Assignment 8 For MAT3210 – Linear Programming:5.2, 5.3

No need to hand in

1. Consider the following problem

$$\begin{aligned} \text{maximize} \quad & x_0 = c^T x \\ \text{subject to} \quad & Ax = b \\ & \ell_i \leqslant x_i \leqslant u_i, \ i = 1, \cdots, n \end{aligned}$$

   (a) Find the dual problem.
   (b) Show that the dual always possesses a FS.
   (c) What can you say about the primal objective function value?

2. Consider the following LPP,

$$\begin{aligned} \text{maximize} \quad & z = x_1 + 5x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + 2x_2 + x_3 = 3 \\ & 2x_1 - x_2 = 4 \\ & x_1, \ x_2, \ x_3 \geqslant 0 \end{aligned}$$

   Given that the optimal basic variables are x_1 and x_3, determine the associated optimal dual solution.

3. Consider the following LP.

$$\begin{aligned} \text{minimize} \quad & z = 5x_1 + 2x_2 \\ \text{subject to} \quad & x_1 - x_2 \geqslant 3 \\ & 2x_1 + 3x_2 \geqslant 5 \\ & x_1, \ x_2 \geqslant 0 \end{aligned}$$

   (a) Without solving the LP, estimate a range for the optimal objective value.
   (b) Let y_1 and y_2 be the dual variables. Determine whether the following pair of prime-dual solutions are optimal: $(x_1, x_2) = (3, 1), (y_1, y_2) = (4, 1)$.

4. Consider the following LPP.

$$\begin{aligned} \text{minimize} \quad & 3x_1 + 5x_2 - x_3 + 2x_4 - 4x_5 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + 3x_4 + x_5 \leqslant 6 \\ & -x_1 - x_2 + 2x_3 + x_4 - x_5 \geqslant 3 \\ & x_1, \ x_2, \ x_3, \ x_4, \ x_5 \geqslant 0 \end{aligned}$$

   (a) Write down the dual problem.
   (b) Given that $(-3, 1)$ is an optimal solution to the dual, find an optimal solution to the primal problem.
   (Hint: use complementary slackness properties)

5. Consider the following LPP

$$\begin{aligned} \text{maximize} \quad & 6x_1 + 7x_2 + 3x_3 + 2x_4 + x_5 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + x_4 = 6 \\ & 2x_1 + 3x_2 + 4x_2 + x_5 = 14 \\ & x_1,\ x_2,\ x_3,\ x_4,\ x_5 \geqslant 0 \end{aligned}$$

The reduced cost row of the optimal tableau is given by

	x_1	x_2	x_3	x_4	x_5	b
x_0	0	0	5	2	0	38

(a) Write down the dual problem.

(b) Making use of the given reduced cost row, find an optimal dual solution. Note that: $(u_1^*, u_2^*) \neq (2, 0)$.

6. True or False

(a) If the dual problem is infeasible, then the primal problem is infeasible

(b) If the dual problem has a finite optimal solution, then the primal has a final optimal solution

(c) If the dual problem has no feasible solution, the primal has an unbounded maximum

(d) If the primal has an unbounded maximum, the dual problem has no feasible solution

(e) If the primal problem has a feasible solution, the dual problem has a finite optimal solution

(f) If the dual problem has an unbounded minimum, the primal problem has no feasible solution