

Solution Keys to MAT 3210 Assignment 4

1. Solution

Tableau 1: $\mathbf{x} = \begin{bmatrix} 0 & 0 & 2 & 1 \end{bmatrix}^T$, $z = 0$.

	x_1	x_2	x_3	x_4	b	
x_3	1	1*	1	0	2	$\frac{2}{1} = 2^*$
x_4	1	-1	0	1	1	-
x_0	1	-2	0	0	0	

Tableau 2: Optimal tableau. $\mathbf{x}^* = \begin{bmatrix} 0 & 2 & 0 & 3 \end{bmatrix}^T$, $z^* = 4$.

	x_1	x_2	x_3	x_4	b
x_2	1	1	1	0	2
x_4	2	0	1	1	3
x_0	3	0	2	0	4

2. Solution

Tableau 1: $\mathbf{x} = \begin{bmatrix} 0 & 0 & 2 & 1 & 1 \end{bmatrix}^T$, $z = 0$.

	x_1	x_2	x_3	x_4	x_5	b	
x_3	1	1	1	0	0	2	$\frac{2}{1} = 2$
x_4	0	1	0	1	0	1	-
x_5	1*	-1	0	0	1	1	$\frac{1}{1} = 1^*$
x_0	-2	1	0	0	0	0	

Tableau 2: $\mathbf{x} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \end{bmatrix}^T$, $z = 2$.

	x_1	x_2	x_3	x_4	x_5	b	
x_3	0	2*	1	0	-1	1	$\frac{1}{2}^*$
x_4	0	1	0	1	0	1	$\frac{1}{1} = 1$
x_1	1	-1	0	0	1	1	-
x_0	0	-1	0	0	2	2	

Tableau 3: Optimal tableau. $\mathbf{x}^* = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}^T$, $z^* = \frac{5}{2}$.

	x_1	x_2	x_3	x_4	x_5	b
x_3	0	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
x_4	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
x_1	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$
x_0	0	0	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{5}{2}$

3. Solution

Tableau 1: $\mathbf{x} = \left[0 \ 0 \ 0 \ 0 \ 40 \ 8 \ 10 \right]^T$, $z = 0$.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	
x_5	1	2	2	4	1	0	0	40	$\frac{40}{4} = 10$
x_6	2	-1	1	2*	0	1	0	8	$\frac{8}{2} = 4^*$
x_7	4	-2	1	-1	0	0	1	10	-
x_0	-3	1	-3	-4	0	0	0	0	

Tableau 2: $\mathbf{x} = \left[0 \ 0 \ 0 \ 4 \ 24 \ 0 \ 14 \right]^T$, $z = 16$.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	
x_5	-3	4*	0	0	1	-2	0	24	$\frac{24}{4} = 6^*$
x_4	1	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	4	-
x_7	5	$-\frac{5}{2}$	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1	14	-
x_0	1	-1	-1	0	0	2	0	16	

Tableau 3: $\mathbf{x} = \left[0 \ 6 \ 0 \ 7 \ 0 \ 0 \ 29 \right]^T$, $z = 22$.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	
x_2	$-\frac{3}{4}$	1	0	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	6	-
x_4	$\frac{5}{8}$	0	$\frac{1}{2}^*$	1	$\frac{1}{8}$	$\frac{1}{4}$	0	7	$7/\frac{1}{2} = 14^*$
x_7	$\frac{25}{8}$	0	$\frac{3}{2}$	0	$\frac{5}{8}$	$-\frac{3}{4}$	1	29	$29/\frac{3}{2} = \frac{58}{3}$
x_0	$\frac{1}{4}$	0	-1	0	$\frac{1}{4}$	$\frac{3}{2}$	0	22	

Tableau 4: Optimal tableau. $\mathbf{x}^* = \left[0 \ 6 \ 14 \ 0 \ 0 \ 0 \ 8 \right]^T$, $z^* = 36$.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
x_2	$-\frac{3}{4}$	1	0	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	6
x_3	$\frac{5}{4}$	0	1	2	$\frac{1}{4}$	$\frac{1}{2}$	0	14
x_7	$\frac{5}{4}$	0	0	-3	$\frac{1}{4}$	$-\frac{3}{2}$	1	8
x_0	$\frac{3}{2}$	0	0	2	$\frac{1}{2}$	2	0	36

4. Solution

(a). (1) x_3 ; (2) x_3 ; (3) x_1 ; (4) x_1 ; (5) x_3 .

(b). (1) $6 \times \frac{5}{2} = 15$; (2) $-6 \times \frac{4}{3} = -8$; (3) 0; (4) 0; (5) 0.

5. Solution

Standardization: maximize $z = -x_2 + 3x_3 - 2x_5$.

Tableau 0: Initial tableau

	x_1	x_2	x_3	x_4	x_5	x_6	b
	1	3	-1	0	2	0	7
	0	-2	4	1	0	0	12
	0	-4	3	0	8	1	10
x_0	0	1	-3	0	2	0	0

Tableau 1: $\mathbf{x} = \begin{bmatrix} 7 & 0 & 0 & 12 & 0 & 10 \end{bmatrix}^T$, $z = 0$

	x_1	x_2	x_3	x_4	x_5	x_6	b	
x_1	1	3	-1	0	2	0	7	-
x_4	0	-2	4*	1	0	0	12	$\frac{12}{4} = 3^*$
x_6	0	-4	3	0	8	1	10	$\frac{10}{3}$
x_0	0	1	-3	0	2	0	0	

Tableau 2: $\mathbf{x} = \begin{bmatrix} 10 & 0 & 0 & 3 & 0 & 1 \end{bmatrix}^T$, $z = 9$

	x_1	x_2	x_3	x_4	x_5	x_6	b	
x_1	1	$\frac{5}{2}^*$	0	$\frac{1}{4}$	2	0	10	$10/\frac{5}{2} = 4^*$
x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	0	0	3	-
x_6	0	$-\frac{5}{2}$	0	$-\frac{3}{4}$	8	1	1	-
x_0	0	$-\frac{1}{2}$	0	$\frac{3}{4}$	2	0	9	

Tableau 3: Optimal tableau. $\mathbf{x}^* = \begin{bmatrix} 0 & 4 & 5 & 0 & 0 & 11 \end{bmatrix}^T$, $z^* = 11$

	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	$\frac{2}{5}$	1	0	$\frac{1}{10}$	$\frac{4}{5}$	0	4
x_3	$\frac{1}{5}$	0	1	$\frac{3}{10}$	$\frac{2}{5}$	0	5
x_6	1	0	0	$-\frac{1}{2}$	10	1	11
x_0	$\frac{1}{5}$	0	0	$\frac{4}{5}$	$\frac{12}{5}$	0	11

6. Solution

Tableau 0: Initial tableau

	x_1	x_2	x_3	x_4	b
	5	20	1	0	400
	5	-5	-1	1	50
x_0	-45	-80	0	0	0

(a). x_3, x_4 are basic variables.

Tableau 1: $\mathbf{x} = \begin{bmatrix} 0 & 0 & 400 & 450 \end{bmatrix}^T$, $z = 0$.

	x_1	x_2	x_3	x_4	b	
x_3	5	20*	1	0	400	$\frac{400}{20} = 20^*$
x_4	10	15	0	1	450	$\frac{450}{15} = 30$
x_0	-45	-80	0	0	0	

Tableau 2: $\mathbf{x} = \begin{bmatrix} 0 & 0 & 400 & 450 \end{bmatrix}^T$, $z = 1600$.

	x_1	x_2	x_3	x_4	b	
x_2	$\frac{1}{4}$	1	$\frac{1}{20}$	0	20	$20/\frac{1}{4} = 80$
x_4	$\frac{25}{4}^*$	0	$-\frac{3}{4}$	1	150	$150/\frac{25}{4} = 24^*$
x_0	-25	0	4	0	1600	

Tableau 3: Optimal tableau. $\mathbf{x}^* = \begin{bmatrix} 24 & 14 & 0 & 0 \end{bmatrix}^T$, $z^* = 2200$.

	x_1	x_2	x_3	x_4	b
x_2	0	1	$\frac{2}{25}$	$-\frac{1}{25}$	14
x_1	1	0	$-\frac{3}{25}$	$\frac{4}{25}$	24
x_0	0	0	1	4	2200

(b). x_1, x_2 are basic variables.

Tableau 1: Optimal tableau. $x^* = [24 \ 14 \ 0 \ 0]^T, z^* = 2200$.

	x_1	x_2	x_3	x_4	b
x_1	1	0	$-\frac{3}{25}$	$\frac{4}{25}$	24
x_2	0	1	$\frac{2}{25}$	$-\frac{1}{25}$	14
x_0	0	0	1	4	2200

We get the optimal tableau immediately.

7. Solution

(a) Standard form: maximize $w = -z = 6x_1^+ - 6x_1^- - 8x_2$.

Tableau 0: initial tableau

	x_1^+	x_1^-	x_2	x_3	x_4	b
	-3	3	2	1	0	5
	0	0	1	2	1	4
x_0	-6	6	8	0	0	0

(b) x_2, x_3 are basic variables.

Tableau 1: $x = [0 \ 0 \ 2 \ 1 \ 0]^T, w = -16$.

	x_1^+	x_1^-	x_2	x_3	x_4	b	ratio
x_2	-2	2*	1	0	$-\frac{1}{3}$	2	1*
x_3	1	-1	0	1	$\frac{2}{3}$	1	
x_0	10	-10	0	0	$\frac{8}{3}$	-16	

Tableau 2: Optimal $x^* = [0 \ 1 \ 0 \ 2 \ 0]^T, w^* = -6$.

	x_1^+	x_1^-	x_2	x_3	x_4	b	ratio
x_1^-	-1	1	$\frac{1}{2}$	0	$-\frac{1}{6}$	1	
x_3	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	2	
x_0	0	0	5	0	1	-6	