A summary of Simplex Method, M-method and Two-Phase Method

1. M-method or Two-Phase method

1.1 If $B = I_m$, $C_B \neq 0 \Rightarrow$ kill C_B first

1.2. If $\exists$ a free variable $\Rightarrow$ get rid of free variable first

1.3. If one of y_i's becomes basic at positive level, or $x_0^* < 0$
$\Rightarrow$ feasible region is empty.

1.4. If one of y_i's becomes basic at zero level and $x_0^* = 0$
$\Rightarrow$ Use Alternate Rule

2. Simplex Method

2.1. If $B = I_m$, $C_B \neq 0 \Rightarrow$ kill C_B first

2.2. If $\exists$ a column $\vec{y}_j = \begin{pmatrix} y_{1j} \\ \vdots \\ y_{mj} \end{pmatrix} \le 0 \Rightarrow$ feasible region is unbounded in x_j-direction

2.3. If $\exists$ a column $\vec{y}_j = \begin{pmatrix} y_{1j} \\ \vdots \\ y_{mj} \end{pmatrix} \le 0$ and $z_j - c_j < 0 \Rightarrow$ feasible region is unbounded in x_j-direction and optimal value is $+\infty$.

3. Check the last simplex table (optimal)

	x_1	$\cdots$	x_j	$\cdots$	x_n	
x_{B_1}						
$\vdots$						
x_{B_m}						
x_0	$z_1 - c_1$	$\cdots$	$z_j - c_j$	$\cdots$	$z_n - c_n$	

3.1. If $\exists$ a column $\vec{y}_j = \begin{pmatrix} y_{1j} \\ \vdots \\ y_{mj} \end{pmatrix} \le 0 \Rightarrow$ feasible region is unbdd in x_j-direction.

3.2. If $z_j - c_j > 0$ for all non-basic variables, then the optimal solution is unique

3.3. If $z_j - c_j = 0$ for some non-basic j, and either $\vec{y}_j \le 0$, or $\min\left\{ \frac{x_{B_i}}{y_{ij}} \,\middle|\, y_{ij} > 0 \right\} > 0$, then $\exists$ too-many optimal solution (some of them may be non basic)

$$(x_{B_1} - \theta y_{1j}, \cdots, x_{B_i} - \theta y_{ij}, \cdots, x_{B_m} - \theta y_{mj}, 0, \cdots, \overset{\hat{x}_j}{\theta}, \cdots, 0), \quad 0 < \theta < \min\left\{ \frac{x_{B_i}}{y_{ij}} \,\middle|\, y_{ij} > 0 \right\}$$