Assignment 2 for MAT 4220 (07/08)
(no need to turn in)

Exercise 2.1, 2, 5, 6, 7, 8, 9

Exercise 2.2, 1, 2, 3, 5

More on 2.2.
Consider the diffusiton equation with Robin boundary condition

$$u_t - ku_{xx} = 0, 0 < x < l, t > 0$$

$$u(x, 0) = \phi(x)$$

$$u_x(0, t) - a_0 u(0, t) = 0, u_x(l, t) + a_l u(l, t) = 0$$

(a) Show that if $a_0 > 0, a_1 > 0$, then the integral $\int_0^l u^2(x, t)dx$ is a decreasing function.
(b) Prove that if $a_0 > 0, a_1 > 0$, then the solution to the above probl em is unique.

Exercise 2.3, 1, 2, 3, 4, 6, 7, 8

More on 2.3:
Extra 1. Consider the diffusion equation $u_t = ku_{xx} + au$ in $(0 < x < 1, 0 < t < \infty)$ with $u(0, t) = u(1, t) = 0$ and $u(x, 0) = sin(\pi x)$, where $k > 0, a$ are real numbers.
(1) Show that $0 < u(x, t) < 1$ for all $t > 0$ and $0 < x < 1$.
(2) Show that $u(x, t) = u(1 - x, t)$ for all $t \geq 0$ and $0 \leq x \leq 1$.

Extra 2:(a) Prove the following generalized Maximum Principle:
If $u_t - ku_{xx} \leq 0$ in $R = [0, \ell] \times [0, T]$, then

$$\max_R u(x, t) = \max_{\partial R} u(x, t).$$

Hint: follow the proof of Maximum Principle.
(b) Show that if $v(x, t)$ satisfies

$$v_t = kv_{xx} + f(x, t), \ -\infty < x < +\infty, 0 < t < T$$

$$v(x, 0) = 0$$

then

$$v(x, y) \leq T \max_{-\infty<x<+\infty, \ 0<t<T} f(x, t).$$

Hint: consider

$$u(x, t) = v(x, t) - t \max_{-\infty<x<+\infty, 0<t<T} f(x, t)$$

and then use (a).

Exercise 2.4, 1, 2,4, 6, 7, 8, 11, 14, 15, 16, 18, 19

Exercise 2.5, 1