Assignment 3 for MAT 3210 – Linear Programming (2.2, 2.3)
(no need to hand in)

(1) Transform the following LPP into standard form

$$\begin{aligned} \text{minimize} \quad & z = x_1 + x_3 \\ \text{subject to} \quad & -x_1 + x_2 \leqslant -1 \\ & x_3 - x_2 \geqslant 1 \\ & x_1 + x_3 \leqslant 3 \\ & x_1 \geqslant 0, x_2 \leqslant 0, \ x_3 \text{ is free} \end{aligned}$$

(2) Transform the following LPP into standard form

$$\begin{aligned} \text{minimize} \quad & z = x_1 - 2x_2 + x_4 \\ \text{subject to} \quad & x_1 + x_4 \leqslant 1 \\ & x_3 - x_4 \geqslant -1 \\ & x_2 + x_3 \leqslant 2 \\ & x_2 + x_5 = -1 \\ & x_1 \geqslant 0, \ x_2 \geqslant 0, x_3 \leqslant 0, \ x_4, x_5 \text{ are free} \end{aligned}$$

(3) Consider the following systems of linear equations

$$\begin{cases} 2x_1 + 3x_2 + x_3 = 6 \\ \ x_1 + 2x_2 \qquad + x_4 = 4 \\ x_1, x_2, x_3, x_4 \geqslant 0 \end{cases}$$

(a) Check $\mathbf{x}_0 = (1, \ 1, \ 1, \ 1)^T$ is a FS but not a BFS.
(b) Starting from $\mathbf{x}_0$, find a BFS.

(4) Consider the following LPP

$$\text{maximize } z = x_1 + x_2 - 4x_3$$

subject to

$$\begin{gathered} x_2 + 3x_3 = 3 \\ x_1 + 3x_2 - 2x_3 = 2 \\ x_1, x_2 \geq 0, \ x_3 \text{ is free.} \end{gathered}$$

Find out all possible BSs **for the standard form**. State if they are BFS, degenerate or nondegenerate. Find the optimal solution and optimal value.

(5) Consider the following set of LPP

$$\textit{minimize } z = 3x_1 - x_2$$

subject to

$$\begin{gathered} x_1 + 2x_2 \leq 4 \\ x_2 - x_1 \geq 0 \\ x_1 \geq 0, x_2 \geq 0. \end{gathered}$$

(a) Transform it into standard form.
(b) Check that $x_0 = (1, 1)$ is a FS but not a BFS. Starting from x_0, find out a BFS.

(c) Check if the BFS found in (b) is optimal. If not, find an improved BFS.
(d) Check if $x_1 = (0, 0)$ is a BFS. If so, check if it is optimal. If not, find an improved BFS.

(6) Consider the following LPP

$$maximize\ z = x_1 + 4x_2 + 7x_3 + 5x_4$$

subject to

$$2x_1 + x_2 + 2x_3 + 4x_4 = 10$$
$$3x_1 - x_2 - 2x_3 + 6x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(a)Check that $x_0 = (3,\ 4,\ 0,\ 0)^T$ is a BFS. Find the basis matrix B and y_1, y_2, y_3, y_4.
(b) Show that x_0 is not optimal.
(c) Determine the entering and leaving column and the improved BFS.
(d) Complete the computations in (c) until optimality is achieved, and find the optimal solution.

(7) Consider the following LPP

$$maximize\ z = 3x_1 + 4x_2$$

subject to

$$3x_1 - x_2 + 2x_3 + x_4 = 7$$
$$2x_1 - 4x_2 + x_5 = 12$$
$$-4x_1 - 3x_2 + 8x_3 + x_6 = 10$$

(a) Check that $x_0 = (0, 0, 0, 7, 12, 10)^T$ is a BFS. Find the basis matrix B and $y_1, y_2, y_3, y_4, y_5, y_6$.
(b) Show that x_0 is not optimal.
(c) Determine the entering and leaving column and the improved BFS.

(8) Consider the following LPP

$$\text{maximize } z = -x_1 + 2x_2$$

subject to

$$x_1 + x_2 + x_3 = 2,$$
$$x_1 - x_2 + x_4 = 1,$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(a) Check that $\mathbf{x}_0 = (1, 1, 0, 1)^T$ is a FS but not a BFS. Start ing from x_0 find a BFS.
(b) Let $\mathbf{x}_1 = (\frac{3}{2}, \frac{1}{2}, 0, 0)^T$. Show that $\mathbf{x}_1$ is a BFS. Find the basis matrix B and the reduced cost vector c_B. Check if $\mathbf{x}_1$ is an optimal sol ution. If not, find a better BFS.
(c) Complete the computations in (b) until optimality is achieved, and find the optimal solution.

(9) Consider the following statement. Is it true or false? Don't attempt to prove or disprove it.
( ) (1) Every supporting hyperplane of a closed convex set must contain at least one extreme point
( ) (2) If there is a feasible solution, then there must be a basic feasible solution.
( ) (3) The optimal value of an LPP can only be attained at an extreme point.
( ) (4) BFSs of a feasible region must lie on the boundary.
( ) (5) The optimal value of an LPP can not be achieved in the interior of the feasible region