MAT4220 PDE——Assignment 5

Exercise 6.1, 2, 4, 5, 6, 7, 10, 11

Exercise 6.2, 1, 2, 3,4, 6, 7(a)

Exercise 6.3, 1, 2, 3

Problem 4. Let $u \geq 0$ and $\Delta u = 0$ in a unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$. Using the Mean-Value Property to prove the following so-called Harnack inequality

$$\frac{1-r}{1+r} u(0,0) \leq u(x,y) \leq \frac{1+r}{1-r} u(0,0)$$

where $r = \sqrt{x^2 + y^2} < 1$.

***Problem 5: Suppose that u satisfies $u_{xx} + u_{yy} = 0$ for all $(x, y) \in B_1(0)$ except $(x, y) = (0, 0)$. Show that if u is a bounded, then $\lim_{(x,y)\to(0,0)} u(x, y)$ exists and by taking $u(0, 0) = \lim_{(x,y)\to(0,0)} u(x, y)$, u is actually smooth in $B_1(0)$.

Hint: Consider the following function $v_\epsilon = \epsilon \log \frac{1}{r}$.

Exercise 6.4, 1, 6, 9, 10, 11, 13

Problem 6: Using the method of separation of variables to solve the following problem

$$\begin{cases} u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0, \text{ in } D = \{(r, \theta) | 1 < r < 2\} \\ u_r(1, \theta) + u(1, \theta) = \sin^3 \theta \text{ for } \quad r = 1 \\ u(2, \theta) = 1 \text{ for } \quad r = 2 \end{cases} \tag{0.1}$$