Assignment 1 for MAT3220 (0.1-0.4)  
(no need to hand in)

**Problem 1** Consider the following transportation problem

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_{11}$ = 2</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>$c_{21}$ = 1</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>$c_{12}$ = 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{22}$ = 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{13}$ = 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{23}$ = 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{14}$ = 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{24}$ = 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

(a) Write down the transportation problem.

(b) Write down the matrix $A$. What is the rank $(A) =$?

(c) Find a square matrix $B$ inside $A$ such that $B$ is non-singular and the size of $B$ is the rank $(A)$.

(d) Use loop method to write all the other $a_{ij}$ in terms of the columns in $B$.

**Problem 2** In Problem 1, suppose that the transportation tableau is

<table>
<thead>
<tr>
<th></th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th></th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{11}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{12}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{21}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{22}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{23}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{24}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find out the basic solution and the basic matrix $B$. Write all other columns of $A$ in terms of columns of $B$. Compute $z_{ij} - c_{ij}$. Find out the entering variable and leaving variable.
Problem 3  Consider the following transportation model

\[
\begin{array}{cccc}
10 & 2 & 20 & 11 \\
12 & 7 & 9 & 20 \\
4 & 14 & 16 & 18 \\
\end{array}
\]

Supply

\[
\begin{array}{c}
15 \\
25 \\
10 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 15 & 15 & 15 \\
\end{array}
\]

demand

(a) What is the rank (A)?
(b) Use least-cost method to obtain a starting BFS.
(c) Compute the starting objective value.
(d) Obtain the transportation tableau. Find out the basic matrix B, write all other columns of A in terms of columns of B. Compute \( z_{ij} - c_{ij} \).
(e) Determine the entering and leaving variables and obtain the next transportation tableau.

Problem 4.  Do the same as in Problem 3 for the following transportation problem

\[
\begin{array}{ccc}
5 & 1 & 8 \\
2 & 4 & 0 \\
3 & 6 & 7 \\
\end{array}
\]

Supply

\[
\begin{array}{c}
12 \\
14 \\
4 \\
\end{array}
\]

demand

9 10 11