Assignment 2 for MAT5011

Due: Oct. 24th (Sat.), 2009

Chapt.1, 10, 12

Extra Problem 3: Suppose that $f_n \geq 0$, measurable and $\int_X f_n d\mu \to 0$ as $n \to +\infty$. Does it follow that $f_n \to 0, a.e.$ in μ ?

Extra Problem 4: Let $E_1 \subset E_2 \subset ... \subset E_n \subset ...$ be measurable sets and $E = \cup_{i=1}^{\infty} E_i$. Supoose that $f \in L^1(E_k, \mu), k = 1, ...,$ and that $\lim_{k\to+\infty} \int_{E_k} |f(x)| d\mu < +\infty$. Show that $f \in L^1(E, \mu)$ and that

$$\int_E f d\mu = \lim_{k\to+\infty} \int_{E_k} f d\mu$$

Extra Problem 5:Let (X, μ) be a measurable space and $f : X \to R$ be meas urable.

(a) If $\mu(X) < \infty$, show that $f \in L^1(X, \mu)$ if and only if

$$\sum_{n=1}^{\infty} 2^n \mu\{x \in X : |f(x)| \geq 2^n\} < \infty.$$

(b) If $\mu(X) = \infty$ but f is bounded, show that $f \in L^1(X, \mu)$ if and only if

$$\sum_{n=1}^{\infty} 2^{-n} \mu\{x \in X : |f(x)| \geq 2^{-n}\} < \infty.$$

Chapt. 2, 1, 2, 3