

### Assignment 7 for MAT4220 (Last Assignment)

Exercise 7.1, 1, 2, 3, 4, 5, 6

Extra Problems:1. Consider the following problem

$$\begin{cases} \Delta u = u^3 & \text{in } D, \\ \frac{\partial u}{\partial n} + a(x)u = h & \text{on } \partial D \end{cases} \quad (0.1)$$

where

$$a(x) \geq 0.$$

Show that the solution to (0.1) (if exists) is unique.

2. Consider the following problem

$$\begin{cases} \Delta u - b(x)u = f(x) & \text{in } D, \\ u = h & \text{on } \partial D. \end{cases} \quad (0.2)$$

Let  $u$  be a  $C^2$  function.

- (a) Define an energy functional  $E[u]$  associated with (0.2).
- (b) Show that  $u$  is a solution to (0.2) if and only if

$$E[w] \geq E[u] \quad \forall w \in C^2, w = h \text{ on } \partial D.$$

Exercise 7.2,1,2,3

Extra Problem: formulate and prove Problem 2 in two-dimensional case.

Exercise 7.3, 1, 3

Exercise 7.4, 1, 2, 3, 5, 6, 7, 9, 10, 13, 15,17(a)