1. (20pts) If $u'' - (1 + x^2)u = 0$ for $0 < x < 1, u(0) = 1, u(1) = 0$, find upper and lower bounds for $u\left(\frac{1}{2}\right)$.

2. (30pts) If $u'' - (1 + x^2)u = 0, 0 < x < 1, u(0) = 1, u(1) = 0$. Find upper and lower bounds for $u'(0)$.
   
   Hint: choose $z_1$ and $z_2$ so that $z_1(0) = z_2(0) = 1$; use $z_1(x) - z_1(a) \leq u(x) - u(a) \leq z_2(x) - z_2(a)$ to find bounds for the difference quotient $\frac{u(x) - u(a)}{x}$.

3. (30pts) Suppose that $u$ is a solution of $u'' + g(x)u' + h(x)u = 0$ for $a < x < b$ with $h(x) \leq 0$. Show how to construct a constant $K$ depending only on bounds for $g, h$ and $b - a$ such that if $|u| \leq M$ on $[a, b]$, then $|u'(a)| \leq MK$.
   
   Hint: construct $z_1$ and $z_2$ so that $z_1(a) = z_2(a) = u(a)$.

4. (20pts) If $u$ satisfies $u'' + xu' - x^2u = 1$ for $x > 0$, $u(0) = u'(0) = 0$, find the upper and lower bounds for $u(1)$ and $u'(1)$.