Assignment 4 for MAT5011 (Due Date: Nov. 21, 2009)

Chapter 3: 4, 5, 7, 11, 14, 16, 18

Extra Problem 1: Let $1 \leq p < +\infty, 1 \leq r \leq +\infty$. Show that $L^p(R^k) \cap L^r(R^k)$ is dense in $L^p(R^k)$.

Extra Problem 2: Suppose that $f \in L^p(R^k)(1 \leq p < \infty)$. Show that $\lim_{t\to 0} \int_{R^k} |f(x+t)-f(x)|^p dm = 0$, $\lim_{|t|\to+\infty} \int_{R^k} |f(x)+f(x-t)|^p dm = 2\int_{R^k} |f(x)|^p dm$

Extra Problem 3: Prove the following generalized Holder's inequality: Let $f_i \in L^{p_i}(X), p_i > 1, i = 1, ..., k, \frac{1}{p_1} + \frac{1}{p_2} + ... + \frac{1}{p_k} = 1$. Then

$$\int_X |f_1 f_2 ... f_k| \leq \|f_1\|_{L^{p_1}} \|f_2\|_{L^{p_2}} ... \|f_k\|_{L^{p_k}}$$