l)$ is the stabilizer subgroup of $l$, the set of all $L^\infty(G/N)$-invariant closed ideals of $L^1_\omega(G)$ with hull $\{\pi_i\}$ is in bijection with the set of translation-invariant, $G$-invariant subspaces of the space of all polynomial functions on $N$ dominated by the weight $\omega|_N$. These ideals may be characterized as kernels of specially built induced representations. If $\omega \equiv 1$ and $\Omega(l)$ is flat, then $\{\pi_i\}$ is a set of spectral synthesis with respect to the group algebra $L^1(G)$. These results are generalized to the situation where the orbit $\Omega(l)$ is not flat, but where $N$ is a closed normal subgroup of $G$ contained in the stabilizer subgroup of $l$.

Crossed products of $C^*$-correspondences by amenable group actions

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The topology on the unitary dual of diamond groups

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Abstract: Let $N$ be a simply connected two-step nilpotent Lie group, and let $S$ be a compact abelian Lie group acting continuously as a group of automorphisms on $N$. For the unitary dual $\hat{G}$ of the semidirect product $G = S \rtimes N$ there is an orbit picture, either by a slight extension of Pukanszky's work (extension because $S$ is not assumed to be connected) or by Lipsman (who studies in general semidirect products of compact Lie groups with simply connected nilpotent Lie groups). In any event, there is a finite-to-one map from $\hat{G}$ into the orbit space $\mathfrak{g}^*/G$, where, as usual, $\mathfrak{g}^*$ denotes the linear dual of the Lie algebra $\mathfrak{g}$ of $G$. We ask for topological properties of this map $\hat{G} \to \mathfrak{g}^*/G$. In the case that $N$ is a Heisenberg group it was shown by J. Ludwig and myself that this map is continuous, but disappointing examples showed that in general this map is not open onto its image. But if one restricts to the open subset $\hat{G}$ of class 1 representations, i.e., representations containing an $S$-fixed vector, one can show that $\hat{G}_1 \to \mathfrak{g}^*/G$ is open onto its image, after proving a theorem in the geometry of numbers. In the talk we discuss possibilities to extend this result to more general representations, to more general two-step nilpotent Lie groups, and, finally, to give a topological orbit picture of $\hat{G}$ by including further parameters.

Some remarks on locally compact quantum groups

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Convolutions on compact groups and Fourier algebras of coset spaces

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Abstract: With my coauthors, Brian Forrest and Ebrahim Samei, I address two related questions. (1) For a compact group $G$, what are the ranges of the convolution maps on $A(G)$ given for $u, v$ in $A(G)$ by $uXv \mapsto u \ast v'(v'(s) = v(s^{-1}))$ and $uXv \mapsto u \ast v'$? (2) For a locally compact group $G$ and a compact subgroup $K$, what are the amenability properties of the Fourier algebra of the coset space $A(G/K)$? The algebra $A(G/K)$ was defined and studied by Forrest.

In answering the first question, we obtain for compact groups which do not admit an abelian subgroup of finite index, some new subalgebras of $A(G)$. Using those algebras we can find many instances in which $A(G/K)$ fails the most rudimentary amenability property: operator weak amenability. However, using different techniques, we show that if the connected component of the identity of $G$ is abelian, then $A(G/K)$ always satisfies the stronger property that it is hyper-Tauberian, which is a concept developed by the second named author. We also establish a criterion which characterises operator amenability of $A(G/K)$ for a class of groups which includes the maximally almost periodic groups. We also establish new sets of spectral synthesis and nonsynthesis for $A(G)$ in some cases.

Operator amenability of Fourier-Stieltjes algebras and amenability constants

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Extensions of Fourier algebra homomorphisms

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Abstract: For a locally compact group $G$, $A(G)$ and $B(G)$ are the Fourier and Fourier-Stieltjes algebras of $G$, and $UCB(\hat{G})^*$ is the dual, with Arens product, of the C*-algebra of uniformly continuous functionals on $A(G)$. When $G$ is amenable, there are completely isometric inclusions $A(G) \subset B(G) \hookrightarrow UCB(\hat{G})^*$. In this case, a complete characterization of the completely bounded homomorphisms $\phi : A(G) \to B(H)$, in terms of piecewise affine maps $\alpha : Y \subset H \to G$, has been given by M. Ilie and N. Spronk. When $H$ is also amenable, we describe how any such homomorphism can be extended first to a completely bounded homomorphism $\phi : B(G) \to B(H)$ which is multiplier–weak* continuous, and then to a completely bounded weak*–weak* continuous homomorphism $\phi : UCB(\hat{G})^* \to UCB(\hat{H})^*$. The dual of $C_\tau^*(M(G))$, the C*-algebra generated by the left regular representation on the measure algebra $M(G)$, also contains a completely isometric copy of $B(G)$. We will show that Arens product on $C_\tau^*(M(G))^*$ is well-defined, and will explain how to extend $\phi : B(G) \to B(H)$ to a cb weak*–weak* continuous homomorphism $\phi : C_\tau^*(M(G))^* \to C_\tau^*(M(H))^*$. These classes of homomorphic extensions of $\phi : A(G) \to B(H)$ are also completely described in terms of piecewise affine maps. Dual versions of these results will also be discussed.

This is joint work with Monica Ilie.

Two problems on separability of linear functionals of two parameters on an $L^2$-space

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Irreducibility of the Principle Series Representations

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Abstract: We provide an alternate proof that the Principle Series Representations of \( SL(2, R) \) are irreducible.

Existence of closed principal ideals

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Abstract: Let \( A \) be an arbitrary Banach algebra. This talk is centered around the following problem. When does the algebra \( A \) have a central element \( a \) such that the principal ideal \( aA \) is closed proper ideal of \( A \)?
Disjointness preserving maps of $C^*$-algebras and group $C^*$-algebras

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Abstract: There are at least four versions to say a linear map $T$ between two $C^*$-algebras is ‘disjointness preserving’:

- $ab = 0$, $a^*b = 0$, $ab^* = 0$ or $a^*b = ab^* = 0$ implies $TaTb = 0$, $(Ta)^*(Tb) = 0$,
- $(Ta)(Tb)^* = 0$ or $(Ta)^*(Tb) = (Ta)(Tb)^*$,

The second, third and the last say $T$ send operators with orthogonal ranges, orthogonal domains, or both, to operators with the same disjointness. The first, however, lacks of such trivial geometrical sense.

Quite an amount of efforts has been put in characterizing such disjointness preserving maps by me in recent years. They are basically a homomorphism or a *-homomorphism followed by a multiplication of a left or right or central multiplier.

In this talk, after a brief report on these results, I will explore the similar problem of group $C^*$-algebras and Fourier algebras.