Solution Keys to MAT 3210 Assignment 7

1. The dual is given by

$$\begin{aligned} \text{minimize} \quad & u_0 = 50u_1 + 10u_2 + 2u_3 \\ \text{subject to} \quad & 3u_1 + u_2 + u_3 \geq 40 \\ & -3u_1 - u_3 \geq 20 \\ & 5u_1 + u_2 + 4u_3 = 2 \\ & u_1, u_2, u_3 \geq 0 \end{aligned}$$

2. The dual is given by

$$\begin{aligned} \text{minimize} \quad & u_0 = 5u_1 + 2u_2 + 20u_3 \\ \text{subject to} \quad & u_1 + u_2 + 7u_3 = 3 \\ & 2u_1 + u_2 + 3u_3 = 1 \\ & -u_2 - 5u_3 = 0 \\ & u_1, u_2, u_3 \geq 0 \end{aligned}$$

3. First convert the second constraint to $x_1 - 5x_2 \leq -3$. Then the dual is given by

$$\begin{aligned} \text{minimize} \quad & u_0 = 5u_1 - 3u_2 \\ \text{subject to} \quad & u_1 + u_2 = 5 \\ & -2u_1 + 5u_2 \geq 6 \\ & u_2 \geq 0, \; u_1 \text{ free} \end{aligned}$$

4. First convert the first constraint to $-x_1 - 2x_2 \geq -5$. Then the dual is given by

$$\begin{aligned} \text{maximize} \quad & u_0 = -5u_1 + 2u_2 + 20u_3 \\ \text{subject to} \quad & -u_1 + u_2 + 7u_3 \leq 3 \\ & -2u_1 + u_2 + 3u_3 \leq 1 \\ & -u_2 - 5u_3 = 0 \\ & u_1, u_2 \geq 0, \; u_3 \text{ free} \end{aligned}$$

5. First convert the first constraint to $-2x_1 - x_2 \geq -5$. Then the dual is given by

$$\begin{aligned} \text{maximize} \quad & u_0 = -5u_1 + 6u_2 \\ \text{subject to} \quad & -2u_1 + 3u_2 = 1 \\ & -u_1 - u_2 = 1 \\ & u_1, u_2 \geq 0 \end{aligned}$$

6. First convert $x_2 \le 0$ to $-x_2 \ge 0$ and the first constraint to $-a_{11}x_1+a_{12}x_2-a_{13}x_3 \le -b_1$. Then the dual is given by

$$\begin{aligned} \text{minimize} \quad & u_0 = -b_1u_1 + b_2u_2 + b_3u_3 \\ \text{subject to} \quad & -a_{11}u_1 + a_{21}u_2 + a_{31}u_3 \ge c_1 \\ & a_{12}u_1 - a_{22}u_2 - a_{32}u_3 \ge -c_2 \\ & -a_{13}u_1 + a_{23}u_2 + a_{33}u_3 = c_3 \\ & u_1, u_3 \ge 0, \ u_2 \text{ free} \end{aligned}$$

7.

$$l_i \le x_i \le u_i \quad \Rightarrow \quad 0 \le x_i - l_i \le u_i - l_i$$

So the problem can be reformulated as

$$\begin{aligned} \text{maximize} \quad & x_0 = \mathbf{c}^T\mathbf{x} + \mathbf{c}^T\mathbf{l} \\ \text{subject to} \quad & A\mathbf{x} = \mathbf{b} - A\mathbf{l} \\ & I\mathbf{x} \le \mathbf{u} - \mathbf{l} \\ & \mathbf{x} \ge 0 \end{aligned}$$

where $\mathbf{u} = [u_1, \cdots, u_n]^T$, $\mathbf{l} = [l_1, \cdots, l_n]^T$.

Denote

$$\tilde{A}^T = [A^T, \ I]; \quad \tilde{\mathbf{b}}^T = [\mathbf{b} - A\mathbf{l}, \ \mathbf{u} - \mathbf{l}]^T$$

Then the dual is given by

$$\begin{aligned} \text{minimize} \quad & \tilde{\mathbf{b}}^T\mathbf{y} \\ \text{subject to} \quad & \tilde{A}^T\mathbf{y} \ge \mathbf{c} \\ & y_1, \cdots, y_m \text{ free}; \ y_{m+1}, \cdots, y_{m+n} \ge 0 \end{aligned}$$

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