**Assignment 6 for MAT 3210 – Linear Programming, Chapter 4**

1. Consider the following LPP:

   Maximize $x_0 = 40x_1 + 20x_2 + 2x_3$

   Subject to
   $$3x_1 - 3x_2 + 5x_3 \le 50$$
   $$x_1 + x_3 \le 10$$
   $$x_1 - x_2 + 4x_3 \le 2$$
   $$x_1, x_2, x_3 \ge 0$$

   1) At which direction, the solution space is unbounded ?

   2) Solve the above LPP.

2. Show how the M-method and two-phase method will conclude that the following problem has no feasible solution.

   Maximize $x_0 = 2x_1 + 5x_2$

   Subject to
   $$3x_1 + 2x_2 \ge 6$$
   $$2x_1 + x_2 \le 2$$
   $$x_1, x_2 \ge 0$$

3. Solve the following LPP:
   $$\text{max. } x_0 = x_1 - x_3$$
   subject to
   $$x_1 + x_2 + x_3 = 1$$
   $$x_1 + 2x_2 + 3x_3 - x_4 = 3$$
   $$-x_1 + x_2 - 2x_3 + x_5 = 1$$
   $$x_1 \text{ is free, } x_2, x_3, x_4, x_5 \ge 0$$
   and answer the following questions:

   (a) Is the feasible region unbounded?

   (b) Is the optimal solution unique? If not, find other optimal solutions which are are not basic.

4. Use Two-phase method to solve the following LPP
   $$\text{Minimize} \qquad x_0 = 4x_1 + 3x_3$$
   Subject to
   $$3x_1 + 6x_2 + 3x_3 - 4x_4 = 12$$
   $$2x_1 + x_3 = 4$$
   $$3x_1 - 6x_2 + 4x_4 = 0$$
   $$x_1, x_2, x_3, x_4 \ge 0$$
   and answer the following questions: is the feasible region unbounded? is the optimal solution unique? If it is not unique, find another optimal solution.

5. The following tableau represents a specific simplex iteration.

| Basic | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | solution |
|---|---|---|---|---|---|---|---|---|---|
| $x_8$ | 0 | 3 | 0 | -2 | -3 | -1 | 5 | 1 | 2 |
| $x_3$ | 0 | 2 | 1 | 3 | 1 | 0 | 3 | 0 | 6 |
| $x_1$ | 1 | -1 | 0 | 0 | 6 | -4 | 0 | 0 | 0 |
| $z$ | 0 | 0 | 0 | 4 | 1 | 10 | 1 | 0 | 60 |

(a) at which direction, the feasible region is unbounded?

(b) Is the BFS unique? If not, find another optimal solution which is not basic.

6. For the following LP, show that the optimal solution is degenerate and that there exist alternative solutions that are all non-basic.

$$\begin{aligned} \text{maximize} \quad & z = 3x_1 + x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leqslant 5 \\ & x_1 + x_2 - x_3 \leqslant 2 \\ & 7x_1 + 3x_2 - 5x_3 \leqslant 20 \\ & x_1,\ x_2,\ x_3 \geqslant 0 \end{aligned}$$

7. Consider the following LP model

$$\begin{aligned} \text{maximize} \quad & z = 3x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 \leqslant 2 \\ & 3x_1 + 4x_2 + 2x_3 \geqslant 8 \\ & x_1,\ x_2,\ x_3 \geqslant 0 \end{aligned}$$

Show by the two-phase method that the optimal solution includes an artificial basic variable. Use alternate rule to solve the problem.

8. True or False

(a) In Phase I of two-phase method, if $x_0^* < 0$, then the feasible region is empty.

(b) In Phase I of two-phase method, if one of the $y_i$ becomes basic, then the feasible region is empty.

(c) If $c_j - z_j < 0$ for all nonbasic varibale $x_j$, then the optimal solution is unique.

(d) If $c_j - z_j \leq 0$ for all $j$ and $c_j - z_j = 0$ for some nonbasic varibale $x_j$, then the optimal solution is not unique.

(e) If there exists $y_j$ such that $y_{ij} \leq 0$ for all $i = 1, ..., m$, then the optimal value is infinity.

(f) If there exists $y_j$ such that $y_{ij} \leq 0$ for all $i = 1, ..., m$ and $c_j - z_j \geq 0$, then the optimal value is infinity.

(g) If there exists $y_j$ such that $y_{ij} \leq 0$ for all $i = 1, ..., m$ and $c_j - z_j > 0$, then the optimal value is infinity.