Assignment 9 For MAT3210 – Linear Programming-5.4, 5.5

No need to hand in

1. Solve the following LPPs by dual simplex method and find out the optimal values of all the primal and dual variables

$$(a) \qquad \min \quad 2x_1 + x_2 + x_3$$

subject to

$$x_1 + x_2 \leq 3$$
$$x_1 - 2x_2 \geq 1$$
$$x_2 + x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

$$(b) \qquad \min \quad 2x_1 + x_2$$

subject to

$$x_1 + x_2 \leq 5$$
$$7x_1 - x_2 \geq 21$$
$$x_1 \geq 4$$
$$x_1, x_2 \geq 0$$

$$(c) \qquad \max \quad 6x_1 + 7x_2 + 3x_3 + 5x_4$$

subject to

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geqslant 10$$
$$x_2 - 5x_3 - 6x_4 \geqslant 12$$
$$2x_1 + 5x_2 + x_3 + x_4 \geqslant 10$$
$$x_1,\ x_2,\ x_3,\ x_4 \geqslant 0$$

2. Consider the following LPP and its optimal tableau shown below.

$$\text{maximize} \quad x_0 = -4x_1 - 3x_2 + x_3 - x_4$$

subject to

$$-x_1 - x_2 + x_4 = 1$$
$$x_2 - x_3 + x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

The final simplex tableau is given as follows:

Basic	x_1	x_2	x_3	x_4	b
x_4	$-\frac{1}{2}$	0	$-\frac{1}{2}$	1	2
x_2	$\frac{1}{2}$	1	$-\frac{1}{2}$	0	1
x_0	3	0	1	0	-5

(a) Suppose $b = (1,3)^T$ is changed to $\hat{b} = (-3,1)^T$. Find the optimal solution. (You must use dual simplex method.)

(b) Suppose that a new constraint $2x_1 - x_2 + 2x_4 \leq 2$ is added. Solve the new problem by dual simplex method.

3. Consider the following LPP and its optimal tableau shown below.

$$\max \quad 2x_1 + 5x_2 - 2x_3$$

subject to

$$x_1 + x_2 - x_3 \leq 3$$
$$x_1 + 3x_2 - x_3 \leq 1$$
$$x_1, x_2, x_3, \geq 0$$

(a) Use simplex method to obtain the final simplex tableau.

(b) Suppose b is changed to $\hat{b} = (1, 2)^T$. Based on (a), solve the new LPP.

(c) Suppose a_2 is changed to $\hat{a}_2 = (2, 2)^T$. Based on (a), solve the new LPP.

4. Consider the following LPP:

$$\max \quad 2x_1 - x_2 + x_3$$

subject to

$$3x_1 + x_2 + x_3 \leq 60$$
$$x_1 - x_2 + 2x_3 \leq 10$$
$$x_1 + x_2 - x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

Let x_4, x_5 and x_6 denote the slack variables for the respective constraints. Suppose the final simplex tableau is given by the following:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	0	1	1	1	-1	-2	10
x_1	1	0	1/2	0	1/2	1/2	15
x_2	0	1	-3/2	0	-1/2	1/2	5
x_0	0	0	3/2	0	3/2	1/2	25

(a) Suppose $b = (60, 10, 20)^T$ is changed to $\hat{b} = (70, 20, 10)^T$. Find the optimal solution.

(b) Suppose $a_1 = (3, 1, 1)^T$ is changed to $\hat{a}_3 = (2, 2, 0)^T$ and c_1 is changed from 2 to 1. Find the optimal solution.

(c) Suppose $c = (2, -1, 1)^T$ is changed to $\hat{c} = (3, -2, 3)^T$. Find an optimal solution.

(d) Suppose that a new constraint $x_1 - 2x_2 + x_3 \leq 20$ is added. Find the optimal solution.

(e) Suppose that a new constraint $3x_1 - 2x_2 + x_3 \leq 30$ is added. Find the optimal solution.