Assignment 2 for MAT 3210 (1.8, 2.1, 2.2)

No need to hand in

(1) Consider the following set

$$C = \{(x_1, x_2) | x_1^2 + x_2^2 \leq 1, x_1 + x_2 \geq 0\}.$$

Is C convex? Is C bounded from below? Find out all extreme points of C. At each extreme point of C, find a supporting hyperplane.

(2) Show that for $\mathbf{b} \geq 0$, that $\mathbf{x} = 0$ is an extreme point of the feasible region $Q = \{\mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq 0\}$.

(3) Consider the set of linear inequalities

$$\begin{aligned} &x_1 \geqslant 0, \quad x_2 \geqslant 0 \\ &3x_1 + 2x_2 \leqslant 18, \\ &x_1 - x_2 \geqslant -6, \\ &5x_1 + 3x_2 \geqslant 20. \end{aligned}$$

3.1) Draw the graph of the above set. Compute the extreme points. Compute the supporting hyperplane at each extreme point.

3.2) Put the problem into standard form. Indicate the corresponding basic feasible solutions.

(4) Put the following Linear program in standard form:

$$\begin{aligned} &\text{minimize} && z = 10x_1 + 2x_2 - x_3 \\ &\text{subject to} && x_1 + x_2 \leqslant 30 \\ &&& x_1 - x_3 \geqslant 1 \\ &&& x_2 + x_3 \leqslant 3 \\ &&& x_2 + x_3 \geqslant -7 \\ &&& x_1 + x_2 + x_3 = 10 \\ &&& x_1, x_3 \geqslant 0 \end{aligned}$$

(5) Given the following set of equations:

$$x_1 + 4x_2 - x_3 = 3,$$

$$5x_1 + 2x_2 + 3x_3 = 4.$$

Determine the basic feasible solution involving x_1 and x_2. Do basic feasible solutions exist for x_1 and x_3 and for x_2 and x_3?

(6) Find all basic feasible solutions for the equations

$$\begin{cases} 2x_1 + 6x_2 + 2x_3 + x_4 = 3 \\ 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2 \end{cases}$$

State if they are degenerate or nondegenerate.

(7) Consider the following LPP

$$\text{maximize } z = x_1 + x_2 - 4x_3$$

subject to

$$x_2 + 3x_3 = 3$$
$$x_1 + 3x_2 - 2x_3 = 2$$
$$x_1, x_2 \geq 0,\ x_3 \text{ is free.}$$

Find out all possible BSs **for the standard form**. State if they are BFS , degenerate or nondegenerate. Find the optimal solution and optimal value.

(8) Consider the following system of equations

$$2x_1 + x_2 + 2x_3 + 4x_4 = 14$$
$$-x_1 + 2x_2 + 4x_3 - 2x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Check that $\mathbf{x}_0 = (3, 2, 1, 1)^T$ is a FS but not a BFS. Starting from x_0 find a BFS $\mathbf{x}_1$.

(9) Consider the following system of equations

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
1	−1	0	1	1	0	0	4
−1	1	2	0	0	1	0	1
−3	1	1	0	0	0	1	−4
−3	1	3	1	1	1	1	1

$$x_j \geqslant 0, \quad j = 1, \cdots, 7$$

(a) Show that $(2, 1, 1, 3, 0, 0, 0)$ is a feasible solution.
(b) Is it a BFS? If not, construct a BFS from it. State also the basis.

(10) Solve the following problem by computing all extreme points manually.
minimize $z = 4x_1 - 5x_2$
subject to

$$\begin{cases} 3x_1 + 2x_2 \leqslant 12 \\ x_1 + 2x_2 \leqslant 6 \\ -x_1 + x_2 \leqslant 1 \\ x_1 \geqslant 0 \\ x_2 \geqslant 0. \end{cases}$$